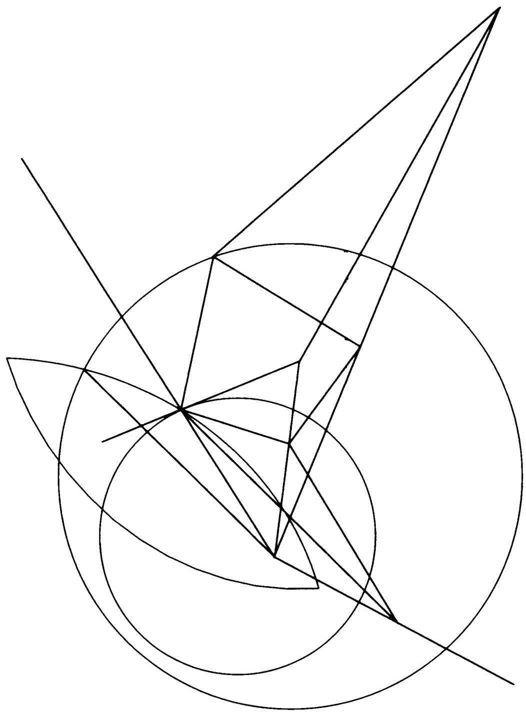
ALHACEN ON IMAGE-FORMATION AND DISTORTION IN MIRRORS

A Critical Edition, with English Translation and Commentary, of Book 6 of Alhacen's *De Aspectibus*, the Medieval Latin Version of Ibn al-Haytham's *Kitāb al-Manāẓir*

Volume One

Introduction and Latin Text



A. Mark Smith

TRANSACTIONS

of the

American Philosophical Society

Held at Philadelphia for Promoting Useful Knowledge

VOLUME 98, Part 1, Section 1

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VOLUME ONE
Introduction and Latin Text

VOLUME TWO
English Translation

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American Philosophical Society
Independence Square • Philadelphia
2008

ISBN: 978-1-60618-981-8
US ISSN: 0065-9746

Library of Congress Cataloging-in-Publication Data

Alhazen, 965-1039.
[Manazir Book 6. English & Latin]
Alhacen's analysis of image-distortion in mirrors: a critical edition, with English translation and commentary, of book 6 of Alhacen's De aspectibus./[edited by] A. Mark Smith.
p. cm. — (Transactions of the American Philosophical Society, v. 98, pt. 1)
Includes bibliographical references (p.) and indexes.
Contents: v. 1. Introduction and Latin text — v. 2. English translation and commentary.
ISBN v. 1 978-1-60618-981-8 (pbk.) ISBN v. 2 978-1-60618-986-3 (pbk.)
1. Optics—Early works to 1800. I. Smith, A. Mark, II. Title. III. Transactions of the American Philosophical Society; v. 98, pt. 1.
QC353
[.A32313 2001]
535'.09'021—dc21 2001041227

To Lois, still and always my sine qua non

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PREFACE

As I remarked in the preface to the previous edition of books 4-5, it is easy to be misled by Alhacen's analytic focus in those books into supposing that his primary aim in studying reflection was to explain how light interacts physically with reflecting surfaces. This supposition is not unreasonable in light of Alhacen's painstaking efforts in book 4 to validate the equal-angles law of reflection experimentally and in book 5 to determine with mathematical precision the point or points on various convex and concave mirrors from which a ray of light emanating from a given spot will reflect to a given center of sight. But these efforts were part of a more comprehensive program to put the cathetus-rule of image-location on the most secure footing possible. That, for instance, is why Alhacen undertook at the beginning of book 5 to demonstrate empirically that the image of any spot on an object facing any mirror will be seen along the normal, or cathetus, dropped from it to the mirror's surface or to a plane tangent to the mirror's surface. Whereas this principle has nothing to do with the physics of light, it has everything to do with the psychology of sight. Without it Alhacen had no meaningful way to define image-location and therefore no meaningful way to explain how and why things appear to us as they do in mirrors.

Viewed from this perspective, Alhacen's study of image-distortion in book 6 takes on dual significance as an end to his reflection-analysis, not simply because it concludes that analysis but because it represents the ultimate goal for it. Accordingly, Alhacen's purpose in this, the sixth book, is to apply the cathetus-rule to an analysis of the various misperceptions that arise in the seven types of mirrors chosen for study in the previous two books. Some of these misperceptions, he informs us, are common to all mirrors, an example being image-displacement. Under no circumstances does an object actually lie where it is perceived to lie in a mirror, no matter where it appears in the reflecting surface. Certain other misperceptions, however, are specific to the type of mirror in which the image appears. These are the ones to which Alhacen devotes most of his attention in book 6. Limited to size, shape, spatial disposition, and number, such misperceptions lead us to see things as diminished or magnified, or as more curved than they are, or as reversed or inverted in orientation. Out of those misperceptions others can arise, a diminution in apparent size leading to an increase in apparent distance, and so forth.

In order to explain such misperceptions, Alhacen extends the analysis of object-points and image-points developed in book 5 to object-lines and image-lines, which he treats according to their constituent points. For the most part, in fact, he restricts his analysis to endpoints and midpoints, leaving it to us to extrapolate from them to the remainder of the line. He also leaves it to us to extrapolate from single lines to the visible surfaces containing them as cross-sections. As a result, the overall analysis in book 6 seems somewhat sketchier than that in book 5. It is also less mathematically demanding. But whatever it may lack in mathematical complexity it makes up for at the level of spatial conceptualization because so many of the constructions and proofs in it involve three rather than two dimensions. It is in this regard, in his ability to think in space, that Alhacen's peculiar analytic genius and imagination shine forth in flashes over the course of book 6. It is in this regard, as well, that this book represents not just an end, but a fitting end to Alhacen's reflection-analysis

As with the previous two editions, so with this one, I have amassed a variety of debts that I am pleased to acknowledge. First and foremost, I wish to thank the NSF for its generous support during 2005-2007 (SES 0521372). The NSF's continuing support for this project from the beginning has lightened my load considerably, and I appreciate it deeply. I also appreciate the advice and encouragement of the two program officers, Michael Sokal and Ronald Rainger, with whom I have dealt from the outset. Thanks are also due to the MU Research Council and the UM Research Board, both of which have provided generous supplementary funding over the course of this project. Thanks, too, to the American Philosophical Society for the various Franklin Grants it has awarded me during the past years. I am deeply indebted as well to Danielle Jacquart, Jean-Marc Mandosio, and various students at the École Pratique des Hautes Études, Paris, for giving me the opportunity to organize and hone my thoughts on Alhacen in a course of lectures I gave there in May and June of 2005. It was an intellectually stimulating experience that I will always cherish.

Warm thanks are also due the librarians and archivists in charge of the manuscript collections I consulted at the following libraries: Bibliothèque Nationale, Paris; Biblioteca Nazionale Centrale, Florence; Bibliothèque de l'agglomération, St-Omer; Royal College of Physicians, London; Corpus Christi College, Oxford; Trinity College, Cambridge; and the Crawford Library of the Royal Observatory, Edinburgh. In one way or another each of these librarians has gone beyond the call of duty not only to facilitate my research but also to make me feel welcome at his or her institution.

As to more personal debts, they are typically legion. Thanks, first, to various colleagues in the history department here at the University of Missouri for serving as sounding boards during the course of this edition. They know

who they are; and they know how much I appreciate their patience. Thanks, too, to my family, especially my wife, Lois, for bearing with my somewhat obsessive pursuit of this editing project, which has at times conflicted with family obligations. Thanks, finally, to my editorial assistant, M. Leo Shaw, of the Anthropology Department here at the University of Missouri, who has been of invaluable help throughout the publication of this edition from the very beginning. The care and intelligence with which he has scrutinized the entire edition for mistakes and infelicities have contributed immeasurably to the final result, as have his punctilious efforts at indexing. I honestly don't know what I would have done without him.

""INTRODUCTION

1. *Alhacen's Analysis of Image-Distortion in Mirrors: An Overview*

Unlike book 5, book 6 is organized in an absolutely straightforward manner that reflects its purpose to explain image-distortion in the seven types of mirrors analyzed in books 4 and 5. Thus, after two introductory chapters, Alhacen devotes the next seven, in order, to: plane mirrors (chapter 3), convex spherical mirrors (chapter 4), convex cylindrical mirrors (chapter 5), convex conical mirrors (chapter 6), concave spherical mirrors (chapter 7), concave cylindrical mirrors (chapter 8), and concave conical mirrors (chapter 9). As in book 5, so in book 6, the two types of spherical mirrors receive the most extensive treatment because their analysis forms the basis for that of the corresponding cylindrical and conical mirrors.

Plane Mirrors: Before examining misperceptions specific to image-formation in each of the seven types of mirrors—i.e., plane; convex spherical, cylindrical, and conical; and concave spherical, cylindrical, and conical—Alhacen discusses those that are common to all mirrors. First and most obvious is the spatial discrepancy between the object and its image, which invariably appears "in" the mirror, whatever its shape, and is perceived as if it were an object directly before the eyes. As such, it is subject to the same perceptual scrutiny and judgment that applies to objects seen in direct vision. Accordingly, we perceive a given image as if it were an object of a certain kind, having a certain shape, size, and color, lying at a certain distance, disposed in a certain way, and so forth.

But, as Alhacen has shown in book 3, these perceptions can be skewed if the threshold conditions for veridical vision are not properly met. Adequate illumination is one of those threshold conditions, and it is affected by the natural weakening of light due to reflection. As a result, the image may be dimmed to the point that certain of its defining features cannot be seen, or its apparent distance cannot be properly judged, or its shape cannot be properly perceived. In addition, the inherent color of the reflecting surface mingles with the color of the object's form, further dimming the image and causing a misperception of its hue. All told, then, these two factors can conspire to cause a host of misperceptions that are independent of, yet added to, the misperceptions arising from the particular shape of the reflecting surface.

Having made these points in the second chapter, Alhacen turns in the third to the simplest case of image-distortion, that due to reflection from plane mirrors. The actual analysis occupies only one theorem (proposition 1), in which Alhacen establishes that the image is the same size as its generating object and lies the same distance below the mirror as the object does above it. Granted, then, that the mirror is virtually colorless and highly reflective, the image will look almost precisely like its object in terms of color, brightness, shape, size, distance, etc. The only distortions that occur are image-displacement and image-reversal, according to which the left-hand side of the object corresponds to the left-hand side of the image, as seen from a viewpoint above the mirror on the side of the object. Thus, if the image were taken as an actual object facing the object that generates it, and if a viewer were posed exactly halfway between it and the object itself so that it could view the two directly, the left-hand side of the one would correspond to the right-hand side of the other, and vice-versa. Otherwise, the two would be perfect replicas of one another. The only distortions specific to plane mirrors are therefore image-displacement and image-reversal, and from these arises a misperception of location and spatial disposition or orientation (*situs*).

Convex Spherical Mirrors: Alhacen opens his analysis of convex spherical mirrors by pointing out that all the distortions or misperceptions arising from reflection in plane mirrors occur in these mirrors as well. This of course includes image-reversal. In addition, spherical mirrors distort both the size and shape of the image. Alhacen addresses size-distortion in the first two theorems of the chapter (propositions 2 and 3), showing in the first of them that images are almost always diminished in spherical convex mirrors. The qualification "almost always" takes us to proposition 3, which is by far the longest, most complex, and most original of the theorems in book 6. Alhacen's purpose in that theorem is to demonstrate that images seen at the very outer edge of a spherical convex mirror can actually appear magnified rather than diminished. The construction and demonstration can be summarized as follows.

Let A in figure 1, p. 261, be the center of a convex spherical mirror whose diameter is AD, and let a plane be passed through the mirror along AD to form a great circle on which arc DB lies. Extend AD to point Z such that ZD is signifcantly smaller than AD. Bisect ZD at H, produce a circle of diameter AH centered on A, and let QH be an arc on that circle. From point H draw chord QH equal to half HD. Find point T on AH such that AH:HD = HD: HT, and connect QT. Find point C such that HC = 3HT, and then find point I such that IA is the mean proportional between CA and HA, which is to say that CA:IA = IA:HA. Produce a circle of diameter IA through I, and

extend line QA to intersect that circle at point N. Draw chord NI. A key consequence of the construction to this point is that NI = QT.

Now find point M on line IH in figure 1a, p. 261, such that IM:MT = AI: AT, which translates to the expression AI:IM = AM:MT, and find point L on that same line such that IL:LH = AI:AH, which translates to AI:IL = AH: LH. From points M and L in figure 2, p. 262, drop tangents MG and LB to the surface of the mirror within plane ADGB. According to proposition 7 of book 5 (in Smith, *Alhacen on the Principles*, 404), if AI:IM = AM:MT, then it follows that M is the endpoint of tangency for reflection-point G, I is the object-point, and T is the image-point. By the same token, if AI:IL = AH: LH, then it follows that L is the endpoint of tangency for reflection-point B, I is the object-point, and H is the image-point. Let us therefore drop normal AGZ_1 to reflection-point G, and let us draw line of incidence IG and line of reflection TG, extending this latter line to point T′. Consequently, angle of incidence IGZ_1 = angle of reflection $T'GZ_1$, and T will be the image of I for any center of sight placed on line T′G. Likewise, if we drop normal ABZ_2 to reflection-point B and draw lines IB and HB, extending this latter to point H′, angle of incidence IBZ_2 = angle of reflection $H'BZ_2$, and H will be the image of I for any center of sight placed on line H′B.

Let us recapitulate figure 2 in figure 3, p. 263, excising tangents MG and LB as well as line of incidence IB and line of reflection HBH′. Let us then mark off arc MB on arc DB equal to arc FD. From the construction we know that points N and Q on line AN correspond to points I and H on line AI, and M has a corresponding position on line AM to that of point B on line AB. Therefore, if N is taken as an object-point and M as a point of reflection, Q will be the image, and it will be seen along line of reflection MQ, which extends to point Q′.

At the beginning of the construction it was specified that ZD be significantly smaller than AD. In figures 1-3, for instance, ZD is somewhat less than half AD, and in that case arc GB is considerably smaller than arc MG. ZD can, however, be as long as we please, and as it augments, so do IH and NQ in concert with DH and QF. Meantime, arcs FD and MB get progressively larger, while arc GB increases in size relative to arc MB until the point is reached at which arc GB = arc MG, as represented in the inset to figure 4, p. 264 (= figure 6.4.3c, p. 102). If, therefore, we continue augmenting ZD, arc GB will become larger than arc MG by tiny increments, as represented in the inset to figure 5, p. 265 (= figure 6.4.3d, p. 103).[1]

Let arc GB (i.e., RC) = arc MG (i.e., RU), as illustrated in figure 4. In that case, Alhacen goes on to demonstrate, lines of reflection QM and TG, when extended to the right, will intersect at point Z on the outer arc NZ. Accordingly, if the center of sight is placed at Z, it will see the entire line QT as the image of line NI. Furthermore, since NI = QT according to the construction,

the image will be the same size as its object. Now take the case illustrated in figure 5. In this instance, with arc GB > arc MG—which it actually is, although by a minuscule amount in the figure—the two lines of reflection QM and TG, when extended to the right, will intersect at point L between points G and Z. Thus, QT will be the image of line NI for a center of sight at L. Finally, let arc GB < arc MG. In that case, as illustrated in figure 6, p. 266 (= figure 6.4.3e, p. 104), it can happen that the intersection of the two lines of reflection will occur to the left of both points of reflection, i.e., at point X between M and NI, so QT will not be visible to a center of sight posed at that point. The same in fact holds for the case in which arc GB is greater than arc MG; where the intersection occurs depends on the relative size of arcs GB and MG. Accordingly, there are innumerable instances in which the lines of reflection will intersect to the left of M and/or G, leaving QT as a whole invisible to a center of sight placed at that point. It follows more or less self-evidently, therefore, that QT will be visible only when the lines of reflection intersect to the right of G, and when it is visible, it will be the same size as its object.

So far the analysis is deficient in two respects. First, it applies to a restricted number of cases: i.e., those in which the lines of reflection from Q and T intersect to the right of G. Second, although it does show that the image can be the same size as its generating object, it does not show that it can be larger. In response to these two issues, the analysis takes a remarkable turn. Suppose that GB is smaller enough than MG that the lines of reflection QM and TG intersect at point X to the left of M, as illustrated in figure 6, p. 266. Extend TG to O and QM to Z, and drop normals AMU and AGR to the points of reflection. Hence, angles of incidence NMU and IGR = angles of reflection ZMU and OGR, respectively.

Now imagine triangle IAO containing normal AG and line of reflection TGO, all highlighted in thick lines in figure 7, p. 267, to be a rigid flap hinged along line IA so that it can be pivoted upward out of the plane of the page toward some point S above line AI. Imagine the same for triangle NAZ containing normal AM and line of reflection QMZ and hinged along line AQN. Bring both flaps together at point S, as illustrated in figure 8, p. 268 (= figure 6.4.3g, p. 106). Thus, as illustrated in figure 9, p. 269 (= figure 6.4.3h, p. 107), line AO will have migrated to AS, normal AG to AY, line of incidence IG to IY, and line of reflection TO to TS. Moreover, point Y will lie on the sphere of the mirror, which is cut along arc O'YD by the plane of reflection IAS. The entire figure SAI with all its constituent points and lines thus corresponds perfectly to the entire figure IAO with all its constituent points and lines, which means that angle of incidence IYP = angle of reflection SYP. T is therefore the image of I for the center of sight at S. By the same

token, the plane of flap ASN cuts an arc on the mirror, and there is a point of reflection on that arc that corresponds to point M on arc DG. From point S, therefore, the image of point N in figure 8, p. 268, will appear at point Q, so from point S image QT of line NI will be seen in its entirety, and it will be the same size as NI.

All that remains is to show that the image can actually be larger than the object. This can be easily demonstrated by recourse to figure 10, p. 270 (= figure 6.4.3k, p. 108). For a start, we know that angle INQA is acute, so if we drop perpendicular IP from I to AQN, it will fall between Q and N and will be shorter than IN. We also know from book 5, proposition 17 (in Smith, *Alhacen on the Principles*, 414-415), that, as the object-point approaches the mirror's surface, its image recedes from the mirror's center—which is to say that it too approaches the mirror's surface from the opposite direction. Thus, since P lies nearer the mirror's surface than N, its image Q′ will lie nearer N than N's image Q. But IP < IN, and Q′T > QT, so *a fortiori* image Q′T > object IP. Moreover, any line between IN and IP will be shorter than IN, whereas the image of any such line will be longer than QT, so there are innumerable cases in which the image can be larger than its generating object.

With the issue of size-distortion out of the way in proposition 3, Alhacen broaches the subject of shape-distortion, opening his account with a series of four fairly rudimentary lemmas. In the first, which occupies proposition 4, he demonstrates that, if T and D in figure 11, p. 271 (= figure 6.4.4, p. 109), represent object-points whose forms are reflected to center of sight E from spherical convex mirror AQB, and if they are both equidistant from the mirror's center G, then the image L of point T, which lies farther from center of sight E than does D, will lie farther from centerpoint G than the image F of point D, which lies closer to E. In short, LG > FG. Furthermore, endpoint of tangency N for T, the point farther from E, will lie farther from centerpoint G than endpoint of tangency S for the nearer point D. In short, GN > GS. Then follow three interrelated lemmas, starting with lemma 2 (proposition 5), where it is demonstrated that, if line AB in figure 12, p. 271 (= figure 6.4.5, p. 109), is divided so that AB:BD = AG:GD, if three lines are produced from points B, D, and G to intersect at E, and if some line AT is drawn from A to intersect BE at T, that second line will be divided according to the same ratio: i.e., AT:TH = AZ:ZH. In lemma 3 (proposition 6), which is the obverse of lemma 2, Alhacen demonstrates that, if line AB in figure 13, p. 272 (= figure 6.4.6, p. 110), is divided according to the ratio AB:BD = AG:GD, and if line AT dropped at a slant from A is divided according to the ratio AT:TH = AL:LH (L being a surrogate for Z from the previous proposition), then, when they are extended, the lines passing from B through T, D through H, and G through L will intersect at some point E. The fifth and

final lemma (proposition 7) is so simple as to be virtually self-evident: if line AB in figure 14, p. 272 (= figure 6.4.7, p. 110), is divided according to the ratio AB:BD = AG:GD, if lines GZ, DH, and BT are parallel, and if line AT is dropped to BT, that line will be cut according to the same ratio—i.e., AT:TH = AZ:ZH.

In the next four propositions, Alhacen deals with the situation in which various object-lines face the center of sight more or less frontally. The first point he wants to establish is that in convex spherical mirrors images are curved with respect to the mirror's center of curvature, not its surface. Were the latter to be the case—that is, were the image to take on the shape of the mirror's surface—it would follow that the image is somehow impressed on that surface, a point that Alhacen was at some pains to refute in book 4.[2] Alhacen addresses this issue in proposition 8, where he starts by supposing that arc AEB in figure 15, p. 272 (= figure 6.4.8, p. 110), is concentric with the mirror, which is centered on G. Arc Z'Y' is the great circle on the mirror formed by the plane passing through AEB and G. From center of sight D drop DG perpendicular to this plane. In that case, Alhacen concludes, image QML of arc AEB will also be concentric with the mirror. However, since it lies nearer than the mirror's surface to centerpoint G, its curvature will be sharper than that of the mirror's surface.

On the other hand, if DG is not perpendicular to the plane of the arc, as in figures 16 and 17, p. 273 (= figure 6.4.8a and 6.4.8b, p. 111), then the curvature of image TQF of arc ECB in figure 17 will clearly not conform to that of the mirror's surface. In order to define the curvature of image TQF, Alhacen argues as follows according to that figure. N is the endpoint of tangency for object-point E, and F is its image.[3] Likewise, M is the endpoint of tangency for point C, and its image is Q, whereas L is the endpoint of tangency for point B, and T is its image. Therefore, according to book 5, proposition 7, GE:EN = GF:FN, GC:CM = GQ:QM, and BG:GL = GT:TL. By proposition 5, lemma 2, then, lines BC, ML, and QT, when extended, will intersect at point O, and when line OQ is extended toward EG, it will intersect it at point K. According to the same lemma, lines EC, NM, and FQ, when extended, will intersect at point P. Since image-point F of E lies below point K, the whole image TQF must be curved. As we will see, Alhacen applies this same analytic technique in several subsequent theorems.

In propositions 9 and 10 Alhacen undertakes to show that the images of curved lines facing the center of sight with their concavity toward the mirror's surface and their endpoints equidistant from the mirror's center (such as ADB and AEB in figure 19, p. 275 [= figure 6.4.9, p. 112]) will appear curved. Likewise, the images of straight lines facing the center of sight with their endpoints equidistant from the mirror's center (such as ACB in figure 20, p. 275 [= figure 6.4.10, p. 112]) will appear curved. Also, given the two

curved lines ADB and AEB in figure 19, the one that is more sharply curved (i.e., AEB) will yield a less sharply curved image than the other. The same holds for arc AEB in figure 20; its image ZTH is less sharply curved than image ZMH of straight line ACB. Alhacen then concludes by showing in proposition 11 that straight line AB in figure 21, p. 276 (= figure 6.4.11, p. 113), whose endpoints A and B are not equidistant from the mirror's center, will yield a curved image, i.e., ZNT, which is more sharply curved than image MZ of curved line AE. In all these cases, moreover, neither the visible line-segments nor their extensions ever touch the mirror's surface.

The next two theorems, propositions 12 and 13, are devoted to showing that, when a given straight line-segment does touch the mirror's surface, that line-segment will generally yield a curved image. For instance, in proposition 12 Alhacen describes the following situation illustrated in figure 22, p. 277 (= figure 6.4.12, p. 114). Let AB be a visible line-segment whose extension BE is tangent to the mirror at E, and let D be the center of sight. Let AB be posed with respect to the center of sight such that plane of reflection DGA cuts arc PZ on the mirror, while plane of reflection DGB cuts arc PH on the mirror. To demonstrate that the image of AB will be curved, Alhacen has recourse to the analytic technique he employed in proposition 8. Accordingly, N on line AG represents the endpoint of tangency for point A, whose image is I, whereas M on line BG represents the endpoint of tangency for B, whose image is O. On that basis, it follows from book 5, proposition 7, that GA:AN = GI:IN, and GB:BM = GO:OM. Since both lines are divided equiproportionally, then, by proposition 5, lemma 2, the lines passing through image-points I and O, endpoints of tangency N and M, and object-points A and B will intersect at some point Q. Find image-point U of Q. Since this point lies below line IOQ, the overall image IOU of line ABQ must be curved, as must be the image of its segment AB. The case in which a rectilinear object-line or its extension actually cuts the mirror's surface is then taken up in proposition 13, whose primary purpose is to show that in every such case but one the image will be curved. The exception occurs when the line or its extension intersects the mirror's center, in which case the image will be rectilinear, a point Alhacen has already established empirically early in the second chapter of book 5.[4]

In the following theorem, proposition 14, Alhacen discusses a range of situations in which object-lines posed in various ways with respect to the mirror and the center of sight may or may not yield a visible image. He then concludes his analysis of shape-distortion with proposition 15, where he demonstrates that, when a rectilinear object-line and the center of sight lie in the same plane, and when the line does not touch the mirror, its image will be curved. Overall, Alhacen's analysis of shape distortion in convex spherical mirrors is organized according to the orientation of the object-line

with respect to the center of sight. First, the orientation may be frontal, as illustrated in the top diagram of figure 23, p. 278, where E is the center of sight and AB the object-line, as seen from above. This is the orientation assumed for propositions 8-11. On the other hand, the orientation may be oblique, as illustrated in the middle diagram of figure 23. This is the orientation assumed for proposition 12 and 13. Or, finally, the object-line and the center of sight may lie in the same plane, as illustrated in the bottom diagram of figure 23. This, of course, is the orientation for proposition 15. Whatever its orientation, the object-line may be disposed in various ways with respect to the mirror's surface according to whether its endpoints are equidistant from the center of curvature, and according to that disposition, it or its extension may or may not touch the mirror's surface.

Convex Cylindrical Mirrors: In chapter 5 Alhacen passes on to convex cylindrical mirrors, pointing out in the introductory paragraph that images in these mirrors are subject to the same distortions of size and shape that arise in convex spherical mirrors, although those distortions are more pronounced in the former. Before addressing actual cases, however, Alhacen offers a prefatory lemma in proposition 16. In this, the fifth and penultimate lemma of the book, Alhacen proposes the following. Let a plane be passed through a cylindrical mirror to form an elliptical section on its surface. Let that section be RBEOA in figure 24, p. 279 (= figure 6.5.16, p. 124), let point B on it be a point of reflection, and choose some other point E on it. Drop normal BD from the point of reflection so as to intersect the cylinder's axis at point D. This normal will thus form the diameter of circle BTO passing through point B, and that diameter will be the minor axis of the elliptical section. Then drop normal EU from point E within the plane of the ellipse. When extended, Alhacen concludes, this normal will intersect the extension of normal BD at some point U, and it will bypass the axis. The purpose of this lemma is to establish that, if the elliptical section lies in the plane of reflection, and if EU is the cathetus of incidence dropped from an object-point, such as N, it will lie farther from the cylinder's axis than the normal EK dropped through point E within circle ES because that normal necessarily intersects the axis at the circle's center.

Alhacen begins his actual analysis of image-distortion in proposition 17 with two cases, in the first of which no distortion in fact occurs. Illustrated in figure 25, p. 280 (= figure 6.5.17, p. 126), this is the case in which straight object-line TH and center of sight E lie in the same plane, which cuts the mirror's surface along line of longitude AG and passes through axis ZK. Image T'H' of line TH will therefore be deformed in neither size nor shape because it is produced in precisely the same way it would have been produced in a plane mirror.[5] In the second case, however, object-line TH, which

is still posed upright and parallel to axis ZK, does not lie in the same plane as the center of sight E. Thus, as illustrated in figure 25a, p. 280 (= figure 6.5.17a, p. 126), E is displaced to the side of TH. That point granted, Alhacen goes on to prove that the entire form of line TH will reflect to center of sight E from some line of longitude GA on the mirror's surface. On that basis he shows in proposition 18 that image ICS of line TH in figure 26, p. 281 (= figure 6.5.18, p. 127), will be curved, although its curvature will be virtually indiscernible to center of sight E because its convexity faces it directly.

To demonstrate this point, Alhacen assumes that object-point Q, reflection-point B, and center of sight E lie in a plane of reflection that forms circle BF on the mirror, whereas planes of reflection TGE and HAE form elliptical sections on the mirror, T and H being equidistant from Q. According to proposition 16, lemma 5, then, catheti TU and HU pass behind the axis, while cathetus QL intersects it, so TU and HU lie farther from E than does QL. From this it follows that image-points I and S also lie farther from E than image-point C—hence the curvature of the image.

Finally, in proposition 19 Alhacen demonstrates that, if line TQH faces the mirror horizontally rather than vertically, as in figure 27, p. 282 (adapted from figure 6.5.19, p. 128), and if the center of sight E lies above it, TQH's image will be convex with respect to E. In this case, Q is assumed to be the midpoint of TH and to lie in the plane formed by line EX, which passes through the center of the top base of the cylinder, and axis DX. Thus, the form of H will reflect to E from point B, and the form of T will reflect to E from point G, and the image of H will lie at R, where line of reflection EB intersects cathetus HU. Likewise, the image of T will lie at point Y on cathetus TU, as represented in the bottom diagram of figure 27, which gives a bird's-eye view of the situation in the top diagram. The form of Q, however, will reflect from point K on line of longitude AZ, which lies in the plane formed by EZ and XD, so its image will lie at point P on cathetus QAC, where it is intersected by line of reflection EK. As is clear from the lower diagram, point P lies significantly closer to the reflecting surface than line RY, and it lies in a lower plane. The resulting image RPY is therefore convex and manifestly curved with respect to center of sight E. It is also smaller than its object. From all this it is therefore clear that the more line TH approaches the vertical—which is to say the closer it is to being parallel to the mirror's axis—the less curved its image will appear. It is also clear that the more the line approaches the horizontal, the more sharply curved its image will appear and, in addition, the more it will shrink in size along the horizontal.

Convex Conical Mirrors: As Alhacen observes at the beginning of chapter 6, convex conical mirrors produce the same sorts of image-distortions as do convex cylindrical mirrors, and they do so according to the same conditions.

Thus, the closer to vertical the object-line, the less curved its image will appear, whereas the closer to horizontal it is, the more curved and shrunken its image will appear. Unlike those in convex cylindrical mirrors, of course, the distortions in convex conical mirrors are affected by the conical shape of the mirror, the distortion being most pronounced at the mirror's vertex and least pronounced at its base.

In the first theorem of this chapter, proposition 20, Alhacen provides the sixth and final lemma, which is to the analysis of convex conical mirrors precisely what proposition 16, lemma 5 is to the analysis of convex cylindrical mirrors. Let E in figure 28, p. 283 (= figure 6.6.20, p. 129), be a point of reflection on convex conical mirror AGZR, and let conic section BFEZ pass through it. That section will thus lie in the plane of reflection. Pick some point Z on the conic section. Then, within the plane of the conic section drop normal ED to point E and normal ZX to point X. Those two normals will intersect at point X, and ZX, which will be the cathetus of incidence for object-point H, will bypass axis AK of the cone to its right.

In the next two theorems Alhacen follows essentially the same line of analysis he followed in propositions 17 and 18 pertaining to convex cylindrical mirrors, albeit with some modifications. Thus, in proposition 21, he demonstrates that, if straight line-segment ON in figure 29, p. 283 (= figure 6.6.21, p. 129), faces the mirror such that its continuation OA intersects the mirror's vertex, and if C is a center of sight facing the mirror to the side of ON, the entire form of ON will reflect to C from some line of longitude AZE on the mirror. Presumably, Alhacen ignores the case in which ON and C lie in the same plane, which passes through the mirror's axis, because it is obvious that in such a case the image of ON will be produced as if the reflection had occurred from a plane mirror. Furthermore, in this case, unlike those in propositions 17 and 18, ON is not parallel to a line of longitude on the mirror's surface because, if it were, its form would not reflect to C from a straight line on that surface.

Having therefore demonstrated both that and how the form of ON will reflect to the center of sight from line of longitude AE, Alhacen shows in proposition 22 that, under these conditions, the image of the entire line AON, as seen from center of sight R in figure 30, p. 284 (= figure 6.6.22a, p. 131), will be slightly curved, that image being represented by line APY. As was the case in proposition 18, the image's curvature will be essentially indiscernible to the eye at R both because it is slight and because its convexity faces the eye directly. However, unlike the image in proposition 18, this one will be slightly shorter than the line itself insofar as straight cross-section AY of APY $<$ AN.

The transition from proposition 21 to 22 marks a shift in translators that is indicated in several ways. First, and perhaps most obvious, the initial half

of proposition 22 is simply a recapitulation of proposition 21 in somewhat different format. Second, from the beginning of proposition 22 on, there are significant changes of both vocabulary and style. And finally, in two of the seven manuscripts collated for the critical text an alternative version of the beginning of book 6 to the end of proposition 20 is interpolated between propositions 21 and 22, this latter theorem flowing naturally from proposition 20 of the interpolated text.[6]

Having finished the demonstration in proposition 22, Alhacen brings chapter 6 to a close with a general description of how and why line-segments appear distorted in size and shape according to their position and orientation with respect to the surface of a convex conical mirror. The nearer to the mirror's vertex a line-segment's image appears, for instance, the more curved and truncated it will be. Likewise, the more horizontal the line-segment is with respect the mirror's surface, the more curved and truncated its image will be.

Concave Spherical Mirrors: Aside from the misperceptions due to the weakening of light and color, Alhacen informs us in the rather extensive introduction to chapter 7, that concave spherical mirrors produce a host of misperceptions that do not occur in plane and convex mirrors. For one thing, size-distortion is far more variable and complex in spherical concave mirrors than in convex mirrors. For another thing, concave spherical mirrors can yield as many as four images of a single object-point, whereas plane and convex mirrors can never yield more than one image. Much of book 5 is devoted to demonstrating these points. For yet another thing, images are always located behind the reflecting surface of plane and convex mirrors, whereas in concave spherical mirrors they can lie behind, on, or in front of the reflecting surface.[7] Furthermore, while reflection from plane and convex mirrors causes image-reversal, reflection from concave spherical mirrors can yield upright and reversed images as well as inverted ones. And, finally, reflection from concave spherical mirrors can cause shape-distortion in the resulting images.

Having laid out the basic order of analysis for chapter 7 in this preliminary section, which occupies the first seven paragraphs of the chapter, Alhacen addresses three of the issues raised there in propositions 23-28: size-distortion, variation in image-location, and variation in image-orientation. Accordingly, he begins in proposition 23 by locating the center of sight T in figure 31, p. 285 (= figure 6.7.23, p. 132), on radius OU of the mirror BUG such that it lies between midpoint O of that radius and the reflecting surface. In that case, he concludes, if line MTN is taken very roughly to represent a cross-section of the surface of an eye facing the mirror, then, as seen by center of sight T, image FQ of that cross-section will appear behind the reflecting

surface and will be larger than its object.[8] With that established, Alhacen goes on in proposition 24 to erect KT at point T perpendicular to plane BUG of the mirror. Let K in figure 32, p. 285 (= figure 6.7.24, p. 132), be a center of sight on that line, and let ABK be a plane of reflection within which the form of some point M reflects to K from point B, and let its image be F. Let AGK be a plane of reflection within which the form of point N reflects to K from point G, and let its image be Q. Therefore, image FQ of line MN will appear behind the mirror, and it will be larger than its object.

In the next three propositions, i.e., 25-27, Alhacen shows that, under the right circumstances, the image seen in a concave spherical mirror can be the same size as, larger than, or smaller than its object. It can also appear reversed or upright, and it can be seen in front of or behind the reflecting surface. In proposition 25, for instance, Alhacen has ZAB in figure 33, p. 286 (= figure 6.7.25, p. 133), be a great circle on a concave spherical mirror centered on E, with GZE a randomly drawn line in the plane of that circle. Line DG is erected perpendicular to line GE, and from point D lines DA, DB, and DE are drawn to the plane of circle ZAB. DE is extended below that plane toward point O. Then, from some point K on DA line KE is passed through point E in the plane of circle ZAB and extended to point L so that KE = EL. Point T is located on line DB such that TE = KE, and TE is extended to point H such that TE = EH. Then lines AE and BE are drawn in the plane of the circle. Consequently, angles DKAE, LAE, DTBE and HBE will all be equal, and lines KT and HL will also be equal.

Let D be a center of sight and LH an object-line inside the mirror. The form of L will thus reflect to D from A, the form of H will reflect to D from B, and the image of line HL will be KT, where lines of reflection DA and DB intersect the extensions of catheti LE and HE. Since KT = LH, therefore, the image will be the same size as its object, and it will lie between the reflecting surface and the center of sight. Moreover, since the right-hand side H of the object will lie on the left-hand side T of the image with respect to center of sight D, and conversely, since the left-hand side L of the object will lie on the right-hand side K of the image, the image will be reversed and, in fact, inverted if LH is oriented parallel to DG.

Now connect HB and AL, and let them intersect at O, through which the extension of DE passes. Let O be a center of sight and KT an object-line behind the center of sight. From O's point of view, then, KT's image will be LH, and it will be the same size as its object. Furthermore, it will lie between the center of sight and the reflecting surface. It will not be reversed or inverted, however, because the right-hand side T of the object as viewed face-on from O is also the right-hand side H of its image as viewed face-on from O, while the left-hand side K of the object is also the left-hand side L of the image from the same point of view.

Using the same approach in proposition 26, Alhacen extends lines BH and AL in figure 34, p. 286 (= figure 6.7.26, p. 133), to points R and M respectively so that RB = AM. He then draws RE and ME, continuing them past point E until they intersect DA and DB at points U and N. Since RE > EN, and ME > EU, it follows that RM > UN. If, therefore, MR is an object-line and D a center of sight, MR's image will be NU, and it will be smaller than its object.[9] It will also be inverted. On the other hand, if O is the center of sight and NU the object-line, NU's image RM will be larger than it and will appear upright. In the same vein, finally, Alhacen shows in proposition 27 that, if CI in figure 35, p. 287 (= figure 6.7.27, p. 134), is the object-line and QF its image as seen from center of sight O, the image will be smaller than the object and will appear upright. On the other hand, if FQ is the object-line and D the center of sight, image CI will be longer than its object and will appear inverted.

In the preceding five propositions Alhacen has dealt with two basic cases. In the first case, which occupies propositions 23 and 24, the center of sight and the object-line lie in a plane between the mirror's center of curvature and the reflecting surface. In the second case, which occupies propositions 25-27, the center of sight and the object-line are located either between the center of curvature and the reflecting surface or on either side of the center of curvature. Alhacen addresses the third and final case in proposition 28, where both the center of sight and the object-line lie beyond the center of curvature. Hence, as illustrated in figure 36, p. 288 (= figure 6.7.28, p. 135), G is the mirror's center of curvature, BDA an arc on the reflecting surface, HZ the object-line, which represents the surface of the eye, and E the center of sight. Under these conditions, Alhacen concludes, image LK of HZ will be smaller than its object and will lie between the center of sight and the reflecting surface BDA. It will also be inverted.

The remainder of chapter 7, which consists of propositions 29-32, is devoted to showing not only how the shape and orientation of images can be distorted in concave spherical mirrors, but also how such distortion can vary according to the number of images formed. In proposition 29, for instance, Alhacen shows that, if we choose two diameters OEA and DEB in concave spherical mirror BDO represented in figure 37, p. 289 (= figure 6.7.29, p. 136), then, when Z is taken as a center of sight and GR as a rectilinear object-line, the image LK will be smaller than its object. It will, however, be rectilinear, like its object, and it will have the same left-to-right and top-to-bottom orientation with respect to the mirror: i.e., point G, which lies to object-line GR's right from the mirror's perspective, will appear at point K, which lies to image-line LK's right from the perspective of E, whereas point R, which lies to GR's left from the mirror's perspective, will appear at LK's left from the perspective of E.

The same holds by extension when the object-line GR is convex or concave. Thus, as Alhacen shows in proposition 30, if the object-line is GNR in figure 38, p. 289 (= figure 6.7.30, p. 138), and if it is convex with respect to reflecting surface ODB, its image LIK will be convex with respect to center of sight E It will also be smaller than its object and will have the same orientation. On the other hand, if the object-line is GQR, which is concave with respect to the reflecting surface, its image LCK will be concave with respect to center of sight E. It too will be smaller than its object and will have the same orientation.

In the last two theorems of chapter 7, Alhacen takes up the problem of multiple images and the various ways in which they can be distorted. He starts in proposition 31 by assuming that E in figure 39, p. 290, is a center of sight and Z an object-point, both of them facing reflecting surface ABDX and both equidistant from centerpoint G of the mirror. It follows, then, that the form of Z can reflect to E from three points on the arc directly facing E: i.e., D, B, and A. Accordingly, Z will have three images, of which two are of special interest here: the one lying at M, where the extension of line of reflection BE intersects the extension of cathetus GZ, the other lying at L, where the extension of line of reflection DE intersects that same cathetus. Granted these conditions, finally, choose some point N on that cathetus such that its form will reflect to center of sight E from some point K′ on the mirror. Its image will lie at Q, where the extension of line of reflection K′E intersects the extension of cathetus GZ. Altogether, then, object-points Z and N will yield three images along cathetus GZ: L and M for Z and Q for N.

Now let us drop a line CZR through point Z perpendicular to plane ABDX, as illustrated in figure 39a, p. 290, and taking NG as radius, let us form circular segment CNR intersecting perpendicular CZR at points C and R such that CZ = ZR. The planes formed by line EG, which lies in the plane of circle ABX, and lines CG and RG oblique to the plane of circle ABX will intersect the sphere of the mirror to form arcs on it equivalent to arc ABDX in figure 39, and within those arcs there will be points perfectly equivalent to K′ from which the forms of C and R will reflect to E, so, from center of sight E's perspective, S will be the image of C within plane of reflection EGC and O the image of R within plane of reflection EGR. Accordingly, since Q is the image of N, the whole of arc SQO will be the image of arc CNR, and it will appear behind the reflecting surface. Moreover, since arc CNR is convex with respect to the reflecting surface, its image will be concave with respect to that same surface as well as to the center of sight, and the image will be larger than its object. On the other hand, if straight line CZR is taken as the object-line, its image will be SLO, which lies behind the mirror and is concave with respect to the center of sight. It too will be larger than its object.

It was established earlier that for center of sight E point Z has at least two images within arc ABD of the mirror, one at L, the other at M. However, from book 5, proposition 37, case 3 (in Smith, *Alhacen on the Principles*, 456-458), we know that, given the conditions of the analysis, according to which center of sight E and object-point Z are equidistant from the center of the mirror's curvature G, object-point Z will produce four images for E. Consequently, as illustrated in figure 39b, p. 291, the two images L and M are produced by the reflection of Z's form from points D and B, respectively, and they lie where lines of reflection DE and BE intersect cathetus ZG. The form of Z also reflects from point A to yield an image at point F, where line of reflection AE intersects cathetus ZG. And the fourth reflection is from point I directly opposite B on diameter BG, its image being T′, where line of reflection IE intersects cathetus ZG. It has also been established that from E's perspective points C and R in figure 39c, p. 291, produce images at S and O, respectively. Accordingly, as we noted before, concave line SLO is an image of straight line CZR.

Imagine that center of sight E is able to scan the entire arc on the mirror just ahead of point B and just beyond line AEF, as represented by the thick lines in both figures 39b and 39c. Imagine, as well, that it can see all four images of Z at L, M, T′ and F, as well as images S and O of points C and R. Accordingly, straight line CZR will yield four separate images according to the four images of midpoint Z, all four images having S and O as their endpoints and all four being concave with respect to the reflecting surface. As we already noted, one of these images will be curved line SLO in figure 39c. Another will be the curved line passing from S through M to O. Yet another will be the curved line passing from S through T′ to O. And the final one will be the curved line passing from S through F to O. Furthermore, according to their placement with respect to center of sight E, points C and R can have as many as four images each within the facing portion of the mirror, so the number of images for straight line CZR will be multiplied commensurately.

Having established in proposition 31 that both straight and convex lines can yield multiple concave images, Alhacen concludes the theorematic portion of chapter 7 by showing in proposition 32 that straight lines can yield convex images, whereas concave lines can yield straight or convex images. Let us start by assuming that arc AG in figure 40, p. 292, lies on the surface of a spherical concave mirror centered on point D. Let H be a center of sight lying on radius DA of the mirror, and on line DGQ let O and U be object-points whose forms are reflected to H from points B and F on the mirror. Point Q, where line of reflection BH intersects cathetus DO, will thus be the image of O as seen from H, and point N will be the image of U as seen from H.

Rotate line DHA 90° counterclockwise out of the plane of circle GBA on axis DQ so that center of sight H is carried to point H′ in figure 40a, p. 293, which represents a three-quarter view of the situation from above. Using DO as a radius, form arc EOZ, and pass line EUZ through U perpendicular to line DO so as to intersect arc EOZ at points E and Z. According to previous analysis, the form of O reflects to center of sight H from point B to yield an image at Q. So too, the form of U reflects to H from point F to form an image at N. Since DH′ is perpendicular to DH, and since the two lines are equal, it follows that the plane formed by DH′ and DNQ will cut an arc on the mirror equivalent to arc AG. This arc is A′G in figure 40b, p. 293, and within it there will be two points B′ and F′ equivalent to points B and F in arc AG. Thus, the form of O will reflect to H′ from point B′ to yield an image at Q, and the form of U will reflect to H′ from point F′ to yield an image at N.

Now extend line DE toward K and DZ toward T in figure 40a so that DK and DT are both equal to DQ. Since points E and Z lie precisely the same distance from the reflecting surface as point O, then within their respective planes of reflection H′DK and H′DT their forms will reflect to H′ from points equivalent to point B′ on arc A′G in figure 40b. K and T will therefore be their respective images, and they will lie the same distance from the center of sight as does Q. The same holds for every point on arc EOZ, so its image will be arc KQT of radius DQ. Hence, line EOZ, which is convex with respect to the reflecting surface, will yield an image KQT that is concave with respect to center of sight H'. Furthermore, since the image of point U is at N, the overall image of straight line EUZ will have its endpoints at K and T and its midpoint at N, which means that the image will be convex with respect to the center of sight. In both cases the image will also be larger than its object.

In order to demonstrate, finally, that a concave line can yield a convex image, Alhacen has us select some random point M on line UZ in figure 41, p. 294 (= figure 6.7.32d, p. 143), and form a circle with radius UM such that it will intersect arc EOZ at points R and F to create concave arc RUF. We are then to draw lines DR and DF, extending them to points C and I on arc KQT. Since points R and F lie on arc EOZ, and since we have shown that every point on that arc yields an image on arc KQT, C and I will be the images of R and F from H′s point of view. On the other hand, point U on concave arc RUF has its image at N according to that same point of view. Therefore, the image of concave arc RUF will pass from C through N to I and will be convex with respect to the reflecting surface.

The reason for this rather elaborate analysis of multiple images and their distortion in concave spherical mirrors comes clear in the closing paragraphs of chapter 7, where Alhacen points out that the surfaces of visible objects consist of various lines and that the way those lines are perceived will de-

termine the way those surfaces will be perceived. Thus, if a given straight line on a flat surface yields a multitude of images of various curvatures and locations, then the surface will appear deformed according to those curvatures and locations. Suffice it to say, when the surface as a whole is analyzed according to several of its constituent lines, the resulting composite image will be confused to the point of incoherence. That, of course, is why objects posed close to the surfaces of concave spherical mirrors so often yield chaotic, blurry images spread out over the reflecting surface.

Concave Cylindrical Mirrors: Occupying chapter 8 of book 6, Alhacen's analysis of image-distortion in these sorts of mirrors unfolds in propositions 33-36. In the first theorem of this series, Alhacen reverts to the construction for proposition 18, which deals with image-formation in convex cylindrical mirrors. That construction is illustrated in figure 26, p. 281. First, let the cylinder represent a convex mirror faced by object-line TQH and center of sight E. According to the conditions set by Alhacen in this context, the form of straight line TQH will reflect to E from line of longitude GBA on the mirror's surface, and the resulting image will be curved line ICS. As part of the construction, moreover, Alhacen has continued lines of incidence TG, QB, and HA to converge at point O and then demonstrated that EG = GO, EB = BO, and EA = AO.

Now let us reverse the situation by designating O as a center of sight and ICS as an object-line, both of them facing the concave surface of the cylinder. By extension from the previous account of reflection from the convex surface of the cylinder, the form of I will reflect from G to O, the form of C will reflect from B to O, and the form of S will reflect from A to O, and the resulting image will be TQH. Thus, the image will lie behind the mirror, it will be larger than its object, and it will be rectilinear, unlike its object, which is convex with respect to the reflecting surface. The amount and type of curvature can vary, however, if point C is moved to and fro along line of reflection EB. Accordingly, the image of C can appear beyond point Q, leaving the whole image TQH concave with respect to the center of sight at O. Or it can appear between points Q and B, leaving the whole image TQH convex with respect to O. Furthermore, depending on its location with respect to center of sight O and the axis of the cylinder, C can have as many as four images, so there can be as many as four images of line ICS with endpoints I and S fixed in the position illustrated in the figure. Meantime, of course, I and S can each have as many as four images, so the number of possible images of ICS will be multiplied commensurately.

As in proposition 33, so in proposition 34 Alhacen reverts to an earlier construction, this one pertaining to proposition 19, dealing with convex cylindrical mirrors. In that theorem it was shown that, when straight line TQH and center of sight E in figure 27, p. 282, face the convex surface of

the cylinder, with E lying above TQH, the resulting image RPY behind the reflecting surface will be noticeably convex and shorter than its object. One step in the construction for that theorem involved extending lines of incidence HB and TG until they intersect at point L, through which line ED is drawn.

Now, if we take L as a center of sight facing the concave surface of the mirror, and if we take straight line RY as an object line, then it follows from symmetry that, just as the form of H reflects to E from B, the form of R will reflect from B to L, and the same holds for point Y, which will reflect from G to L. Accordingly, the image of R will lie at H, where the extension of line of reflection BL intersects cathetus HU, and the image of Y will lie at T, where the extension of line of reflection GL intersects cathetus TU. Let M in figure 42, p. 295 (= figure 6.8.34b, p. 149), be the point at which line RY is bisected by the plane formed by EX′ and axis X′XD. The form of M will therefore reflect from point F on line of longitude AZ within that plane, as represented in the lower diagram of the figure. Its image will lie at S, where the extension of line of reflection FL intersects cathetus SQM, which lies on the diameter of the circle passing through Q with its center at axis X′XD. Furthermore, as is clear from the diagram, S lies beyond line TH and higher than it. It follows, then, that image HST of straight line RMY will be larger than its object, and it will be manifestly concave with respect to center of sight L.

Proposition 35 addresses the case in which the plane of reflection is elliptical, as represented in figure 43, p. 296 (= figure 6.8.35, p. 150), where BG represents an arc on an ellipse that also passes through point A on the opposite side of the cylinder. Since G and A lie at the intersection of the ellipse and the circle passing through G and A, diameter GA of that circle will be the minor axis of the ellipse. Let BK be normal to the ellipse at point B. Thus, BK and GA will intersect at some point E beyond the axis of the cylinder, and line BK will lie behind that axis. Let LMK represent an object-line, let G and A represent points of reflection at the respective ends of minor axis GA, and let D be a center of sight lying on DG, which is constructed parallel to BK. According to D's particular placement, then, the form of K will reflect to D from O, and its image will be N. Meantime, the form of M will reflect to D from A, and its image will be T, and the form of L will reflect to D from G, yielding an image at G on the very surface of the mirror.[10] All told, therefore, the resulting image TGN of straight line LMK will be concave with respect to D.

In proposition 36, finally, Alhacen reverts back to the analysis of size-distortion and image-reversal in concave spherical mirrors provided in propositions 26 and 27. Thus, in figure 44, p. 297 (= figure 6.8.36, p. 151), D and O can be taken as respective centers of sight, and IC, KT, and UN can

be taken as cross-sections of either objects or images tied, respectively, to QF, HL, or RM as the corresponding images or objects. Thus, if IC is taken as an object-line and D as the center of sight, the image QF will be smaller than its object, and it will appear reversed (but not inverted) in a concave cylindrical mirror. With IC as an object-line and O as a center of sight, on the other hand, the image QF will still be smaller than its object, but it will not be reversed. By the same token, if QF is taken as the object-line and IC its image, then the image will be larger than the object for both D and O, but for D the image will be reversed, whereas for O it will not. And the same sort of analysis can be applied to the other two object-image pairs KT and LH and UN and RM.

Concave Conical Mirrors: Consisting of only two theorems, i.e., propositions 37 and 38, chapter 9 on concave conical mirrors is by far the shortest of book 6. In the first of these theorems, Alhacen reverts back to proposition 22, which deals with the formation of convex image APY of straight line AON in the top diagram of figure 30, p. 284. According to his analysis there, the form of the entire line AON reflects to center of sight R from line of longitude AZE. Accordingly, lower endpoint N reflects to R from E to yield an image at Y, upper endpoint A is identical with its image at the cone's vertex, and midpoint O reflects to R from Z to yield an image at P. In proposition 37 Alhacen merely reverses the analysis, as illustrated in figure 45, p. 298 (= figure 6.9.37, p. 152), where APY is the object-line and F the center of sight, both facing the concave surface of the cone. Hence, the form of Y reflects to F from E to yield an image at N, the image of A lies at point A itself, and the form of P reflects to F from Z to yield an image at O. In this case, then, line APY, which is convex with respect to the reflecting surface, will yield a rectilinear image. If, however, midpoint P is moved to and fro in line with RZP, then the resulting image AON can take on a convex or concave curvature with respect to the center of sight. Furthermore, in this case, with A and N the endpoints of whatever image is produced, the image will be larger than its object.

The thirty-eighth and final proposition of book 6, is essentially a recapitulation of proposition 36, where Alhacen provides a general analysis of size-distortion and image-reversal in concave cylindrical mirrors. The same analysis applies to concave conical mirrors and is based on figure 44, p. 297, where CI and QF, KT and HL, and NU and RM can be taken as object-image couples according to whether the center of sight is posed at D or O. On that basis it can be shown that a given image can be equal to, smaller than, or larger than its object, depending on the placement of the object and the center of sight with respect to the mirror's center of curvature at E. On that same basis, moreover, it can be shown that the image will sometimes

be reversed and sometimes unreversed—again depending on the relative placement of the object and the center of sight with respect to the mirror's center of curvature.

2. *The Sources for Alhacen's Analysis and Its Reception in the Latin West*

Not surprisingly, the mathematical and optical sources for Alhacen's analysis of reflective image-distortion in book 6 do not go beyond those used in book 5.[11] In fact, unlike his analysis in book 5, that of book 6 makes no explicit use of Apollonius' *Conics*, so the proofs in book 6 are based entirely on Euclidean reasoning. As regards optical sources, there is no need to look past Ptolemy's account of image-distortion in plane and convex mirrors in books 3 and 4 of his *Optics*.[12] Aside from elaborating on, and re-organizing that account to include cylindrical and conical mirrors, Alhacen's only real innovation—and he admits as much—lies in his demonstration in proposition 3 that images seen at the very edge of convex spherical mirrors can be magnified. Otherwise, his analysis is essentially Ptolemaic in style, apart, of course, from his reliance on light-rays rather than visual rays and his more rigorous approach to mathematical analysis.

Like his general account of vision, Alhacen's analysis of image-distortion in mirrors followed the path along which *De aspectibus* as a whole was disseminated either directly, through manuscript copying, or indirectly, through derivative works, such as Roger Bacon's *Perpsectiva*, Witelo's *Perspectiva*, and John Pecham's *Perspectiva communis*.[13] As was the case with his study of reflection in book 5, the complex mathematical details of Alhacen's analysis of image-distortion in book 6 seem to have excited little or no interest among his Latin followers—apart, that is, from Witelo.[14] Thus, although both Bacon and Pecham were cognizant of Alhacen's demonstration in proposition 3 that images can sometimes be larger than their objects when seen in convex spherical mirrors, neither of them made any effort to explain that conclusion geometrically, despite the fact that it is the only truly original contribution Alhacen made in book 6.[15] The same applies to their analyses of image-distortion in general; although they drew upon Alhacen for certain salient points, they ignored most of the analytic details of his account. As a result, their accounts of both image-formation and image-distortion in mirrors are both sketchy and relatively unsophisticated. Neither author, in fact, took the topic of image-distortion much further than Ptolemy.

The question of precisely how Alhacen's account of image-distortion in mirrors might have influenced certain practical and theoretical developments in catoptrics during the fifteenth and sixteenth centuries has come to the fore recently in two arenas. On the one hand, the artist David Hockney, with

the collaboration of Charles Falco, an optical researcher at the University of Arizona, has suggested that, from the early fifteenth century, Renaissance artists routinely used spherical concave mirrors or spherical convex lenses to project real images of the scenes or portraits they wanted to paint directly onto their boards or canvasses and then used those images to establish landmarks for the actual painting.[16] This thesis has aroused considerable controversy among art historians and historians of technology, the latter arguing that suitable concave mirrors were not technologically feasible at the time.[17] Perhaps most problematic is the lack of textual evidence, or at least convincing textual evidence, to support Hockney's and Falco's claim. In response to this issue, the two have appealed to the fact that a full theoretical justification of image-projection from concave spherical mirrors and convex lenses was well within the range of Perspectivist optics. Hence, the ability of concave spherical mirrors to project real images could easily have been discovered and explained through the application of Perspectivist ray-analysis.

Meantime, several scholars have been looking closely at technological and theoretical developments leading to the invention and improvement of optical devices, telescopes in particular, during the sixteenth century. Among such developments, according to Sven Dupré, was the recognition in that century of a correlation between the focal point of concave spherical mirrors and the "inversion point" (*punctum inversionis*) at which images seen in such mirrors flip upside-down.[18] Since the focal point determines where real, inverted images will begin to be physically projected by concave spherical mirrors, establishing the correlation between that projection and the inversion of images *seen* beyond that point linked the formation of both real (physical) and virtual (psychological) images in concave mirrors. The same applies to convex lenses, where the focal point also constitutes the inversion-point of images seen in them.

The very concept of a focal point (from Latin *focus* = hearth, fire place) harks back to antiquity and the effort to perfect burning mirrors. Within that context, the focal properties of concave spherical mirrors had been accurately determined by at least Archimedes' time according to the ability of such mirrors to ignite combustible material at or around a particular point on its diameter. Such is clear from the treatise of Diocles on burning mirrors. Most likely written in the early second century B.C., this work provides a rigorous mathematical analysis of the focal properties not only of concave spherical mirrors but also of paraboloidal mirrors.[19]

Whether Alhacen was familiar with Diocles' account of burning mirrors is an open question, although there are grounds to suppose that he was.[20] Beyond doubt, however, is that Alhacen was fully aware of the focal properties of concave spherical mirrors discussed and demonstrated in that

account and, moreover, that he was also well aware of the focal properties of concave paraboloidal mirrors.[21] Clearly, then, Alhacen had at his disposal the analytic tools needed to explain the projection of real images by concave spherical mirrors, although the context within which he applied those tools was specific to the physical capacity of light-rays to create heat rather than images. The question thus arises whether Alhacen recognized the correlation between the two capacities, and the answer lies in how he might or might not have brought that correlation to bear in his analysis in book 6, chapter 7, of image-formation and image-distortion in concave spherical mirrors.

That such a correlation is at least implicit in his analysis of image-formation in concave spherical mirrors can be seen from the account he provides in book 5, proposition 32 (in Smith, *Alhacen on the Principles*, 446-448). The point of this proposition is to show that image-locations for object-points vary according to how and whether the line of reflection and the cathetus of incidence dropped from the object-point intersect. To illustrate, let the circle in figure 47, p. 300 (adapted from figure 5.2.32 [in Smith, *Alhacen on the Principles*, 250] upon which the proposition is based), be a concave spherical mirror centered on D, and let A be a center of sight posed on diameter IB. Let M be an object-point on diameter HDO, and let its form reflect to A from N. As seen from A, therefore, its image will lie at L behind the reflecting surface. Now let T be an object-point on the same diameter, and let its form be reflected to A from point E such that reflected ray EA is parallel to cathetus of incidence TD. Since the two lines will never intersect, it follows that there will be no definite image; instead, a confused image will appear at reflection-point E.[22] Then let Q be an object-point, and let its form reflect to A from G. Its image O will therefore lie behind the center of sight. If, on the other hand, Z on diameter IAB is the object-point, and if its form reflects to A from E, its image will lie at the center of sight itself. And, finally, if K is the object-point, and if its form reflects to A from C, its image S will lie between the center of sight and the mirror.

These same points emerge from an analysis of figure 47a, p. 300, which is adapted from the diagram accompanying Ptolemy's account of the same phenomena in book 4 of the *Optics*.[23] Let the arc represent a portion of a concave spherical mirror centered on C, and let E represent the center of sight, R a point of reflection defining reflected ray ER, and O_6R a line of incidence upon which object-point O is posed at various locations from O_1 to O_6. Accordingly, if O_1 is an object-point whose form reflects to E from R, its image, as seen from E, will lie at I_1 behind the mirror. On the other hand, if O_2 is taken as the object-point, and if its cathetus of incidence O_2C is parallel to reflected ray RE, its image will appear at R on the mirror's surface, according to Alhacen's account in book 5, proposition 32. When the object is O_3,

however, and when the center of sight is shifted to I_3, the image will lie at the center of sight itself. In the same vein, finally, if the center of sight is at point I_4 with the object-point remaining at O_3, the resulting image I_3 will lie behind the center of sight, whereas if the center of sight remains at I_4 with the object-point at O_5 or O_6, the resulting image I_5 or I_6 will lie between the center of sight and the reflecting surface.

Let us modify figure 47a somewhat as follows. First, let us pass radius CF through line of incidence O_6R so that it is bisected at point F. F is therefore a focal point of the mirror. Then let us find point R′ on the other side of radius CF such that FR = FR′, as illustrated in figure 47b, p. 301. Likewise, let us locate point O_1' on line FR′ such that it lies the same distance from F as O_1, and let center of sight E lie well beyond center of curvature C, where the extension of CF intersects reflected rays RE and R'E. As we just determined in the previous analysis, the form of O_1 will reflect to E from R to yield an image at I_1 behind the mirror. Symmetry dictates that the form of O_1' reflect from R′ to E to yield an image at I_1' behind the mirror. Hence, if we take O_1O_1' to represent an object-line posed between the mirror's surface and focal point F, its image I_1I_1' will appear behind the mirror from E's perspective, and it will be larger than its object. If we assume that O_1 is at the top of the object and O_1' at the bottom, the image will appear upright, insofar as top-most point O_1 of the object will appear at top-most point I_1 of the image.

Now, following the same analytic model, let O_2O_2' in figure 47c, p. 301, represent an object-line whose top and bottom endpoints O_2 and O_2' reflect from R and R′ such that the two reflected rays RE and R′E are parallel, respectively, to the two catheti O_2C and $O_2'C$. In that case, there will be no definite image, only a confusion at R and R′. Consequently, when the object is placed just above the focal point, it will yield a blurry image anchored to the mirror's surface at those two points.

What happens when the object-line falls between center of curvature C and focal point F is illustrated in figures 47d and 47e, p. 302, where the two object-lines O_3O_3' and O_4O_4', with points O_3 and O_4 at the top, lie between the focal point and the center of curvature. Their respective images I_3I_3' and I_4I_4' will be larger than their objects and will appear inverted, insofar as the top-most points O_3 and O_4 of the objects will appear at the bottom-most points I_3 and I_4 of the images. When the object-line reaches the center of curvature, as in figure 47f, p. 303, and when O_5 is at the top and O_5' at the bottom, its image I_5I_5' will coincide with it. That image, however, will have diminished in size so as to be smaller than its object, although it will remain inverted. Finally, in figure 47g, p. 303, when the object O_6O_6' passes beyond the center of curvature (i.e., between the center of curvature and the center of sight), its image I_6I_6' will have gotten even smaller relative to the

object itself, but it will remain inverted. As the object moves progressively farther from the center of curvature toward the center of sight, its image will get progressively smaller relative to the object, but it will continue to be inverted.

Several features of this analysis merit attention. First, it is consistent with experience. If a viewer faces a concave spherical mirror from a position well beyond the center of curvature, and if an object of moderate size is placed close to the reflecting surface, the viewer will see its image behind the mirror, upright and magnified. As the object is drawn away from the reflecting surface toward the viewpoint, its image becomes progressively larger as it recedes farther and farther behind the surface until a point is reached at which the image becomes blurry and incoherent. Then, as the object is drawn just a bit farther from the reflecting surface toward the viewpoint, its image resolves back into a clear representation that is inverted and larger than its object. From that point on, as the object is drawn ever farther from the reflecting surface, its image gets progressively smaller while remaining inverted. A second feature of the analysis that bears remarking is that it ties the inversion of images seen in concave spherical mirrors clearly and explicitly to the focal point of those mirrors. The final and perhaps most significant feature to consider is that the analysis itself follows directly and logically from Ptolemy's account of image-location and its variation in book 4 of the *Optics*. That account in turn forms the basis for Alhacen's own account of the phenomenon in book 5, proposition 32. Within this context, therefore, the connection among heat-production, the projection of real images, and the inversion of images seen in concave spherical mirrors seems every bit as evident as its grounding in the focal point of such mirrors.

Did Alhacen in fact recognize this seemingly obvious connection? As we have seen from our examination of propositions 23-28, where Alhacen deals specifically with image-inversion and magnification in concave spherical mirrors, there is no clear indication that he did. True, the construction in proposition 23 calls for locating the object-line, as well as the center of sight, between the mirror's focus and the reflecting surface, as illustrated in figure 31, p. 285, where O lies at the midpoint of radius AU, with MTN representing the object-line and T the center of sight. But the fact that O is the focal point for the mirror is incidental to the resulting demonstration. Other than specifying the diameter of circle ZOE to which tangents TE and TZ are dropped, it has no explicit function in that demonstration. The same, of course, applies to proposition 24, which is merely a variant of proposition 23. In the remaining four propositions devoted to image-inversion and magnification, moreover, the cardinal reference-point for analysis is the mirror's center of curvature, not the focal point. No more is needed to

confirm this point than a brief look at figures 33-36, pp. 286-288, and their construction in propositions 25-28. All told, propositions 23-28 are designed to show *that* image-inversion, along with magnification or diminution, can occur in concave spherical mirrors under particular circumstances, not to explain precisely how and why they occur according to general conditions, as defined by the focal properties of such mirrors.

Does this mean that Alhacen failed to see the connection between the inversion of images seen in concave spherical mirrors and the focal properties of such mirrors implied by his analysis in book 5, proposition 32? Perhaps, but to pose the question in terms of failure is misleading because the two phenomena lie in entirely different domains from an Alhacenian perspective. The gathering of light to a burning point in concave spherical mirrors is a physical phenomenon, as is the projection of real images beyond that point, both phenomena being tied to the mirror's focal point. On the other hand, the inversion of images seen in such mirrors is a psychological phenomenon involving not real, physical images but imaginary, or virtual, ones. Granted, the two phenomena are "optical" in that they involve light, but they are absolutely different in terms of their respective effects.[24] So Alhacen was bound by the framework of his analysis in the *De aspectibus*—where the focus is upon the psychological (or "subjective") effects of light rather than its physical (or "objective") effects—to treat image-inversion as a visual rather than a physical phenomenon. There is simply no place for real images within such an analytic framework.[25] Nor, for that matter, is there any clear functional place for the focal point within it. Consequently, the analysis of reflection in general and image-inversion in particular according to Alhacen and his Perspectivist disciples did not lend itself to the discovery, much less a coherent explanation of the image-projecting capacity of spherical concave mirrors.

3. *Conclusion*

Our examination of Alhacen's analysis of image-formation and distortion in the seven types of mirrors, concave spherical mirrors in particular, reinforces the conclusion drawn in the previous edition of books 4 and 5 that, from a post-Keplerian perspective, Alhacen's treatment of reflection represents little more than wasted effort as far as the development of modern physical optics, catoptrics in particular, is concerned.[26] The reason is not far to seek. Modern, post-Keplerian catoptrical analysis is based primarily on focal points and real images, whereas Alhacenian catoptrics is based on viewpoints and virtual images. Hence, as far as the evolution of modern physical optics is concerned, Alhacen's account in the *De aspectibus* of image-

formation and distortion in mirrors represented a dead end because it had no practical ramifications for the improvement of optical devices, such as the *camera obscura* or the reflecting telescope, which was of utmost concern to sixteenth- and seventeenth-century opticians.

Proposition 3 of book 6 exemplifies this lack of concern for practical application. On the one hand, the fact that images can be magnified in convex spherical mirrors opens the possibility of using such mirrors as telescopic devices. On the other hand, the circumstances under which such magnification occurs are so contrived and narrowly constrained that such an application would be fruitless. Why, then, bother to analyze the phenomenon at such excruciating length? The answer lies in the analysis itself. For one thing, it is brilliant in conception, fully commensurate with Alhacen's extraordinary imagination and skills as a mathematician. For another, it was possible to do and therefore had to be done for the sake of completeness. Alhacen thus approached reflection as a mathematician concerned with explaining visual anomalies as rigorously and as comprehensively as possible, not as a physicist or technical practitioner bent on exploiting those anomalies. It is both unfair and irrelevant, then, to judge him and his contribution to catoptrics from this latter perspective. To do so—and to find him deficient according to it—is to judge him not for what he actually accomplished but for what we, as post-Keplerians, would prefer him to have accomplished. What he did accomplish was to take the science of catoptrics, as conceived by his Greco-Arabic predecessors, Ptolemy foremost among them, to the highest level of sophistication and mathematical elegance it had ever attained and would ever attain. That, in its own right, is a remarkable achievement.

NOTES

[1]See note 24 to the translation, pp. 236-237, for an explanation of these points.

[2]Book 4, chapter 4, in A. Mark Smith, *Alhacen on the Principles of Reflection*, Transactions of the American Philosophical Society, 96.2 (Philadelphia: APS, 2006), 324-325.

[3]This point is illustrated in figure 18, p. 274, where the sphere of the mirror is delineated by the gray arc ZY passing through line EG. EGD is the plane within which the form of E reflects to D, and it forms the great circle Z′Y′ on the mirror's surface. Accordingly, the form of E reflects to D from point R on that arc, and N is the endpoint of tangency insofar as the line drawn from it to arc Z′Y′ is tangent to that arc at reflection-point R.

[4]See book 5, chapter 2, paragraphs 2.6-2.9, in Smith, *Alhacen on the Principles*, 387-388.

[5]It is of course true that, if the mirror faces the center of sight with its axis horizontal rather than vertical, the image of line TH will suffer left-right reversal.

[6]For a more detailed account of the change in translators and the interpolated text, see the section on manuscripts and editing, pp. xlv-xlviii.

[7]See book 5, proposition 32, in Smith, *Alhacen on the Principles*, 446-448.

[8]It should be noted that, when comparing image-size to object-size, Alhacen is comparing rectilinear cross-sections rather than actual object-lines or image-lines. Hence, in this proposition line MTN represents a cross-section of the surface of the eye, not that surface itself, and likewise line FQ represents a cross-section of MN's image, not the image itself. In fact, as is clear from figure 31, the image of point X, as seen from T, will lie far beyond the reflecting surface where line of reflection RT and cathetus AX intersect. Later on in book 6, when Alhacen deals with actual object-lines and their resulting image-lines, he defines them according not only to their two endpoints but also to their midpoints.

[9]Suffice to say, if MR is to serve as an object-line, it must be short enough to fit inside the mirror. Hence, ER and MR can be no longer than the radius of the mirror; otherwise, it will extend outside the mirror. On the other hand, if MR serves as an image, it can be as long as we please, since it can extend outside the mirror, which is to say that it will appear behind the reflecting surface.

[10]According to Alhacen's account in book 5, paragraph 2.312 (in Smith, *Alhacen on the Principles*, 448), when the line of reflection and the cathetus are parallel, the image is seen by the eye at the point of reflection. In this case, of course, BK, being normal to the ellipse at point B, is the cathetus, whereas GD is the line of reflection, which has been constructed parallel to BK. Hence, the image of point L will appear at point G.

[11]For an account of these sources, see Smith, *Alhacen on the Principles*, lxvi-lxxvii.

[12]See A. Mark Smith, *Ptolemy's Theory of Visual Perception*, Transactions of the American Philosophical Society, 86.2 (Philadelphia: APS Press, 1996), theorems

III.13-III.16, pp. 166-171, and theorems IV.30-IV.38, pp. 205-222. Ptolemy's analysis of convex mirrors is restricted solely to spherical mirrors, and his analysis of concave mirrors is slanted mostly toward spherical mirrors.

[13]For a brief account of this line of dissemination, see A. Mark Smith, *Alhacen's Theory of Visual Perception*, Transactions of the American Philosophical Society, 91.4 (Philadelphia: APS, 2001), lxxxii-lxxxiv.

[14]For more on this point, see Smith, *Alhacen on the Principles*, lxxxii-lxxxvi.

[15]Bacon's acknowledgement of this conclusion has already been discussed in Smith, *Alhacen on the Principle*s, cii, n. 82; for Pecham, see David C. Lindberg, ed. and trans., *John Pecham and the Science of Optics* (Madison, WI: University of Wisconsin Press, 1970), proposition 33, 187-189. Although both Bacon and Pecham rely heavily upon geometrical diagrams, their analyses are generally descriptive rather than rigorously demonstrative.

[16]See David Hockney, *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters* (New York: Viking, 2001). The primary optical device used by Renaissance artists, according to Hockney, was the *camera obscura*, within which a spherical concave mirror could be placed facing the small opening that allows light into the chamber so as to project a real image of the scene beyond that opening onto a screen. Much the same thing can be accomplished by placing a spherical convex lens at the opening.

How a concave spherical mirror projects real images can be easily understood by recourse to figure 46, p. 299. Let EAK be an arc on a concave spherical mirror centered on C. Let F lie at the midpoint of radius AC of that mirror, let the parallel lines O_1E, O_2D, etc., striking the mirror's surface at E, D, B, A, G, H, and K, represent light-rays reaching the mirror from a distant luminous object, and let AE = AK, AD = AH, and AB = AG. According to the law of equal angles, each of the light-rays reaching the mirror's surface will reflect at equal angles from the point at which it strikes the mirror, so it follows that ray O_1E will reflect along EX, which intersects diameter AC at point X, such that angle of incidence O_1EC = angle of reflection CEX, and so forth. Moreover, since AE = AK by construction, it follows that ray O_7K will reflect at equal angles from point K to form reflected ray KX, which also intersects diameter AC at point X. The same holds by extension for ray AC (which reflects back onto itself), as well as for ray-couple O_2D and O_6H and ray-couple O_3B and O_5G. Furthermore, it is clear from the diagram that, as the incident ray approaches diameter AC, the reflected ray intersects diameter AC at a point approaching focal point F. Thus, as far as the focusing of rays is concerned, the effective portion of the mirror is quite small, limited, say, to arc BG. Moreover, focal point F represents the limit toward which the reflected rays all tend, so there is a small area between points X and F where enough rays congregate to provide the heat of ignition.

One other point comes clear from the diagram: all the reflected ray-couples, such as EX and KX, continue past their respective points of intersection, so if a screen were to be placed beyond point F, as represented by line K'E', the continuing rays would reach it at points corresponding to the points of reflection from which they originate. Accordingly, the ray reflected from E would strike the screen at E',

and so forth on down the line to K, which would reach the screen at K′. Imagine a viewer facing the screen and, therefore, also facing the object. From his perspective the right-hand side O7 of the object would appear at the left-hand side K′ of the image, whereas the left-hand side O1 of the object would appear at the right-hand side E′ of the image. By the same token, if O1 were taken to represent the top of the object, that point would appear at E′, which lies at the bottom of the image. Hence, the image would appear inverted. In modern parlance such images are called "real," whereas images seen behind a reflecting surface are called "virtual."

[17]A representative sample of reactions to Hockney's and Falco's thesis among both academics and non-academics from various disciplines can be found in *Early Science and Medicine,* 10.2 (2005), a special issue containing papers given at a European Science Foundation workshop entitled "Optics, Optical Instruments and Painting: The Hockney-Falco Thesis Revisited" that was held at Ghent University in 2003. For a critique of Hockney's thesis on technological grounds, see Sara J. Schechner, "Mirrors and their Imperfections in the Renaissance," ibid., 137-162. While Schechner's argument that the technology of mirrors was inadequate to produce suitable concave mirrors for image-projection in the early fifteenth century has some merit, Vincent Ilardi's long-awaited study of early eyeglasses, *Renaissance Vision from Spectacles to Telescopes* (Philadelphia: American Philosophical Society, 2007), makes it abundantly clear that adequate convex lenses for such purposes were readily available by that time. Over the past several years Charles Falco has vigorously defended the Hockney-Falco thesis on the basis primarily of computer analyses of various paintings from the fifteenth and sixteenth centuries. See, e.g., "The Art of the Science of Painting," Proceedings of the Symposium on Effective Presentation & Interpretation in Museums, The National Gallery of Ireland, Dublin, 2003, available in pdf format at www.optics.arizona.edu/SSD/NatlGallery.pdf.

[18]See, e.g., Sven Dupré, "Ausonio's Mirrors and Galileo's Lenses: The Telescope and Sixteenth-Century Practical Optical Knowledge," *Galileana,* 2 (2005): 145-180.

[19]See G. J. Toomer, *Diocles on Burning Mirrors* (Berlin/Heidelberg/New York: Springer-Verlag, 1976), esp. proposition 3, 56-62. Toomer's edition and translation of Diocles' work is based on an Arabic version, which exists in a single manuscript completed in AH 867 (1462/3 A.D.) according to the testimony of the scribe. In all likelihood, the original Arabic translation predates this particular copy, but by how much is subject to speculation. Toomer opines that it may have been done by Quṣṭā ben Lūqā, which would take it back to around 900 A.D. (see ibid., 21).

[20]See Toomer, *Diocles,* 18-23. Cf., however, A. I. Sabra, *The Optics of Ibn al-Haytham: Books I-III on Direct Vision* (London: Warburg, 1989), p. 52, where he emphasizes that Diocles' proof of the focusing properties of paraboloidal mirrors "*may* have been known to" Alhacen. Presumably this would apply to Diocles' proof for the focusing properties of concave spherical mirrors as well.

[21]See H. J. J. Winter and W. 'Arafat, "A Discourse on the Concave Spherical Mirror by Ibn al-Haitham," *Journal of the Royal Asiatic Society of Bengal,* 16 (1950): 1-16, and "Ibn al-Haitham on the Paraboloidal Focussing Mirror," *Journal of the Royal Asiatic Society of Bengal,* 15 (1949): 25-40.

[22]See note 10 above.

[23]See figure IV.21 accompanying experiment IV.1 of Ptolemy's *Optics* (in Smith, *Ptolemy's Theory*, 194-195), where Ptolemy draws the same conclusions that Alhacen does in book 5, proposition 32: namely that, depending on where the object-point is located on the line of incidence and where the center of sight is located on the line of reflection, the resulting image may appear behind the mirror, on the mirror's surface (i.e., when the cathetus of incidence and the line of reflection are parallel or when the image lies at the center of sight itself), between the center of sight and the reflecting surface, or behind the center of sight. Even a cursory comparison of Ptolemy's and Alhacen's approach to this issue and the diagrams upon which their accounts are based shows that Alhacen was following Ptolemy's analysis quite closely.

[24]Sabra echoes this point in *Optics*, xlii, where, in discussing Alhacen's two treatises on spherical and paraboloidal burning mirrors, he observes that they are "concerned with the behavior of solar rays . . . as agents of combustion, not of vision." See also Sven Dupré, "Optics, Pictures and Evidence: Leonardo's Drawings of Mirrors and Machinery," *Early Science and Medicine*, 10 (2005): 211-236.

[25]For elaboration on this point, see A. Mark Smith, "Reflections on the Hockney-Falco Thesis: Optical Theory and Artistic Practice in the Fifteenth and Sixteenth Centuries," *Early Science and Medicine*, 10 (2005): 163-185.

[26]See Smith, *Alhacen on the Principles*, lxxxix.

MANUSCRIPTS AND EDITING

Textual Issues in the Manuscripts: The Latin text in this current edition is based on the same seven manuscripts as the Latin text in the previous edition of books 4-5: i.e., *F*, *P1*, *S*, *E*, *L3*, *O*, and *C1*. Having already explained in detail both how and why I chose this particular group from the seventeen extant manuscripts containing all or most of the *De aspectibus*, I will not repeat the rationale here.[1] Suffice it to say, I am still satisfied that those seven manuscripts yield an appropriately critical text.

Unlike that of books 4 and 5, the text of book 6 presents an anomaly reminiscent of the one encountered in chapter 3 of the third book. In three of the seventeen manuscripts just mentioned (i.e., *O*, *L3*, and *C2*), chapter 6 on convex conical mirrors is interrupted early on by a clearly demarcated text that opens with the title *capitulum sextum de fallaciis que accidunt in speculis piramidalibus convexis erectis* ("chapter six [literally, "the sixth chapter"] on misperceptions that occur in right convex conical mirrors"). Then follows an extremely brief résumé of the misperceptions arising in such mirrors, which concludes with the phrase *ad huius autem demonstrationem premittamus hanc propositionem* ("and for the purpose of this demonstration let us set forth this proposition"). The theorem that follows is essentially the same as proposition 20, lemma 6 (Latin, pp. 48-50; English translation pp. 196-198), which occurs before the interpolation. From that point on, all the manuscripts continue with proposition 22 (Latin, pp. 52-58; English, pp. 199-204), whose opening phrase *hoc ergo declarato* ("now that this has been demonstrated") presumably refers to the introductory lemma in proposition 20.

Quite clearly, then, this swatch of interpolated text represents an alternate version of the introductory résumé and initial theorem of chapter 6 as it opens in all seventeen manuscripts, including the three anomalous ones. But its placement in those three manuscripts is puzzling. Whereas it should have been inserted right after proposition 20, it actually follows proposition 21, which draws on proposition 20 to support its argument. Equally puzzling is that roughly the first half of proposition 22 (paragraphs 6.20-6.30) recapitulates proposition 21, albeit in a somewhat different narrative form.[2]

Aside from its odd placement, the interpolated text has some peculiar features that set it apart from everything that precedes it. For a start, there is its title. Up to that point, the chapter headings, when they are given, are

designated by *pars* rather than *capitulum*. In addition, only one such chapter—chapter 4—includes a descriptive subtitle: *pars quarta in speculis spericis* ("chapter four on spherical mirrors"). Chapters 5 and 6 have no heading at all. Also, the introductory résumé of the alternate version of chapter 6 is longer and commensurately more detailed than that of the initial version contained in all the manuscripts. Finally, there are marked differences in style and vocabulary between the two versions, as witness the shift from *pars* to *capitulum*. In the initial version, for example, the term *sectio* is used to denominate "conic section," whereas in the second the replacement term is *sector*. These alterations in vocabulary and format are matched by a noticeable shift in narrative style, that of version 2 being choppier than that of version 1.[3]

Taken as a whole, these incongruities in style and vocabulary suggest strongly that two distinct translators were at work here. More to the point, the entire text from the beginning of book 6, proposition 22, to the very end of book 7 bears all the stylistic and terminological hallmarks of translator 2—which means that from the point of insertion to the very end of book 7, the text forms a continuous whole produced by translator 2. That being the case, the two puzzling features mentioned earlier are easily explained. First, the recapitulation of proposition 21 in the initial part of proposition 22 makes sense if we view proposition 22 as an amalgam of two distinct theorems, the first of which constitutes proposition 21 and the second of which constitutes proposition 22 proper. Within its context in version 2 of the text, therefore, proposition 22 with its two parts forms the appropriate sequel to proposition 20. Second, the placement of version 2 after proposition 21 of version 1 can be accounted for by assuming that translator 1 was suddenly replaced at this point by translator 2, who took it upon himself to retranslate the opening portion of chapter 6 for the sake of continuity.

The situation described here has an obvious parallel in the one encountered in book 3, chapter 3, where paragraph 3.13 is suddenly interrupted by an interpolation unique to six of the seventeen manuscripts containing all or most of the *De aspectibus* (i.e., *P1*, *F*, *S*, *O*, *V2*, and *Vat*).[4] On close examination, the inserted text proved to be a rough, often maladroit translation of the concluding portion of chapter 2 and the opening section of chapter 3 up to paragraph 3.13. It also proved to be continuous in style with the remainder of book 3 as it appears in all the manuscripts. I therefore concluded that there was an abrupt switch of translators at the point of interpolation, the master (translator 1) yielding to the apprentice (translator 2). I also concluded that the section of version 2 that overlaps with version 1 represented a sort of practice run for translator 2 to prepare him to take over in earnest at paragraph 3.13 of book 3. On that basis I decided to relegate the

overlapping portion of version 2 to an appendix rather than include it as an integral part of the critical text. Following the same rationale, I have placed the overlapping portion of version 2 of book 6, chapter 6, in the appendix on pp. 330-335.

While there is no doubt that both cases involve a change in translators, there may be more at play than that. For instance, as remarked in the analysis of book 3, the Latin text from paragraph 3.13 to the end of the book represents less a translation than a paraphrase of the Arabic counterpart established by A. I. Sabra.[5] This of course could be due to the ineptitude or haste with which translator 2 rendered that portion of the text into Latin. But it could also be due to his having used an Arabic version that falls outside the manuscript tradition upon which Sabra based his edition.

This possibility is borne out, at least to some extent, by the second version of book 6, chapter 6. The introductory résumé in both versions is a case in point. In the first it consists of a single paragraph (6.1, p. 48 [Latin] and p. 196 [English]), whereas in the second it consists of three (Appendix, pp. 330 and 331). It could of course be argued that version 1 is a mere distillation or paraphrase of version 2, but to argue that is to ignore the omission in version 1 of pertinent and significant details provided in version 2.[6] Moreover, although there are obvious disparities between the two versions of proposition 20, those disparities are not easily explained on the basis of paraphrasing. For one thing, both versions are roughly the same length. For another, the structure of argument is noticeably different in both, even though each argument leads to the same conclusion along a valid train of logic. If both translators were working from the same text, then surely they would have followed the same train of logic, particularly when that train is as rigidly controlled as it is in a mathematical proof.[7] Finally, there is the issue of the heading and subtitle for chapter 6. Even if he was paraphrasing, why did translator 1 omit it entirely if it was actually there? After all, in the overlapping portion of the second version of chapter 3, book 3, translator 2 (who presumably *was* paraphrasing) included such a heading for chapter 3 (i.e., *de modis quibus error accidit visui*), and he continued to provide such headings (with a few exceptions) throughout the rest of the book.

If nothing else, this raises the possibility that the Latin translation of the *De aspectibus* was based on at least two different, perhaps markedly different, Arabic versions of the text; that at least two, but perhaps no more than two translators were at work on it; and that each translator was drawing on a different Arabic version. That in turn raises the possibility that the manuscript tradition of the Arabic text was more complex and variegated than current scholarship might lead us to suppose. As of now, only five manuscript copies of the Arabic text are known to exist. Of those only one

contains the entire treatise, and that one is quite late, dating from 1493-94. The earliest known copy (dating from 1083-84) lacks books 4 and 5; the next in chronological order (dating from 1239) contains books 4 and 5 alone; the third in chronological order (dating from the fourteenth century) has only segments of books 5-7; and the last in chronological order (dating from 1509) is limited to books 1-3.[8] Given the paucity and defective state of these manuscript copies, it would be rash to conclude that they represent the sole, authoritative witnesses to either the text itself or its dissemination in the Arabic world. It would be equally rash, therefore, to conclude that the disparities between the Latin text and the Arabic version established on the basis of these five manuscripts (in disparate groups of three, no less) is due solely to the Latin translators.

The Critical Text: The topical organization of book 6 is articulated clearly at the beginning, where the number of chapters (nine) and a brief description of their content is given explicitly in most of the manuscripts. Yet despite this indication, six of the seventeen aforementioned manuscripts add chapter-breaks at various points in the text. One case has already been discussed: i.e., the break at the beginning of the alternate version of chapter 6 that is found in *O*, *L3*, and *C2*. In addition, *S* and *Er* introduce a chapter-break at the beginning of proposition 22, and *M* introduces one in the middle of chapter 7. On the other hand, most of the manuscripts omit various chapter-breaks where they are appropriate or else signal them weakly.[9] Nevertheless, the topical structure of book 6 is obvious in the organization of the text itself.

For the most part, book 6 follows the organizational structure of book 5, the lion's share of it being devoted to a mathematical analysis of the seven mirrors in order from plane (chapter 3), through convex spherical, cylindrical, and conical (chapters 4-6), to concave spherical, cylindrical, and conical (chapters 7-9). Since that structure is explicit in book 6, I had no need to impose it, as I did for book 5. As to the remaining levels of organization, which I did have to impose, I followed the pattern established earlier for book 5, breaking the text into propositional elements (38 in all), which on occasion I further subdivided into cases. As with the Latin text of book 5, so with this one, I have inserted fairly strong spacing-breaks between propositions and weaker ones between cases. Each proposition is further demarcated by a numerical designation (e.g., [PROPOSITIO 1]). At the next, lower level of organization, I have imposed a paragraph structure in order to make the text easier to follow, each paragraph being numbered according to its chapter (e.g., [6.1], which designates the first paragraph of chapter 6). At the lowest level of organization I have punctuated as I thought appropriate in order to ease the modern reader's way through the narrative, most of which consists of mathematical argument.

Diagrams: Here, again, I followed the pattern established for book 5, paring the diagrams in the manuscripts down to what I consider to be a canonical set. In this case that set consists of only 36, as opposed to 83 for book 5. As in book 5, so in this one, I traced the figures to accompany the Latin text directly from diagrams scanned for the most part from manuscript *O*, although I have relettered them with a modern font and have occasionally reoriented them. As before, I have designated the resulting text-diagrams in capital letters according to the format "FIGURE 6.7.31," the number-series indicating book (6), chapter (7), and proposition (31). For a detailed description of the criteria I followed in choosing and representing the appropriate diagrams, see *Alhacen on the Principles*, cxi-cxv.

The Critical Apparatus: For the conventions used in the critical apparatus of this edition, I refer the reader to *Alhacen's Theory*, clxxii-clxxiv.

The Translation and Commentary: The general guidelines I followed in translating book 6 are those discussed in *Alhacen's Theory*, clxxiv-clxxvi and *Alhacen on the Principles*, cxv-cxvii. Accordingly, I have reinforced the organizational breaks discussed earlier by interpolating parenthetical headings, such as [CASE 1], to clarify the analytic structure of the text. I have also provided external and internal citations at appropriate points in the translation, inserting them parenthetically and setting them in brackets. As before, I have inserted brief explanations set off in brackets at spots in the text so as to streamline the commentary, which I have relegated to endnotes. Unlike the diagrams that accompany the Latin text, those that are matched to the translation are meant to reflect as faithfully as possible the actual conditions specified in the constructions and proofs. In order to distinguish these diagrams from their counterparts in the Latin text, I have designated them according to the lower-case format "figure 6.7.31," the number-series indicating book, chapter, and proposition. As in the previous edition, so in this one, I have placed the figures that go with volume 2 at the end of volume 1 and vice-versa so that the reader can match the logical flow of each propositions with its appropriate diagram(s) without flipping back and forth between proposition and diagram within the same volume. The reference-aids provided in this edition are the same as those provided in the previous two: i.e., a Latin-English index keyed to technical terms in both Latin text and English translation; an English-Latin glossary for cross-referencing to that index; and a general index keyed primarily to the introduction and commentary.

NOTES

[1]For a full description of the seventeen manuscripts, See Smith, *Alhacen's Theory*, clv-clxi. For a discussion of the criteria I used for sorting these manuscripts into families and family-representatives, see ibid., clxi-clxvii. And for the rationale behind my choice of manuscripts upon which to base the text of books 4-5, see Smith, *Alhacen on the Principles*, cvii-cviii.

[2]The points made in this paragraph are illustrated in the schematic below. The left-hand column gives the format of chapter six from the introductory résumé to the end of proposition 22 as it appears in all the manuscripts except *O, L3,* and *C2*. The right-hand column gives the format for those three manuscripts, with the interpolated text in gray. Ending with proposition 20, lemma 6, that interpolation mirrors sections A and B in all the manuscripts, so logic would demand that it be placed right before proposition 21 (section C in all the manuscripts), which is based in part on proposition 20. Furthermore, the first part of proposition 22 (section D in all the manuscripts) is clearly redundant insofar as it recapitulates proposition 21 in section C.

All Except O, L3, and C2	**O, L3, and C2**
A. Introductory résumé B. Proposition 20, lemma 6 C. Proposition 21 D. Proposition 22 1. Part 1 (Prop. 21) 2. Part 2 (Prop. 22')	A. Introductory résumé B. Proposition 20, lemma 6 C. Proposition 21 *Title* *A. Introductory résumé* *B. Proposition 20, lemma 6* D. Proposition 22 1. Part 1 (Prop. 21) 2. Part 2 (Prop. 22')

[3]See note 87, p. 248 below for a more detailed accounting of these terminological and stylistic differences.

[4]See Smith, *Alhacen's Theory*, clxi and clxviii-clxix, for a discussion of this interpolation. Note, by the way, that *O* is the only manuscript containing both interpolations, a fact that may indicate its privileged status as a witness to the original formation of the Latin versions of the *De aspectibus*.

[5]Cf. Sabra, *Optics*, 250-367, and Smith, *Alhacen's Theory*, 588-627.

[6]The main differences in detail are as follows: 1) Version 1 likens the misperceptions that occur in convex conical mirrors to those occurring in convex spherical mirrors; version 2 likens them more appropriately to the misperceptions that occur in convex cylindrical mirrors. 2) Version 2 notes that images in convex conical mirrors take on the shape of those mirrors; version 1 does not. 3) Version 1 remarks

that, as visible lines approach the vertical with respect to such mirrors, their images appear less curved, whereas the closer they approach the horizontal, the more curved their images appear; version 2 makes no mention of this fact. 4) Version 2 points out that the images of lines posed horizontally with respect to such mirrors are noticeably shortened; version 1 says nothing on this matter. More significant than its inclusion of more information, however, is that, in providing that particular information, version 2 offers a far more insightful summary of chapter 6 than does version 1.

[7]The most obvious difference between the two versions of proposition 20 lies in the order of steps taken in the respective constructions. In both versions, for instance, BFZ (figures 6.6.20 and 6.6.20alt, pp. 129 and 153, respectively) is identified as the relevant conic section quite early in the first paragraph of the proof (i.e., paragraph 4, p. 330, of version 2 and paragraph 6.2, p. 48 of version 1). As soon as it is so identified in version 2, we are instructed to drop normal ED from E and then draw tangent TQ to point Z. At the same point in version 1, on the other hand, we are instructed to pass a plane through Z to form circle GBZ; the instruction to draw tangent TQ comes considerably later, in paragraph 6.5, p. 49. Such differences are far too numerous to detail here, but even a fairly superficial comparison of the two texts will reveal significant disparities between the two at the procedural level. It is of course possible that one or both of the translators were playing somewhat fast and loose with the narrative structure of the same Arabic text.

[8]See Sabra, *Optics*, lxxx-lxxxiii. Just how problematic the Arabic manuscript tradition is comes clear from the schematic Sabra provides on p. lxxxii. If we designate the five manuscripts according to chronological order from A (earliest) to E (latest), then the entire text of the Arabic treatise breaks down as follows: books 1-3, represented by A, D, and E; books 4-5, represented by B, C, and D (C consisting of excerpts only); and books 6-7, represented by A, C, and D. Thus books 1-3 exist in only three manuscripts; books 4-5 in only three, of which one is defective; and books 6-7 in only three, of which one is defective.

[9]For an overview of the addition and omission of chapter-breaks in book 6, see Table 6 of Appendix 2 in Smith, *Alhacen's Theory*, 660.

ALHACEN'S
DE ASPECTIBUS

LATIN TEXT

LIBER SEXTUS

Liber iste in novem partes partitur. Pars prima, titulus libri; secunda, quoniam error accidit visui propter reflexionem; tertia, in errore evenienti in speculis planis; quarta, in errore qui oritur in speculis spericis exterioribus; quinta in speculis columnaribus exterioribus; sexta, in piramidalibus exterioribus; septima, in spericis concavis; octava, in columpnis concavis; nona, in piramidalibus concavis.

PARS PRIMA

[1.1] Patuit ex libris superioribus modus adquisitionis formarum in speculis per visum, situs linearum reflexionis vel accessus, situs ymaginum et loca ipsarum. Verum per reflexionem non semper comprehenditur forme veritas. In concavis enim speculis apparet ymago faciei distorta, et occultatur visui dispositio ipsius vera, unde planum errorem incidere in comprehensione formarum per reflexionem. Huius erroris modum et modi causam propositum est in libro presenti explanare et secundum diversitates speculorum disquirere varietates errorum.

1 liber sextus *om. FP1SO* 2 liber *om. E* / iste: sextus *R* / partitur: dividitur *R* / pars prima *om. FP1* / *post* prima *add.* est *R* / libri *corr. ex* pun *C1* 3 quoniam: quod *R* / accidit: accidat *R* / reflexionem *corr. ex* reflexonem *L3* / in: de *R* 4 evenienti: venienti *L3E*; eveniente *R* / *post* planis *scr. et del.* concavis *L3* / in[2]: de *R* / qui *corr. ex* que *C1* / *post* qui *scr. et del.* in *L3* 5 *post* quinta *add.* de errore *R* 6 *post* sexta *add.* de errore *R* / *post* septima *add.* de errore *R* 7 *ante* in[1] *add.* de errore *R* / columpnis: columpnaribus *R* / *post* nona *add.* de errore *R* 8 pars prima *om. FP1; transp. L3; inter. a. m. E*; prooemium libri capitulum primum *R* 9 ex: in *O* / libris superioribus *transp. R* 10 vel: et *R* 12 concavis enim speculis: speculis enim concavis *L3ER* / *post* speculi *add.* an *S*; *add.* aliquando *corr. ex* ali *O* 13 occultatur: occulta *L3E* / ipsius: eius *L3R* 14 *post* planum *add.* est *OR*; *inter.* est *a. m. C1* / in *om. S* / per: propter *L3ER* 15 huius *rep. et del. L3* / libro *corr. ex* loco *F* 16 presenti: presente *R* / post presenti *add.* determinare *P1* / disquirere: discurrere *P1R*; *corr. ex* discurrere *E* 17 errorum *corr. ex* eorum *L3*

PARS SECUNDA

[2.1] Comprehensionem formarum in visu directo liber secundus docuit, et singula que propter egressum a temperantia in visu illo errorem inducunt liber tertius diligenter exposuit. Fit autem comprehensio formarum per reflexionem sicut et directe, et quorum fit adquisitio in directione fit etiam in reflexione, utpote lucis, coloris, figure, magnitudinis, distantie, et similium.

[2.2] Et quemadmodum in directione rerum prefixarum et cognitarum ad alia fit collatio, et inde oritur coniecturatio, et sumitur iudicium in anima, similiter accidit in reflexione. Unde quecumque temperamentum egressa in visu directo errorem efficiunt, in reflexione similiter inducunt. Et secundum singula maior accidit error in reflexione propter lucem debilem quam debilitat ipsa reflexio.

[2.3] Ut autem generaliter loquamur, non potest in reflexione comprehendi veritas forme sicut potest in directione propter triplex impedimentum reflexioni speciale. Primum est quoniam in reflexione apparet rei forma pre oculis visui opposita, cum non sit re vera. Secundum quoniam lux et color corporis visi miscentur cum colore speculi, quam mixturam visus percipit non verum rei vise colorem vel lucem. Tertium quoniam ipsa reflexio, ut in superioribus est assignatum, lucem et colorem debilitat, quare in reflexione latebit visum veritas lucis et coloris plus quam in directione.

[2.4] Amplius superiora docuerunt quoniam quantitas temperamenti eorum que in visu directo errorem inducunt fortitudinem lucis et coloris respicit, fortiore enim luce vel colore erit maior, debiliore minor. Cum autem per reflexionem debilitentur lux et color, erit latitudo temperamenti singulorum errorem inducentium minor in reflexione quam in directione, et temperantie diminuta latitudo pluralitatem erroris inducit. Preterea quedam minutie corporum com-

18 pars secunda *om. FP1L3; inter. a. m. E*; quod error accidat visui propter reflexionem capitulum secundum *R* 19 formarum in visu *corr. ex* in visu formarum *C1* / liber secundus *transp. C1* 21 autem *corr. ex* esse *L3* 23 coloris *corr. ex* corporis *C1* / *post* coloris *add.* et *L3* 26 alia: alias *OL3C1R* / collatio *corr. ex* collectio *L3* 27 iudicium: in deum *S* / *post* similiter *scr. et del.* inducunt et secundum singula *S* / quecumque: quicumque *O* 28 in: secundum *O* 29 inducunt: indocuit *F*; docuit *P1* 32 potest *om. ER* 33 reflexioni *corr. ex* reflexionis *F*; reflexionis *S* / speciale: speculi *E* / *post* primum *add.* enim *C1E* / quoniam: quod *R* 35 quoniam: quod *L3R*; quem *C1* 36 *post* speculi *inter.* propter *O* 37 quoniam: quod *R* / in *inter. O* 38 lucem . . . debilitat *inter. O* 40 docuerunt: docuerit *FP1* / quoniam: quam *FP1E*; quod *R* 42 enim *inter. O* / *post* luce *scr. et del.* erit *E* / debiliore: debiliorem *O* 43 debilitentur: debilitantur *E* 45 pluralitatem *om. O* 46 erroris: errorem *O* / *post* minutie *scr. et del.* me *E*

prehendi poterunt per directionem que nullatenus comprehensibiles sunt per reflexionem. Palam ergo quod directionem superat reflexio in maioritate errorum et numero.

PARS TERTIA
[In speculis planis]

[3.1] In singulis speculis erronea formarum accidit comprehensio, sed iuxta varietatem speculorum fit varietas errorum. In speculis planis minor accidit error quam in aliis. In hiis etenim comprehenditur veritas figure, situs, et quantitatis, sicut in directione, quod per probationem patebit.

[3.2] **[PROPOSITIO 1]** Proponatur speculum planum [FIGURE 6.3.1, p. 304], et sit AB linea in superficie illius speculi communis superficiei speculi et superficiei orthogonali super superficiem speculi. Sint H, Z duo puncta in superficie illa orthogonali, E centrum visus, et a puncto H ducatur perpendicularis super superficiem speculi, que sit HL. Et producatur ut LG sit equalis LH. Similiter, producatur perpendicularis ZF ut DF sit equalis FZ.

[3.3] Planum ex superioribus quoniam H refertur ad E a puncto speculi, et locus ymaginis ipsius est G, tantum distans a superficie speculi quantum H. Similiter, Z refertur ad E, et locus ymaginis est D.

[3.4] Ducta autem linea ZH, et similiter linea GD, quodcumque punctum linee ZH refertur ad E. Locus ymaginis eius est tantum distans a superficie speculi quantum ipse punctus, et ita quilibet punctus linee ZH tantum videtur distare quantum distabit. Unde, si

47 poterunt *corr. ex* potuit *F*/comprehensibiles *om. E*/comprehensibiles sunt (48) *transp. R* 48 *post* sunt *inter.* comprehensibiles *a. m. E* 49 errorum: erroris *E* 50 pars tertia *om. FP1L3; mg. a. m. E;* de errorre qui accidit in speculis planis capitulum tertium *R* 51 comprehensio: comprehenso *O* 52 *post* varietatem *scr. et del.* formarum accidit comprehensio *E*/*post* varietas *scr. et del.* eo *C1* 53 accidit: accidet *FP1*/etenim: enim *C1* 54 situs: sicut *P1E; corr. ex* sicut *a. m. L3; om. R*/et *om. P1*/quantitatis: quantitas *C1; corr. ex* quantitas *L3*/*post* sicut *add.* et *ER* 55 probationem *corr. ex* propositionem *S* 57 speculi: circuli *S; corr. ex* circuli *a. m. L3*/superficiei . . . et (58) *om. O*/*post* superficiem *add.* scilicet *E*/speculi²: sibi *S; mg. a. m. L3; inter. a. m. E* 59 H Z: L F *R*/post ortogonali *scr. et del.* est *L3* 60 H: L *R* 61 HL: BL *S;* LH *R*/ut . . . producatur *om. FP1*/LG: HG *R*/producatur²: producitur *C1*/producatur perpendicularis (62) *transp. L3* 62 ZF: FZ *R*/ut: et *S*/FZ: ZF *R* 63 planum: palam *R*/post planum *add.* est *S*/ex *corr. ex* et *SO*/quoniam: quod *R*/H: L *R*/a: ab *R* 64 *ante* puncto *add.* aliquo *R*/ymaginis: ymaginum *FP1OL3E*/ipsius *om. R* 65 H: L *R*/Z: F *R* 66 ymaginis: ymaginum *L3* 67 ZH: FL *R*/et *om. P1*/linea² *om. R*/quodcumque: quodque *E* 68 ZH: FL *R*/est *inter. O*/est tantum *transp. O* 69 ipse punctus: ipsum punctum *R* 70 punctus: punctum *R*/ZH: FL *R*/distabit: distat *ER*

linea ZH fuerit recta, erit linea DG recta. Si fuerit arcus, erit DG arcus et eiusdem curvitatis, quare linea ZH apparebit eiusdem quantitatis, eiusdem figure cuius fuerit, quod est propositum.

[3.5] Verum, si in punctis linee ZH fuerit varietas colorum minutim variata, forsan non discerneretur variatio; sed una pretendetur visui coloris confusio. Unde erit error in luce et in colore, et hoc in numero propter reflexionem. Illa etenim colorum et lucium varietas forsan comprehendi posset directe, sed egressus est color a temperantia respectu reflexionis, non respectu directionis. Similiter, minutie occultantur aut confunduntur in reflexione que discerni possent in directione.

[3.6] Et propter debilitationem lucis vel coloris ex reflexione accidit error in longitudine qui quidem non accideret directe.

[3.7] In situ manifeste accidit error ex sola reflexione, in ymagine enim sinistra comprehendimus ea que in corpore viso, si esset in loco ymaginis, dextra videremus. Cum enim aliquid alii opponitur, contrarius est eis adinvicem situs, quod enim uni fuerit dextrum alii erit sinistrum. Igitur quod rei vise dextrum est ymagini sinistrum, et sinistrum in ymagine dextrum erit videnti, sed comprehenditur in ymagine sinistrum.

[3.8] Et generaliter in modo lucis, vel coloris, vel situs error semper accidit ex sola reflexione. In hiis et in aliis que errorem inducunt directe inducunt similiter in reflexione, et facilius, quoniam temperamentum singulorum minus est visui reflexo quam in directo. Horum omnium unum apponatur exemplum, et idem in ceteris intelligatur.

[3.9] In visu directo, cum fuerit corpus visum remotum ab axibus visualibus, accidit ipsum videri duo; idem evenit in speculis re visa ab axibus elongata.

71 ZH: FL *R*/erit DG arcus *om. FP1* 72 ZH: HZ *L3E*; LF *R* 74 in *inter. a. m. C1*/ZH: FL *R*/minutim *corr. ex* minutum *L3* 75 forsan: forsitan *FP1E*/discerneretur: discernetur *FP1ER* 76 erit: erat *L3*/erit error *transp. R*/et[1] *om. SL3E; inter. O*/in[2] *om. FP1C1R*/*post* et[2] *scr. et del.* in *L3*/hoc in *corr. ex* in hoc *E*/in[3] *inter. L3*/*post* in[3] *scr. et del.* no *F* 77 numero: termino *FP1* 78 comprehendi *corr. ex* comprehenderi *L3* 79 respectu[1] *rep. et del. F*/reflexionis *corr. ex* directionis *C1*/non *om. P1*/*post* similiter *add.* particule *ER* (*inter. a. m. E*)/minutie: minute *ER* 80 confunduntur *corr. ex* confundantur *P1*/possent: posset *S* 82 debilitationem: debilitatem *R* 83 qui *om. FP1; inter. L3*; que *O*/directe *om. P1* 84 manifeste *corr. ex* manifefeste *F*/*post* manifeste *scr. et del.* quod *L3*/sola reflexione *transp. R* 85 si *om. S*/*post* si *scr. et del.* o *P1* 86 aliquid: quid *E* 87 contrarius: concursus *S*/est eis *transp. E*/est . . . situs: eis situs est adinvicem *R*/adinvicem *corr. ex* advicem *a. m. C1*/alii . . . dextrum (88) *inter. E* 88 *post* est *add.* erit *P1*/ymagini . . . erit (89) *inter. a. m. L3* 89 *post* dextrum *scr. et del.* est *F*/erit: est *R*/sed . . . sinistrum (90) *om. P1R* 91 error semper *transp. R* 92 *post* reflexione *add.* et *FP1*/in hiis *om. R*/hiis et in *om. FP1*/inducunt directe (93) *om. FP1* 93 inducunt: inducit *E*; *om. C1*/et *om. FP1*/quoniam: quam *P1* 94 *post* est *add.* in *R*/visui: visu *ER*/reflexo: reflexio *P1*; *corr. ex* reflexio *L3*; *corr. ex* reflectio *O*/in *scr. et del. O* 95 apponatur: proponatur *ER* 97 idem: item *L3E*/re *corr. ex* rei *L3*

[3.10] In speculis ab aliqua longitudine videbitur corpus minus quam sit, quod forsan directe a tanta longitudine videretur minus quam esset in veritate, sed non adeo minus. Et hoc minoritatis additamentum in speculis provenit propter minus longitudinis temperamentum.

[3.11] In figura non numquam accidit error in speculis per causas per quas in directo, sed maior et frequentior propter situm.

[3.12] Si aliquid ab aliqua longitudine opponatur speculo, et eius capita non percipiantur a visu, ut funis vel aliquid tale, videbitur forsan continuum speculo. Idem accidit in visu directo. Si opponatur funis aliquis foramini et non videantur capita funis, non apparebit distantia inter funem et foramen, licet magna sit, et est propter situm. Si autem alterum capitum videatur, aliud vero non, videbitur fortassis illud caput continuum. Et in singulis ubi directe accidit similiter in reflexione.

PARS QUARTA

In speculis spericis [exterioribus]

[4.1] Universitas errorum in speculis planis accidentium evenit similiter in spericis exterioribus, et preter hoc, in spericis speculis res visa videtur minor quam sit. Et generaliter in hiis speculis nichil ex re visa comprehenditur in veritate preter ordinationem partium, que talis apparet in speculo qualis est in corpore viso.

[4.2] **[PROPOSITIO 2]** Quod autem semper videatur res minor in hoc speculo quam ipsa sit probatur.

[4.3] Sit AB [FIGURE 6.4.2, p. 304] linea visa, ZP speculum, D centrum circuli, E centrum visus. A reflectatur ad E a puncto H, B a

99 videbitur: videtur *S* 100 *post* sit *add.* corpus *C1* / *post* videretur *add.* etiam *R* / minus *inter. a. m. E* 101 *post* minus *add.* sed *L3E* / et: sed *O* / *post* et *add.* in *S* 102 provenit: pervenit *C1* / *post* minus *add.* in *R* / longitudinis: longitudine *ER* 104 per: propter *R* 105 per: propter *R* / *post* in *add.* visu *R* 106 et eius . . . speculo (108) *mg. a. m. L3* 107 percipiantur: percipitur *L3* / a visu *inter. a. m. E* / funis *corr. ex* finis *P1* / forsitan: forsan *SOL3C1* 108 *post* continuum *add.* in *L3C1* / visu: viso *C1* 109 videantur *corr. ex* videtur *a. m. E* 110 funem *corr. ex* finem *F* / sit *om. O* 111 aliud: alium *FP1*; alterum *R* 112 ubi *inter. a. m. E* / *post* directe *add.* error *R* / *post* accidit *scr. et del.* i *O*; *add.* error *L3C1E* (*inter. a. m. L3E*) 113 in *inter. a. m. E* / *post* reflexione *add.* pax *F*; *add.* par *P1* 114 pars . . . spericis (115) *om. FP1SL3*; *mg. a. m. E*; de errore qui accidit in speculis spericis convexis capitulum quartum *R* 117 in *inter. O* / in spericis speculis *om. R* / spericis speculis *transp. O* / speculis *inter. O* 118 hiis speculis *transp. S* 120 corpore viso: imagine *R* 121 semper . . . res: res semper videatur *R* / videatur res *transp. E* 122 in hoc speculo *om. R* / post speculo *scr. et del.* qualis est in corpore viso quod *P1* / ipsa *om. R* 123 ZP: ZX *R* 124 *post* centrum *inter.* scilicet *a. m. E* / circuli *om. R* / circuli E *inter. a. m. E* / centrum[2]: punctum *R*; *corr. ex* punctus *a. m. E* / a puncto H B *inter. O* / B a *inter. a. m. E* / *post* B *scr. et del.* D *P1*

puncto N. Linea AB producta aut transibit per centrum speculi, aut non.

[4.4] Transeat. Et ducatur a puncto N linea contingens circulum, que sit NL; a puncto H contingens HM. Et ducantur linee reflexionis BN, EN, AH, EH, et producantur linee EH, EN donec cadant in perpendicularem, que est AD, et puncta casus sint T, Q. Palam quoniam T est locus ymaginis A; Q est locus ymaginis B. Dico quoniam AB maior est QT.

[4.5] Patet ex superioribus quoniam proportio AD ad DT sicut AM ad MT. Similiter, proportio BD ad DQ sicut proportio BL ad LQ. Sed AD maior BD, et DT minor DQ. Erit igitur maior proportio AM ad MT quam BL ad LQ.

[4.6] Secetur AM in puncto F ut proportio FM ad MT sit sicut BL ad LQ. Erit igitur minor BM ad MT quam BL ad LQ. Secetur MT in puncto K ut proportio BM ad MK sit sicut BL ad LQ. K necessario cadet inter M et Q, quoniam LQ minor MQ, et BL maior BM. Cum igitur FM ad MT sicut BL ad LQ et sicut BM ad MK, erit proportio FB ad KT sicut BL ad LQ. Sed BL maior LQ. Igitur FB maior KT, quare AB maior QT, quod est propositum.

[4.7] Si vero linea AB producta non perveniat ad centrum, ducatur a puncto A [FIGURE 6.4.2a, p. 304] linea ad centrum, que sit AG, et sit G centrum, et a puncto B ducatur linea BG. Locus ymaginis A sit punctus D, locus ymaginis B sit E, et ducatur linea ED, que quidem est ymago linee AB. Dico quoniam AB maior est ED, quoniam ED aut est equidistans AB aut non.

[4.8] Si fuerit equidistans, planum quoniam est minor. Si non fuerit equidistans, producatur usquoque concurrat cum ea. Sit concursus Z, et a puncto E ducatur equidistans AB, que sit EH. Angulus EDH aut est acutus, aut rectus, vel maior.

125 *post* N *add.* ducatur *P1*/linea *om. L3ER*/aut[1] *inter. a. m. E* 127 N *om. O*/*ante* linea *add.* et ducatur *O* 128 *post* contingens *add.* circulum *ER*/ducantur: ducatur *S*/*post* linee *add.* accessus et *R* 129 et . . . EH *mg. F*/in *om. O* 130 sint: sunt *O*/quoniam: quod *R* 131 est[1]. . . Q *om. P1*/Q: quod *S*/Q . . . ymaginis[2] *mg. O*/dico quoniam *transp. L3*/quoniam: quod *R; scr. et del. L3* 133 patet: palam *R*/quoniam: quod *R*/DT: TD *FP1* 135 BD: DB *R*/erit . . . maior: ergo maior est *R*/igitur *inter. a. m. C1* 137 secetur . . . LQ[2] (138) *scr. et del. F; om. P1*/linea *scr. et del. F*/MT[2] *om. S* 138 *post* minor *add.* proportio *R* 139 sit *om. FP1*/necessario cadet (140) *transp. R* 140 cadet *corr. ex* cadat *a. m. E*/et[1] *om. FSL3; inter. OL3*/quoniam: quia *R* 142 FB: FL *SO* 144 ducatur . . . centrum (145) *om. P1* 145 *post* linea *add.* a puncto *C1*/AG: AD *R* 146 G: D *R*/BG: BD *R*/*ante* locus *add.* et *R* 147 punctus D: punctum G *R*/E: P *R; corr. ex* EE *F*/*post* linea *scr. et del.* et *E*/ED: GP *R; corr. ex* ZE *L3* 148 *post* dico *scr. et del.* quod *P1*/quoniam[1]: quod *R*/ED: GP *R* 149 ED: GP *R* 150 quoniam: quod *R* 151 usquoque: usquequo *FSL3;* quousque *R* 152 Z: EZ *P1; corr. ex* EZ *F*/E: P *R*/ducatur: producatur *ER*/EH: PH *R* 153 EDH: PGH *R*/vel: aut *R*

[4.9] Si rectus vel maior, erit latus EH maius ED. Sed EH minus AB, et ita propositum.

[4.10] Si fuerit acutus, poterit accidere quod forma sit maior ipsa re cuius est forma, quam licet excedat raro accidet. Et cum acciderit, forsan comprehendetur forma a longitudine tali quod minor videbitur quam sit, quoniam ipsum corpus ab hac longitudine forsan videtur minus.

[4.11] **[PROPOSITIO 3]** Quod autem forma in hiis speculis aliquando videatur maior re visa, scilicet cum maior fuerit, et comprehendatur a tali longitudine a qua certa eius quantitas possit discerni declarabitur.

[4.12] Sit A [FIGURE 6.4.3, p. 305] centrum speculi, et superficies sumatur reflexionis que secabit speculum super circulum. Sit circulus ille EDB, ED dyameter illius circuli, et producatur dyameter ED usque ad Z ut multiplicatio EZ in ZD non sit maior quadrato AD, quod planum, cum sit possibile dyametro ED talem addi lineam ut ductus totalis in partem additam sit equalis quadrato AD. Et dividatur linea ZD in partes equales in puncto H. Erit igitur AH medietas EZ. Ductus ergo AD in HD non erit maior quarta parte quadrati AD, et quoniam ductus AH in HD maior est quadrato HD, sit ductus AH in HT equale quadrato HD.

[4.13] Fiat circulus secundum quantitatem AH, et a puncto H ducatur corda equalis medietati linee HD, que sit HQ. Et producantur linee QA, QT, et supra punctum Q fiat angulus equalis angulo QAH, qui sit HQN. Cum ergo hiis duobus triangulis hii duo anguli sint equales, et unus communis, scilicet QHA, erit tertius tertio equalis, scilicet AQH angulo HNQ. Et erunt trianguli similes, et erit proportio AH ad HQ sicut HQ ad HN. Igitur quod fit ex ductu AH in HN equale quadrato HQ.

154 EH[1, 2]: PH *R*/maius: maior *FP1*/ED: PG *R* 155 *post* ita *add.* est *R* 156 poterit: potest *R*/quod: ut *R* 157 est *inter. a. m. C1*/accidet: accidit *L3*/cum: si *R* 158 comprehendetur: comprehendentur *L3*/a *inter. O* 159 ab . . . minus (160) *mg. a. m. L3*/forsan *corr. ex* forsian *F*/videtur: videbitur *R* 161 aliquando videatur maior (162): videatur maior aliquando *C1* 162 maior fuerit et *om. R*/comprehendatur: comprehenditur *R* 163 a qua *om. L3*/certa *corr. ex* terra *a. m. C1*/certa eius *transp. L3ER*/*post* quantitas *add.* non *R* 166 *post* secabit *scr. et del.* circulum *O* 168 multiplicatio: multiplicato *FP1*/ZD *corr. ex* D *E*/non sit *corr. ex* sit non *E*/non sit maior: sit equalis *R*/quadrato: quam *S* 169 *post* planum *add.* est *P1R*/planum cum sit *rep. S* 171 *post* H *add.* AD *F* 172 AD[1]: AH *L3C1ER*/*post* HD *scr. et del.* sit ductus H *L3*/non erit maior: erit equalis *R*/parte: parti *R* 173 HD[1] *corr. ex* KD *mg. a. m. F*; *corr. ex* HT *L3*/sit . . . HD (174) *scr. et del. E* 174 equale: equalis *L3ER* 175 H *corr. ex* HS *F* 176 ducatur: producatur *SL3C1ER*/corda *inter. a. m. L3*/*post* corda *scr. et del.* eius *E*/HQ: BQ *O* 177 supra: super *R* 178 HQN *corr. ex* HNQ *F*/*post* HQN *scr. et del.* et *E*/*post* ergo *add.* in *R* 179 sint: sunt *L3C1*/scilicet *om. L3*/tertio *om. P1* 180 trianguli similes *transp. P1*; triangula similia *R* 181 AH *corr. ex* QH *E*/HQ[2] *corr. ex* AD *F*/ex *inter. O* 182 HN: NH *C1*/*post* HN *add.* erit *SL3C1*/*post* equale *add.* erit *O*; *add.* est *R*

[4.14] Sed quadratum HQ est quarta pars quadrati HD, cum HQ sit medietas HD. Igitur multiplicatio AH in HN equalis est quarte parti multiplicationis AH in HT, quare HN est quarta pars HT. Igitur N cadit inter H et T. Restat ut ductus HT in TN sit tres quarte quadrati HT.

[4.15] Verum angulus QHD acutus, et equalis angulo HQA, quia respiciunt equalia latera in maiori triangulo. Igitur angulus QHN equalis angulo HNQ, et ita HQ equalis QN.

[4.16] Et angulus HNQ acutus, quare angulus QNT obtusus. Quadratum igitur TQ superat quadratum QN et quadratum TN ductu linee TN in NH, quoniam, ut dicit Euclides, quadratum lateris oppositi obtuso superat quadrata duorum laterum quantum est quod fit ex ductu unius lateris bis in partem ei adiunctam procedentem usque ad locum casus perpendicularis a capite alterius lateris ducte. Et si a puncto Q ducatur perpendicularis super lineam HT, cadet in puncto medio linee HN, et ductus TN in medietatem HN bis equipollet ductui TN in HN.

[4.17] Igitur quadratum TQ superat quadratum QN et TN ductu TN in NH. Sed ductus HN in NT cum quadrato NT equalis est ductui HT in TN. Igitur ductus HT in TN est excessus quadrati TQ supra quadratum HQ.

[4.18] Amplius, sit proportio AI ad AH sicut QT ad QH [FIGURE 6.4.3a, p. 305]. Erit quadratum ad quadratum sicut quadratum ad quadratum, et erit proportio excessus quadrati AI super quadratum AH ad quadratum AH sicut ductus HT in TN ad quadratum

183 sed quadratum: per quantum *F*/quadrati: que *S*; *mg. a. m. L3*/*post* HD *scr. et del.* igitur *S* 184 AH: HA *FP1*/in HN *corr. ex* ad HN *C1* 185 est *inter. O* 186 H et T: HZT *S*/TN *corr. ex* TL *E*/sit: sint *R*/quarte: quadrate *S*; in quantitate *P1*; *corr. ex* quantum *F*/quadrati: quadrate *S*; quantum *alter. in* quarti *F* 187 QHD: QHA *R*/*post* acutus *add.* est *R*/et *om. C1*/*post* equalis *inter.* est *a. m. C1* 188 equalia latera *transp. FP1*/maiori triangulo *transp. R*/angulus *corr. ex* angulo *F* 189 *post* equalis[1] *add.* est *O*/equalis angulo HQN *rep. F*/*post* HQN *add.* quia utraque equatur HQA *P1*/*post* et *scr. et del.* L *L3* 190 HNQ *corr. ex* HQN *a. m. E*/angulus[2] *om. FP1*/QNT: GNT *P1S*; *corr. ex* GNT *mg. a. m. F* 191 quadratum[1]: quantum *FP1S*/igitur *om. FP1*/superat quadratum QN *om. FP1*/*post* QN *scr. et del.* i *L3*/quadratum[3]: quantum *FP1*/ductu *corr. ex* ducti *O* 192 NH: HN *R*/lateris: laterum *SL3*/oppositi: opposita *FP1*; opponi *S* 193 obtuso *corr. ex* obtusa *F*/quadrata *corr. ex* quadrato *L3*/laterum: alterum *FP1* 194 lateris: laterum *FP1SO*/procedentem: precedentem *FP1SE* 196 et: nam *R*/post puncto *scr. et del.* medio linee HN *C1*/cadet: cadent *FP1* 197 *post* puncto *scr. et del.* per ZE pri *E*/medio: medium *R*/linee *om. O*/equipollet: equipollens *FP1*; *corr. ex* equipoll *O* 198 ductui *corr. ex* ductu *O*/*post* ductui *scr. et del.* p *C1* 199 TQ superat quadratum *mg. F*/quadratum[2]: quadrata *RL3* (*alter. in L3*)/et *om. FP1SOL3ER*; *inter. a. m. C1*/TN: in *SO* 200 TN *rep. S;* tamen *P1*/*post* NH *inter.* et quadratum NT *a. m. O*/HN *corr. ex* HNM *L3*/HN in NT: TN in HN *R*/in *inter. O*/NT[2]: TN *R*/ductui: actui *P1* 201 TN *corr. ex* ITN *F*/quadrati: quantitati *FP1*/supra *mg a. m. F;* super *SOL3C1E* 202 quadratum *corr. ex* quantum *mg. a. m. F* 203 AI: AL *L3E* 204 erit *om. S*/quadratum[1]: quadrati *FSOL3C1E*; quantitati *P1*/quadratum[2]: quantum *FP1*; quantitatum *O*/sicut . . . quadratum[1] (205) *inter. a. m. E*/quadratum[3]: quadrati *FSOL3C1E;* quantitati *P1*/ad[2] *om. S*/*post* ad[2] *add.* d *P1* 205 quadratum[1]: quantitatum *FP1*; *om. S*/quadrati: quantitati *P1*/super: supra *R*/quadratum[2]: quantum *P1* 206 ad quadratum AH *mg. F; rep. O*

QH. Et quoniam quadratum QH quater sumptum efficit quadratum HD, et ductus HT in TN quater sumptus efficit triplum quadrati HT, erit ductus HT in TN ad quadratum QH sicut tripli quadrati HT ad quadratum HD.

[4.19] Sit autem HC tripla ad HT. Erit ductus CH in HA triplus ad quadratum HD, sed quoniam proportio AH ad HD sicut HD ad HT, erit HT ad HA sicut quadratum HT ad quadratum HD. Verum proportio CH ad HA sicut ductus CH in HT ad ductum HA in HT, et ita CH ad HA sicut tripli quadrati HT ad quadratum HD. Sed hec erat proportio excessus quadrati AI super quadratum AH ad quadratum AH. Igitur CH ad HA sicut excessus quadrati AI super quadratum AH ad quadratum AH. Igitur coniunctim proportio CA ad AH sicut quadrati AI ad quadratum HA, excessus enim quadrati AI super quadratum HA cum quadrato HA efficit quadratum AI.

[4.20] Igitur IA erit media in proportione inter CA et HA, cuius rei conversam paulo ante tetigimus. Igitur proportio CA ad IA sicut IA ad HA, et eadem erit proportio residui ad residuum, id est CI ad IH, et cum IA maior HA, erit CI maior IH.

[4.21] Amplius, ductus AH in HD minor quarta parte quadrati AD. Igitur HD est minor quarta parte linee AD. Igitur est minor quinta parte AH. Cum ergo AH sit maior quam quintupla ad HD, et ductus eius in HT efficiat quadratum HD, erit HT minor quinta parte HD, et ita HT erit minor vicesima quinta parte HA. Sed proportio CI ad IH sicut IA ad HA, ut dictum est. Igitur, coniunctim erit CH ad IH sicut IA cum AH ad AH. Igitur tertia primi ad secundum sicut tertia tertii ad quartum.

211 HC: HCA *F*; HAT *P1*; HE *C1*; HO *R* / CH: HO *R* / HA: HQ *S*; HT *R* 212 HD^{1}: HT *R* / *post* HD^{2} *add.* est *ER* / HT *corr. ex* HA *E* 213 HA: AH *L3ER* / quadratum: quadrati *FP1SOL3C1E* / quadratum HD *transp. deinde corr. C1* / verum *inter. a. m. L3* 214 CH^{1,2}: OH *R* / HA^{1,2}: AH *R* / *post* HA^{1} *add.* est *E* / HA^{2} *corr. ex* HT *E* / ita: proportio *R* 215 CH: OH *R* / *post* sicut *add.* proportio *R* / erat *inter. a. m. L3* 216 super: supra *R* / ad quadratum AH (217) *inter. a. m. E* 217 CH: OH *R* / HA: AH *L3ER* / super: supra *R* 218 *post* AH^{2} *scr. et del.* excessus enim quadrati AI *C1* / igitur . . . HA (219) *mg. a. m. L3* / proportio *rep. S* / CA: OA *R* 219 HA: AH *L3C1ER* / enim *inter. L3* / super: supra *R* 220 HA^{1}: AH *C1R*; *corr. ex* AH *E* / quadrato: qua *L3* / HA^{2}: AH *R* 221 CA: OA *R* / HA: AH *R* / cuius . . . tetigimus (222) *om. R* 222 CA: OA *R* / sicut IA *mg. F* 223 CI *inter. E*; OI *R* 224 et . . . IH *om. R* 225 AH: AD *R* / *post* HD *inter.* est *a. m. C1* / *post* minor *add.* est *R* 226 igitur^{1} *om. FP1* / *post* igitur^{2} *add.* HD *R* 227 ergo *corr. ex* erga *L3* / quintupla: cum tripla *L3*; quadrupla *P1*; quadra *alter. in* quadrupla *F* / *post* ad *add.* AH *S*; *scr. et del.* AH *C1* 228 eius *inter. O* / HD *rep. FP1* / HT minor *transp. deinde corr. O* 229 HD: HT *L3* / vicesima: et *FP1*; *om. S*; 3 *E* / quinta: quarta *L3* / *post* quinta *scr. et del.* parte quinta *C1* / parte *inter. a. m. O* / CI: OI *R* 230 IH^{1}: CH *S* / sicut IA *om. L3*; *inter. a. m. C1* / IA: IH *SE* / ad^{2}. . . IA (231) *om. FP1* / HA: AH *R* / coniunctim: coniunctinctim *S* / CH: OH *R* / IH: FH *S* 231 IA *corr. ex* IH *a. m. C1* / ad AH *om. FP1OL3*; ad HA *C1* / primi: prime *R* / secundum: secundam *R* 232 tertii: tertie R / post ad *scr. et del.* quantum *F* / quartum: quartam *R*

[4.22] Sed HT est tertia pars linee CH. Igitur TH ad IH sicut tertia pars linee IA cum AH ad lineam AH. Igitur TH ad IH sicut due tertie linee AH cum tertia linee IH ad lineam AH. Sed quoniam CI maior IH, erit IH minor medietate CH, et erit tertia IH minor sexta parte CH, et ita tertia IH erit minor medietate TH. Igitur due tertie AH cum minori parte medietate HT se habebunt ad AH sicut TH ad IH. Igitur IH ad HT sicut AH ad duas sui tertias cum minori parte medietate HT.

[4.23] Sed HT minor vicesima quinta AH, et eius medietas minor medietate vicesime quinte. Sed linea AH in viginti quinque partes divisa; due eius tertie cum medietate vicesime quinte non efficiunt octodecim eius partes. Igitur proportio IH ad HT maior quam sit proportio viginiti quinque ad octodecim.

[4.24] Item, cum HT sit minor vicesima quinta AH, erit AT maior viginti quattuor vicesime quinte AH. Sed linea IH minor medietate CH, et ita minor HT cum medietate HT, et ita minor una et dimidia viginti quinque partium AH, et ita IA minor viginti sex et dimidie partis sumptis partibus secundum divisionem HA in viginti quinque. Ergo proportio IA ad AT sicut minoris viginiti sex et dimidii ad maius viginti quattuor. Igitur proportio IA ad AT minor quam viginti sex et dimidii ad viginti quattuor. Sed IH ad HT maior quam viginiti quinque ad octodecim. Igitur IH ad HT maior quam IA ad AT.

[4.25] Sit proportio IM ad MT sicut IA ad AT. Cadet quidem M inter I, H. Item, maior erit proportio IM ad MH quam IA ad AT, et ita

233 CH: OH *R* / CH . . . linee (234) *mg. a. m. E* / *post* IH *add.* est *ER* / tertia *om. FP1* 234 *post* cum *scr. et del.* tertia linee AH *C1*; *add.* tertia parte *R* / *post* ad[1] *scr. et del.* a *S* / IH: IA *R* 235 linee: linea *FP1*; line *S* / *post* quoniam *add.* linea *R* / CI: OI *R* / *ante* maior *add.* est *R* 236 erit IH *om. L3* / IH[2] *corr. ex* CH *E* / CH: OH *R* 237 CH: OH *R* / TH *corr. ex* HT *E* 238 minori: minore *R* / parte *inter. a. m. E* / medietate: quam sit medietas *R* / *post* IH *scr. et del.* sicut TH ad IH *F* 239 minori: minore *R* / parte *om. R* / medietate: quam sit medietas *R* 241 sed HT *om. S* / vicesima quinta: vicesimo quinto *S*; viginti quinque *FP1L3* / AH: HA *C1* 242 medietate: quam medietas *R* / vicesime quinte: viginti quinque *FP1L3C1* / *post* quinte *add.* partis *R* / sed *inter. a. m. C1* / viginti quinque: vicesime quinte *E* / partes: parte *FP1* 243 divisa: commissa *L3* / eius *inter. E*; *om. R* / *post* medietate *add.* quadrate *FP1SE* / vicesime quinte: viginti quinque *FP1*; *om. S* / *post* quinte *add.* partis *R* 244 octodecim *om. FP1*; octo *L3E* / eius partes *transp. FP1* / ad *rep. P1* / *post* HT *inter.* est *O* / *post* maior *add.* est *R* 245 *post* proportio *scr. et del.* quam sit proportio *C1* / viginti quinque: vicesime quinte *E* 246 vicesima quinta: vicesimo quinto *SC1*; vicesime quinte *E* / *post* quinta *add.* parte *R* 247 viginti quattuor: vicesima quarta *E* / *post* quattuor *add.* partibus quarum AH est viginti quinque *R* / vicesimis quintis: viginti quinque *S*; vicesima quinta *L3E* / vicesimis . . . AH *om. R* / *post* minor *add.* est *R* 248 CH: OH *R* / HT cum *om. R* / cum medietate HT *mg. F* / HT *mg. P1* 249 viginti quinque: vicesima quinta *E* / ita IA *transp. R* / viginti sex: vicesima sexta *E* 250 partis *om. R* / partibus *om. FP1* / AH: HA *FP1* / in . . . quinque *om. R* / viginti quinque: ZH *FP1*; vicesime quinte *E* 251 IA . . . proportio (252) *om. FP1* / *post* AT *scr. et del.* minor *C1* / *post* minoris *add.* linee *R* / viginti sex: viginti octo *S* / maius: maiorem *R* 252 AT *corr. ex* ADT *FP1* / *post* minor *add.* est *R* 253 *post* sed *add.* proportio *R* / ad[1] *om. FP1* / *post* maior *add.* est *R* 254 *post* igitur *add.* proportio *R* / *post* maior *add.* est *R* 255 IM: NM *S* / IA: NL *FP1L3E*; IN *S* / cadet *corr. ex* cadat *E* 256 *post* I *add.* et *OR* (*inter. O*) / IM: MI *L3*; LM *C1*

maior quam IA ad AH. Sit igitur proportio IL ad LH sicut IA ad AH. Cadet quidem L inter M et I.

[4.26] Amplius, a punctis L, M ducantur contingentes LB, MG, et ducantur linee IB, HB, IG, TG, AB, AG, que ultime producantur usque ad exteriorem circulum.

[4.27] Et habebis ex quinta quinti libri quod angulus IBZ sit equalis angulo HBA, cum enim sit proportio IL ad LH sicut IA ad AH, erit H locus ymaginis in reflexione a puncto B. Et si dicatur contrarium ut sumatur alius locus ymaginis, improbabis per impossibile, sumpta impossibilitate a proportione quam verum est esse IA ad lineam a puncto A ad locum ymaginis sicut IL ad lineam a puncto L ad locum ymaginis.

[4.28] Cum igitur H sit locus ymaginis, et LB sit contingens super AB, producta HB faciet angulum reflexionis equalem sibi collaterali, et quoniam LB perpendicularis super ABZ, restabit angulus IBL equalis angulo LBH. Eodem modo erit angulus IGZ equalis angulo TGA, et cum MG sit perpendicularis, erit angulus IGM equalis angulo MGT.

[4.29] Amplius, producatur a puncto H ad lineam AB linea equidistans IB, que sit HP, et a puncto T equidistans IG, que sit TR. Erit angulus IBZ equalis angulo HPB. Sed angulus IBZ equalis, ut dictum est, angulo HBA, et ita duo anguli HBA, HPB sunt equales, quare duo latera HB, HP sunt equalia. Similiter, TR equalis TG. Verum angulus HPB acutus, cum sit equalis angulo reflexionis; erit angulus HPA obtusus, et erit HA maior HP, et ita maior HB. Similiter, erit TA maior TG.

[4.30] Amplius, quoniam HP equidistans IB, erit proportio IA ad AH sicut AB ad AP; erit similiter proportio IA ad AT sicut GA ad AR,

257 IL *om. FP1*/LH: IH *FP1*/IA^2: LA *E*/AH^2 *corr. ex* HA *E* 258 M et I: MTI *FP1*/I: L *L3* 259 MG: ING *F* 260 ducantur: ducant *L3*/*post* que *add.* due *R* 262 habebis: habebit *SOL3E*; habebitur *R*/quinta: quarta *S*; quinque *L3*; *om. FP1R*/*post* quinta *add.* figura *C1*/quinti libri: quinto libro *P1*; quarto libro *R*/angulus *om. S*/sit . . . enim (263) *rep. S* 263 enim: igitur *R*/IL: AL *L3*/LH: HL *FP1* 264 *post* ymaginis *add.* I dum *R*/in reflexione: reflectitur *R* 265 improbabis: I probabis *R*/*post* improbabis *add.* quod *FP1* 266 *post* quam *add.* enim *L3E*; *add.* non *R*/verum: necessarium *O*/verum est *transp. R*/IA . . . A (267) *inter. O*/a^2: ab *O* 267 puncto^1 *om. O*/A ad locum *om. R*/*post* ymaginis *add.* ductam ad punctum A *R*/IL: ID *FP1* 269 sit contingens: contingat *R*/super AB (270): circulum in B *R* 270 HB: AB *R*/*post* angulum *add.* LBZ *R*/reflexionis *om. R*/equalem *om. P1*; *corr. ex* qualem *O*/sibi: suo *R*/collaterali: collaterabit *FP1* 271 LB: IB *E*/*post* LB *add.* est *P1*; *inter.* est *O*/restabit: restabat *SL3*/IBL: BL *L3E* 272 *post* LBH *add.* faciet *S*/IGZ: IG *L3E* 273 TGA: TAG *E*/*post* perpendicularis *add.* super AGZ *R*/IGM: IGMS *F* 275 producatur: ducatur *R* 276 *ante* IB *scr. et del.* IB *E*/*post* IG *add.* ad lineam AG *R* 277 IBZ^1 *corr. ex* IBH *S*/IBZ^2: QBZ *L3*/*post* equalis^2 *add.* angulo HBA *R* 278 angulo HBA *om. R*/et . . . HBA^2 *mg. F*/HPB *om. L3*; HP *alter. in* HPA *E*/sunt . . . HPB (280) *om. S*/equales *corr. ex* equalia *E* 280 *post* HPB *add.* est *R*/acutus: acucutus *S*/*post* angulo *add.* IBZ *R*/reflexionis *om. R*/*post* erit *add.* igitur *R* 281 et^1 *om. O*/HA: AH *R*/et^2 . . . HB *om. P1L3R* 282 TG *inter. a. m. E* 283 equidistans: equidistat *L3ER* (*alter. in a. m. L3*)/IB *corr. ex* HP *S*/proportio *om. R*/ad *om. P1* 284 AT *corr. ex* AH *E*/GA: AG *R*

et erit proportio AH ad AI sicut AP ad AB. Sed IA ad AT sicut AB ad AR, cum AB sit equalis AG. Igitur a primo erit proportio AH ad AT sicut AP ad AR.

[4.31] Verum, cum angulus HPA obtusus, quadratum HA excedet quadratum HP et quadratum AP cum multiplicatione AP in lineam ductam a puncto P usque ad locum perpendicularis ducte a puncto H bis. Sed perpendicularis ducta a puncto H cadet in puncto medio linee PB, cum HB, HP sint equales, et ita quadratum HA excedet quadratum HP et quadratum AP in multiplicatione AP in PB. Et ita quadratum AH excedit quadratum HP in multiplicatione AB in AP, quoniam ductus AP in PB cum quadrato AP valet ductum AB in AP. Similiter, quadratum AT excedit quadratum TR in ductu AG in AR, sive AB in AR, quod idem est.

[4.32] Ducatur ergo linea AB in duas lineas AP, AR, et provenient duo excessus. Igitur proportio excessus ad excessum sicut AP ad AR, quare proportio excessus quadrati AH super quadratum HP ad excessum quadrati AT super quadratum TR sicut AH ad AT. Et cum HP equalis HB, et TR, TG, erit proportio excessus quadrati AH super quadratum HB ad excessum quadrati AT super quadratum TG sicut AH ad AT.

[4.33] Sed multiplicatio AH in HU equalis est quadrato HB. Igitur multiplicatio AH in AU erit excessus quadrati AH super quadratum HB. Igitur proportio AH ad AT sicut multiplicationis AH in AU ad excessum quadrati AT super quadratum TG. Et si due linee AH, AT ducantur in AU, erit proportio AH ad AT sicut multiplicationi AH in

285 AH: AB *FP1* / AI: AL *L3*; IA *R* / AB: AH *FP1E* 286 a primo *om. R* / AH *corr. ex* AG *L3* / AT: AI *alter. in* AR *a. m. E* 287 AR *corr. ex* AI *E* 288 *post* HPA *add.* sit *R* / excedet *corr. ex* excedit *E* 289 cum: in *O*; *om. R* / lineam ductam (290): linea ducta *L3E* 290 puncto[1] *corr. ex* puncta *F* / P *om. E* / ducte *corr. ex* dute *F* 291 bis *om. L3* / bis . . . H *om. FP1* / puncto[2] *om. R* / medio: medium *R* 292 PB: PH *L3E* / *post* quadratum *scr. et del.* HP et quadratum *S* 293 HP: AP *O* / et[1] *inter. O* / et[1] . . . HP (294) *om. L3* / quadratum[2] *om. O* / AP[1]: HO *O* / AP[2] . . . multiplicatione (294) *mg. O* 294 excedit: excedet *S* / in[1]: cum *L3* / AB: APB *E* 296 AT . . . quadratum *mg. F* / quadratum: quadraturam *FP1* 297 AB *om. FP1* / AR: AT *S* / in . . . idem *inter. a. m. E* 298 *post* AB *scr. et del.* et *F* / duas: dua *S* / *post* AP *add.* et *R* 299 excessus *corr. ex* accessus *S* / *post* proportio *scr. et del.* AD *F* / ad excessum *mg. F* 300 AR: R *S* / *post* AR *scr. et del.* quare proportio excessus ad excessum sicut AP ad AR *E* / quare: erit ergo *R* / *post* proportio *add.* hoc sequitur quia dictum est quod proportio AH ad HT est sicut proportio *FP1* / quadrati *mg. a. m. E*; qua *L3* / super: supra *R* / *post* HP *add.* et *L3* / ad: ED *L3* 1 super: supra *R* / TR *corr. ex* HB *S* / AT[2]: HT *L3* 2 *post* HP *add.* sit *R* / HB: MB *L3E* / AH: HA *S* / super: supra *FP1R* 3 HB . . . quadratum *mg. C1* / super: supra *R* 5 sed . . . AT *om. S* / HU: HB *FP1OC1* / *post* multiplicatio *add.* EH in HD est equalis quadrato linee a puncto H ad circulum DBE contingenter ducte et erit minor HB et ita multiplicatio EH in HD minor est quadrato HB et fiat ductus *R* / equalis est *transp. L3E* / quadrato: quadrati *L3* / *post* HB *add.* ergo HU minor est HA et quadratum AH est equale multiplicationi AH in AU et HU *R* 6 AU: AHU *FP1* / AH[2]: HA *ER* / super: supra *R* 7 AH[1] *corr. ex* HA *E* / *post* sicut *add.* proportio *R* / AU: AB *L3* / ad[2] . . . AU[1] (10) *mg. C1* 8 super: supra *R* / TG: AG *FP1*; TQ *L3E* 9 erit . . . AU[1] (10) *om. FP1* / *post* sicut *add.* proportio *R* / multiplicationi: multiplicationis *R*

AU ad multiplicationem AT in AU. Igitur multiplicatio AT in AU est excessus quadrati AT super quadratum TG. Igitur erit multiplicatio AH in HU equalis quadrato HB, et multiplicatio AT in TU equalis quadrato TG.

[4.34] Amplius, arcus BG dividatur per equalia in puncto O, et ducantur tres perpendiculares super lineam HA, scilicet BF, OY, GK, et a puncto G ducatur linea equidistans HA, que sit GS, et a puncto B ducatur perpendicularis super AG, que sit BX. Hoc quidem BX, si produceretur usque ad circulum, divideret linea AG ipsam per equalia, et arcum cuius esset corda. Et ita secaretur alius arcus equalis arcui BG, quoniam alium arcum respiceret angulus GBX, et ita angulus GBX est medietas anguli supra centrum respicientis eundem arcum, secundum Euclidem. Igitur angulus GBX est medietas anguli BAG quem dividit linea OA per equalia. Igitur angulus GBX est equalis angulo OAG. Duo autem anguli BSG, BXG recti.

[4.35] Si intelligatur circulus super BG transiens per S, transibit per X, et fiet arcus SX super quem cadent duo anguli XBS, XGS. Igitur hii duo anguli sunt equales. Sed angulus GAY equalis angulo XGS propter equidistantiam linearum, et ita angulus GAY equalis angulo XBS. Et ut dictum est, angulus GBX equalis angulo OAG. Erit angulus OAY equalis angulo GBS, et erit triangulus OAY similis triangulo GBS. Igitur proportio GB ad BS sicut OA ad AY.

[4.36] Amplius, cum angulus AHB sit acutus, quadratum AB minuit ex quadratis AH, HB quantum est illud quod fit ex ductu AH in HF bis, secundum quod dicit Euclides. Igitur quadratum AH cum quadrato HB superat quadratum DA, que est equalis AB, in ductu

11 super: supra *R*/TG: DG *O*/igitur erit *transp. R* 12 AH: HA *L3ER*/HU: HB *S*/HB ... quadrato (13) *om. L3*/*post* et *add.* p *F* 14 *post* dividatur *scr. et del.* dimidiatur *E*/*post* O *add.* et ducatur AO *R* 16 linea *om. R* 17 *post* ducatur *scr. et del.* linea *P1*/sit *mg. F*/BX[1]: BG *SL3E*; BC *R*/BX[2]: LG *L3*; BG *E*; BC *R* 18 produceretur: producantur *FP1*; producatur *L3E*; producantur *alter. in* producatur *O*/divideret: dividet *FP1C1*; dividat *L3E*; dividat *alter. in* dividet *O*/AG: AH *P1*/ipsam: ipsum *L3* 19 cuius *inter. O*/*post* ita *scr. et del.* secavi *F* 20 BG *inter. O*/quoniam: quem *C1*/alium: illum *R*/GBX: GLG *L3*; GBG *E*; CBG *R* 21 GBX: GLG *L3*; GBG *E*; CBG *R*/supra: super *R* 22 Euclidem: eundem *FP1*; *corr. ex* eundem *E*; Euclid *O*/GBX: GBG *L3E*; CBG *R*/BAG: GAB *R* 23 quem: quoniam *L3*; *corr. ex* quoniam *E*/OA: EA *S*; IA *O*; GA *L3E*; AO *R*/*post* angulus *scr. et del.* angulo *E*/GBX: GBG *L3*; GBE *E*; CBG *R*/est *inter. a. m. E*/est equalis *transp. E* 24 BSG: BLG *FP1SL3E*/*post* BSG *add.* et *FP1*/BXG: BEG *E*; BCG *R*/*post* recti *add.* sunt *R* 25 *post* si *add.* igitur *R*/*post* per *add.* centrum *P1*/transibit: transiens *FP1* 26 X: Q *FP1*; E *E*; C *R*/fiet: fiat *FP1*/SX: Q *FP1*; SE *E*; SC *R*/cadent: cadunt *FP1*/XBS: XLS *L3*; EBS *E*; CBS *R*/XGS: EGS *E*; CGS *R* 27 hii *om. FP1*/equales: equale *L3*/*post* equalis *add.* est *OR* (*inter. O*)/XGS: EGS *E*; CGS *R* 28 propter ... et (29) *mg. a. m. E*/*post* GAY *add.* est *L3* 29 XBS: GS *FP1*; GBS *SO*; EBS *E*; CBS *R*/et *inter. C1*/GBX: GBQ *FP1*; GBE *E*; GBC *R*/equalis ... OAY (30) *om. P1*/*post* angulo *scr. et del.* T *L3* 30 triangulus: angulus *FP1*; triangulum *R*/similis: simile *R* 31 igitur *om. L3*/GB: BG *C1*/BS: US *L3E*/AY *corr. ex* TA *E*/*post* AY *add.* et proportio GB ad GS sicut OA ad OY *R* 32 *post* amplius *inter.* est *O*; *scr. et del.* est *C1*/angulus *inter. O*/quadratum: quam *FP1* 33 minuit ex: minus est *R*/HB: LB *FP1* 34 bis *inter. O*; hoc *L3*/quadratum: quantum *FP1* 35 superat: super AT *L3*/DA: DH *S*; AD *R*

AH in HF bis, et ita in ductu AH in HD bis et AH in DF bis. Sed multiplicatio AH in HD bis cum quadrato AD est equalis quadrato AH cum quadrato HD. Et ita ablato communi quadrato AB [cum multiplicatione AH in HD bis], restabit quadratum HD cum ductu AH in FD bis equalis quadrato HB.

[4.37] Sed multiplicatio AH in HT equalis est quadrato HD, et multiplicatio AH in HU equalis quadrato HB. Erit ergo multiplicatio AH in HU equalis multiplicationi AH in HT et multiplicationi AH in DF bis. Subtracto ductu AH in HT, quem communem ponimus utrique multiplicationi, restabit multiplicatio AH in TU equalis multiplicationi AH in DF bis. Igitur TU est dupla DF.

[4.38] Amplius, cum angulus ATG sit acutus, secundum predictum modum erit quadratum AT cum quadrato TG equale quadrato AD cum ductu AT in TK bis, et ita cum ductu AT in TD bis, et in DK bis. Et probatur modo predicto quod quadratum TG equale est quadrato TD cum ductu AT in DK bis. Sed ductus AT in TU equalis quadrato TG, et ita equalis quadrato TD cum ductu AT in DK bis.

[4.39] Sit ductus AT in TE equalis quadrato TD. Restat ergo ut ductus AT in EU sit equalis ductui AT in DK bis, per ablationem communis, quod est ductus AT in TE. Igitur EU est dupla DK. Sed iam dictum est quod TU est dupla DF. Restat ergo TE duplum FK.

[4.40] Amplius, proportio AH ad HT sicut AH ad HD duplicata, HD enim media est in proportione inter illas, cum eius quadratum sit equale ductui AH in HT. Et similiter proportio AT ad TE sicut AT ad TD duplicata. Sed maior est proportio AT ad TE quam AH ad HD. Igitur maior est proportio AT ad TE quam AH ad HT, et cum AH maius AT, erit HT maior TE. Sed TE dupla FK.

36 HD: HA *L3* 38 AB: AH *FP1OE;* AD *R*/multiplicatione: ductu *R* 39 restabit: restat *FP1*/AH: AB *S*/AH . . . multiplicatio (41) *mg. C1* 40 bis *inter. E*/equalis: equale *R*/HB: B *S* 41 sed . . . HB (42) *om. L3*/HT: HD *S*/equalis est *transp. S*/est *om. O* 42 *post* HU *inter.* est *O* 43 AH^{2} . . . multiplicationi^{2} *om. L3* 44 bis *corr. ex* bbis *P1*/*post* subtracto *add.* que *R*/quem *om. OL3;* quam *C1E* 45 restabit: restat *FP1*/equalis *scr. et del.* est *E* 47 *post* acutus *add.* erit *R*/predictum: supradictum *L3E* 48 erit *om. R*/equale: equalis *L3E; corr. ex* equales *C1* 49 TK . . . in^{2} *om. L3*/AT in TD: ACN TD *FP1*/in *om. S*/TD: DT *C1; corr. ex* T *O*/TD bis et *om. S*/DK: TK *L3* 50 et . . . bis (51) *om. S*/probatur: probabitur *L3C1ER*/equale est *transp. FP1* 51 cum *rep. C1*/AT^{1}: A *FP1*/DK: TK *L3*/sed: sit *FP1*/TU: TE *F;* TR *P1*/*post* equalis *add.* est *R* 52 TG: TD *FP1* 53 sit *corr. ex* si *O*/*post* sit *add.* autem *R*/TE: TO *SO* 54 EU: OU *FC1;* TU *P1;* GU *S; corr. ex* GU *O* 55 quod: qui *R*/TE: TO *FP1SC1; alter. ex* T *in* TO *O*/EU: IOU *FP1;* OU *SC1;* GB *O*/sed . . . dupla (56) *mg. a. m. E*/DK: KD *R* 56 DF: TF *L3E*/TE: TO *FP1SOC1*/duplum: dupla *R*/FK: FQ *FP1;* KF *R* 57 *post* HT *add.* est *L3R* 58 HD: HA *L3*/enim *om. FP1*/quadratum: quam *L3* 59 ductui *corr. ex* ductu *O*/TE: TO *FP1SOC1* 60 sed *inter. O*/*post* maior *add.* AT ad DO quam AH ad HD igitur maior *FP1*/TE: TO *FP1;* TD *SOL3C1ER* (*inter. O*)/HD *alter. in* HT *a. m. E* 61 igitur . . . HT *scr. et del. E; om. R*/TE: TO *FP1SOC1*/*post* quam *add.* AH ad HD igitur maior est proportio AT ad CO quam *FP1*/et: ergo *inter. O*/*post* AH *add.* sit *R* 62 maius: maior *R*/TE^{1,2}: TO *FP1SOC1*/sed TE *om. L3*/*post* dupla *add.* ad *R*/FK: KF *R*/*post* FK *add.* ergo HT maior est quam dupla ad KF *R*

[4.41] Item, ut dictum est, proportio BG ad GS sicut OA ad OY. Erit proportio BG ad OA sicut GS ad OY. Sed OA equalis BA, et GS equalis FK. propter equidistantiam. Erit proportio BG ad BA sicut FK ad OY.

[4.42] Amplius, IH minor medietate CH, et CH tripla HT. Erit IH minor HT et medio ipsius. Sed HT minor quinta HD. Igitur IH minor TD multo, quare IH multo minor ND, quare MI minor ND. Et palam per hoc quod I cadet inter H et Z.

[4.43] Amplius, quod fit ex ductu EZ in ZD non est maius quadrato AD; igitur quod fit ex ductu EM in MD est minus quadrato AD. Sed quoniam MG est contingens, quod fit ex ductu EM in MD est equale quadrato MG, secundum quod dicit Euclides. Igitur MG est minor AD; igitur MG est minor AG.

[4.44] Amplius, duo trianguli AGM, MGK habent unum angulum communem, et uterque eorum habet angulum rectum. Igitur sunt similes, quare proportio MK ad KG sicut MG ad GA, et ita MK minor est KG. Et cum OY sit maior GK, erit HD minor OY.

[4.45] Amplius, AH ad HD sicut HD ad HT; erit ergo sicut medietas HD ad medietatem HT. Et ita AH ad HD sicut QH ad medietatem HT, cum QH sit medietas HD, et ita AH ad QH sicut HD ad medietatem HT, et ita QH ad AH sicut medietas HT ad HD. Sed medietas HT maior FK, et HD minor OY. Erit igitur medietas HT ad HD maior quam FK ad OY, quare erit QH ad AH maior quam FK ad OY.

[4.46] Amplius, linea AQ secat circulum EBD. Sit punctus sectionis Q, et ducatur linea DQ, que erit equidistans QH. Eritque proportio QH ad HA sicut QD ad DA, et ita QD ad DA maior quam FK ad

63 ut dictum *om. L3* / *post* proportio *scr. et del.* proporti *S* / *post* sicut *scr. et del.* CK *FP1* / OA: CA *FP1S*; EA *L3C1E* 64 proportio *om. R* / OA: GA *FP1*; GOA *S* / OA: CA *FP1S*; EA *L3E* 65 FK: SK *L3* 67 *post* amplius *add.* quia *R* / minor *om. FP1* / *post* minor *add.* est *R* / CH^{1,2}: OH *R* / et CH *om. FP1* / HT: HTD *FP1*; TH *R* 68 medio: medietate *R* / *post* quinta *add.* parte *R* 69 minor^{1}. . . IH *rep. S* / *post* minor^{1} *add.* est *R* / TD: ID *L3E* / quare IH multo *mg. a. m. E* / multo minor *transp. deinde corr. C1* / MI *scr. et del. E* / *post* MI *add.* multo *R* 70 per: propter *O* / hoc: hec *S* / cadet: cadit *R* / Z: S *L3E* 71 EZ: Z *FP1*; ES *L3E* / ZD: SD *L3E* / non est maius: est equale *R* 72 AD igitur *inter. O* / igitur . . . AD^{2} *mg. a. m. E* / minus: maius *FP1* 73 sed *om. FP1* / *post* MG *add.* circulum DBE *R* / contingens: continens *L3E*; contingit *R* / contingens . . . est (75) *mg. a. m. O* / EM: AM *FP1SOL3E* 74 quadrato *inter. E* / secundum: scilicet *S* 75 igitur: et ita *O* / MG *om. R* / est minor *transp. R* / AG: AS *L3* 76 duo *om. R* / trianguli: triangula *R* / AGM *corr. ex* AMGM *L3* 77 uterque: utrunque *R* / habet: habent *S* / *post* habet *add.* unum *L3C1ER* 78 similes: similia *R* / minor est (79) *transp. L3* 79 HD: HG *L3* 80 *post* amplius *add.* quia *R* / ergo *om. R* / *post* ergo *add.* AH ad HO *FP1*; *inter.* AH ad HD *O* / sicut: sic *R* / *post* sicut *add.* HD ad HT *L3* 81 *post* QH *scr. et del.* ad *E* 82 cum . . . HT^{1} (83) *mg. a. m. E* / HD^{1}: HT *S* / AH: HA *L3* / HD^{2}: HT *L3* 83 *post* HD *add.* sed medietas HT ad HD *P1* 84 *post* maior^{1} *add.* est *R* / *post* et *add.* ita *P1* / HD^{1}: HB *FP1* / OY: COY *FP1*; AY *O* / *post* igitur *add.* proportio *R* / *post* HT *scr. et del.* maior FK *F* 85 *post* erit *add.* proportio *R* 86 AQ: AK *FP1* / punctus: punctum *R* 87 Q: quod *FP1*; que *O*; X *SC1*; Œ *R* / DQ: DX *SC1*; DŒ *R* / que erit *transp. deinde corr. S* / eritque: erit OS *FP1*; erit et *L3* 88 QD^{1}: ZD *FP1*; XD *SC1*; Œ *R* / et . . . DA^{2} *om. FP1* / *post* ita *add.* proportio *R* / QD^{2}: XD *SC1*; Œ *R*

OY. Sed FK ad OY sicut GB ad BA. Erit igitur maior QD ad DA quam BG ad BA, et ita QD maior BG, et arcus QD maior arcu GB.

[4.47] Amplius, producatur AQ usque ad punctum S ut sit AS equalis AI, et ducatur linea SI, que erit equidistans QH, et erit proportio SI ad QH sicut IA ad AH. Sed supra positum est quod IA ad HA sicut TQ ad QH; erit igitur SI equalis TQ.

[4.48] Amplius, mutetur figura ad evitandam linearum intricationem multiplicem et propter defectum litterarum ad distinctionem nominum. Cum ergo IA sit equalis linee quam diximus AS, fiat circulus secundum quantitatem ipsarum [FIGURE 6.4.3b, p. 306]. Et loco S ponamus nomen N, et producantur AG, AB usque ad hunc circulum, et sint ABC, AGR; loco littere Q ponamus F. Dictum est quoniam arcus DF maior est arcu BG. Sit arcus BM equalis arcui DF, et ducatur linea AMU et linee IM, NM, et linea QM, que producatur usque ad exteriorem circulum. Et cadat in punctum Z, et ducantur linee ZA, ZG.

[4.49] Cum arcus BM sit equalis arcui DF, addito communi, erit arcus MF equalis arcui DB. Erit angulus NAM equalis angulo IAB, et latera lateribus equalia; erit MN equalis IB. Et cum positum sit supra, AQ equale AH, erunt AQ, AM equalia HA, AB, et angulus angulo. Erit QM equalis HB, et erit angulus QMN equalis angulo HBI, quoniam duo eius latera duobus illius equalia. Et basis que est IH equalis basi NQ, et angulus NMU equalis angulo IBC.

[4.50] Sed angulus IBC equalis angulo HBA, et angulus HBA equalis angulo QMA; erit angulus NMU equalis angulo QMA. Et quoniam QMZ est linea recta, ut posuimus, erit angulus QMA equa-

89 *post* maior *add.* proportio *R*/QD: quod D *F;* XD *SC1;* QZD *L3E;* ŒD *R*/DA *corr. ex* ADA *S* 90 BG[1]: GB *C1*/QD[1]: quod D *FP1;* XD *SC1;* ZQD *L3E;* ŒD *R*/*ante* maior[1] *scr. et del.* ad *S*/*post* maior[1] *add.* est *O*/BG[2]: G *FP1*/QD[2]: OD *FP1;* XD *SC1;* ZQD *L3E;* ŒD *R* 92 *post* equalis *add.* arcui *P1*/AI: AR *FP1*/proportio *om. R* 93 AH: HA *R*/positum: dictum *FP1* 94 HA: AH *SOC1ER* (*alter. in E*)/igitur *inter. E* 95 mutetur: mittetur *S*/evitandam *corr. ex* vitandam *E*/intricationem: intritionem *S* 96 et *inter. C1*/distinctionem: instinctionem *O* 97 nominum: linearum *R* 99 S: scilicet *L3*/ponamus: ponatur *R*/nomen: litera *R*/hunc circulum (100) *transp. R* 100 AGR: AGK *L3E*/loco: loct *E* /Q: quod *FP1S;* que *O;* quam *L3E;* X *C1;* Œ *R*/dictum: deinde *L3E* 101 quoniam: quod *R*/arcus *om. FP1*/sit *corr. ex* sicut *C1* 102 *post* linee *add.* IB IG *R*/IM: YM *S*/producatur *corr. ex* producantur *O* 103 Z: S *L3E* 105 *post* cum *add.* autem *R*/arcus: arcu *FP1*/communi: quoniam *FP1* 106 erit: eritque *R*/IAB: RAB *FP1* 107 latera: latria *S*/*ante* lateribus *add.* et *S*/equalis *om. FP1*/*post* IB *add.* et angulus NMA equalis angulo IBA et angulus NMU angulo IBC *R*/positum: positus *L3* 108 AQ[1]: AR *P1*/*post* AM *add.* latera *R* 109 angulus *inter. a. m. E*/QMN: QMA *R*/ HBI: HBA *R*/*post* HBI *add.* et QMN equalis angulo HBI *R* 110 que: qui *L3*/IH: QN *R*/*ante* equalis *add.* est *R* 111 NQ: HI *R*/NMU: MNR *FP1*/IBC: TBC *FP1*/IBC sed angulus (112) *om. SL3E* 112 sed: et *R*/angulus[1] *om. R*/angulo *om. L3* 113 QMA[1]: quia *S*/*post* QMA[1] *add.* et quoniam QMS est linea recta ut posuimus *mg. a. m. E*/erit: ergo *R*/angulus *om. R*/erit . . . QMA[2] *inter. O*/NMU: MNB *P1;* MNS *E*/angulo[2] *om. R* 114 quoniam *om. O*/*post* quoniam *add.* ut posuimus *R*/QMZ: QMS *L3E*/*post* recta *scr. et del.* QMA *C1*/ut *inter. O*/ut posuimus *om. R*

lis angulo UMZ, quare punctus N refertur ad Z a puncto M, et locus ymaginis ipsius Q. Hoc tamen deest probationi ut pateat MZ totam esse extra circulum, quod sic patebit.

[4.51] Palam quoniam contingens ducta a puncto B cadet inter I et H, et remotio puncti B a puncto H quanta est puncti M a puncto Q, et IH equalis NQ. Igitur contingens ducta a puncto M cadet inter N et Q. Igitur QM secat circulum, quare tota MZ extra circulum, et ita propositum.

[4.52] Amplius, quoniam angulus NMU equalis angulo UMZ, erit arcus NU equalis arcui UZ. Erit angulus NAU equalis angulo UAZ. Sed iam patuit quod angulus NAU equalis est angulo IAC. Erit angulus IAC equalis angulo ZAU.

[4.53] Angulus BAG aut erit equalis angulo GAM, aut minor, aut maior. Sit equalis. Si igitur ab angulo IAB subtrahatur angulus BAG, et ab angulo ZAU angulus MAG, remanebit angulus IAG equalis angulo ZAG. Erit IG equalis ZG, et triangulus triangulo, et erit angulus IGA equalis angulo ZGA. Restabit angulus IGR equalis angulo ZGR. Sed angulus IGR equalis angulo TGA. Erit angulus TGA equalis angulo ZGR. Si igitur TG producatur, veniet ad Z, quare TGZ linea recta. Igitur I a puncto G refertur ad Z, et locus ymaginis eius T.

[4.54] Sit ergo Z visus. Reflectentur ad ipsum duo puncta N, I a duobus punctis M, G, et loca ymaginum puncta T, Q. Igitur linea TQ erit ymago linee IN, et probatum est supra quoniam TQ equalis est IN, et ita potest accidere in hiis speculis ymaginem esse equalem rei vise.

[4.55] Si vero angulus BAG fuerit maior angulo GAM, erit angulus ZAG maior angulo IAG. Sit angulus KAG equalis angulo IAG.

115 UMZ: MZ *FP1*; MNG *L3*; NMS *E*/punctus: punctum *R*/refertur: reflectitur *R*/Z: S *L3E* 116 ipsius *om. P1*/MZ: MS *L3*; *alter. ex* MG *in* MS *E* 117 extra *inter. E* 118 palam: pam *P1*/quoniam: quod *R*/B: H *L3*/cadet: cadat *R*/I: L *E* 119 *post* et[2] *add.* tanta est *R*/remotio: remoto *L3*/Q: quod *FP1* 120 IH *corr. ex* H *P1*/NQ: NH *L3*/cadet *om. FP1* 121 MZ: MC *L3*; MS *E*/*post* MZ *add.* est *R*/et *om. FP1*/et ita propositum (122) *om. R* 123 *post* angulus *scr. et del.* minor *C1*/*post* equalis *add.* est *R*/UMZ: UMS *L3*; NMS *E* 124 *post* arcui *scr. et del.* UR *F*/UZ: AS *L3*; NS *E*/*post* UZ *add.* et *R*/UAZ: NAZ *FP1S* ; NAS *L3E* 125 patuit: placuit *L3*/equalis *om. F*/equalis est *transp. P1*/est *inter. E*/angulo IAC *transp. C1*/erit: igitur *R*/angulus[2] *om. R*/angulus IAC (126) *om. L3E* 126 *post* IAC *add.* erit *R*/ZAU: SAU *L3E*; UAZ *R* 127 *ante* angulus *add.* amplius *C1*/angulus: amplius *L3E*/*post* angulus *add.* vero *R*/aut maior (128) *om. P1* 128 ab: BA *L3E*/IAB *corr. ex* IBA *a. m. E*; IAC *R* 130 ZAG: SAG *L3E*/*post* ZAG *add.* et *R*/*post* IG *add.* equalis angulo ZAG erit IG *FP1*/ZG: SG *L3E*/et[1] *inter. a. m. C1*/triangulus: triangulum *R* 131 ZGA: SAG *L3E*; ZAG *R*/*post* restabit *add.* igitur *R*/IGR: IGT *C1*; *alter. ex* AGT *in* IGT *S*/angulo[2] . . . equalis[1] (132) *om. P1*/ZGR: SGI *L3E* 132 sed: fiat igitur *R*/angulus[1]: angulo R/angulus IGR: equalis EGR *F*/angulo[1]: angulus *R*; *om. L3*/TGA[1]: IGA *P1*; *corr. ex* TAG *E*/*post* TGA[1] *add.* ut patuit in precedenti figura longe ante *FP1*/TGA[2] *corr. ex* TAG *E* 133 ZGR: SGR *L3E*/Z: S *L3E*/TGZ: TGS *L3E* 134 refertur: reflectitur *R*/Z: S *L3E*/*post* eius *add.* est punctum *R* 135 *ante* sit *add.* si *R*/sit ergo Z: ergo Z sit *R*/ipsum *mg. F*/*post* ipsum *inter.* visum *a. m. C1*/N I *transp. R* 136 igitur linea TQ *om. FP1* 137 *post* linee *scr. et del.* e *C1*/et *om. R*/probatum: propriatum *L3E*/*post* probatum *add.* autem *R*/quoniam: quod *R*/TQ: RQ *C1* 139 GAM: GABA *FP1* 140 sit: si *S*/IAG: VAG *FP1*

Quoniam punctus K dimissior puncto Z, et punctus M dimissior G, linea KG secabit lineam ZM. Secet in puncto L, Igitur, existente visu in puncto L, refertur N ad ipsum a puncto M, et locus ymaginis Q; refertur ad ipsum I a puncto G, et locus ymaginis T, secundum priorem probationem. Et ita TQ ymago IN, quod est propositum.

[4.56] Si vero angulus BAG fuerit minor angulo GAM, erit angulus ZAG minor angulo IAG. Sit angulus OAG equalis angulo IAG, et producatur linea OG. Palam quoniam I refertur ad O a puncto G. Linea OG aut secabit lineam ZMQ extra circulum speculi aut non.

[4.57] Si secet extra, et in puncto sectionis fuerit visus, refertur ad ipsum duo puncta I, N, et loca ymaginum T, Q, et ita redit propositum.

[4.58] Si forsan linea OG secabit lineam ZMQ intra circulum, nec poterit aptari predicta probatio. Sed dico quoniam extra hanc totalem superficiem erit invenire punctum ad quod refertur duo puncta I, N a duobus punctis speculi, et ymago TQ.

[4.59] Verbi gratia, palam quoniam angulus NAZ duplus ad angulum IAB, et angulus IAO duplus ad angulum IAG, secundum predicta. Et angulus NAZ non excedit angulum IAO in angulo maiori angulo NAI. Et duo anguli OAI, ZAN maiores tertio, quod est IAN, et duo OAI, IAN maiores tertio NAZ, et duo ZAN, NAI maiores tertio IAO. Habemus ergo tres angulos, quorum quilibet duo maiores tertio.

[4.60] Igitur ex illis est facere angulum corporalem. Fiat angulus ille super A, et sit linea SA erecta super A, et angulus IAS sit equa-

141 punctus[1, 2]: punctum *R*/*post* K *inter.* est *O*/dimissior: demissius *R*/puncto Z: punctorum *FP1SL3E*/M: ZM *SL3E*/*ante* G *add.* puncto *R* 142 *post* linea *add.* G linea *F*/*post* secabit *scr. et del.* KG secabit *C1*/lineam *om. C1*/*post* L *scr. et del.* refertur *F* 143 refertur: reflectetur *L3ER*/a *om. L3*/Q . . . ymaginis (144) *inter. E*/*post* Q *add.* similiter *R* 144 refertur: reflectetur *E*/refertur . . . I: I reflectetur ad ipsum *R*/*post* ipsum *add.* vel refertur *E*/a puncto G *om. R*/ymaginis: ymaginum *S*/*post* ymaginis *add.* est *R*/T *om. P1* 145 probationem: proportionem *OL3E*/*post* TQ *inter.* est *O*/*post* ymago *add.* est *R* 146 angulus BAG *transp. FP1* 147 IAG[1]: YAG *S*/*post* IAG *scr. et del.* et ita *F*; *add.* et *P1*/sit *mg. F*/OAG: SAG *FP1*; EAG *SL3E*/*ante* equalis *scr. et del.* equalis *E* 148 producatur: ducatur *R*/OG: CG *FP1*; LG *L3*; TG *E*/quoniam: quod *R*/refertur: reflectitur *R*/*post* O *scr. et del.* a puncto *E* 149 OG: EG *FP1*; TG *L3E*/ZMQ: ZMG *P1S* 150 in . . . visus: visus fuerit in puncto sectionis *R*/refertur: reflectentur *R*; *corr. ex* refen *O*; *alter. in* referuntur *E* 151 I N *transp. R*/*post* ymaginum *add.* erunt *R*/*post* redit *scr. et del.* ad *E* 153 si: sed *OC1*/secabit: secet *R*/ZMQ: ZMG *FP1*/intra: inter *L3*/nec: nichil *L3E*; non *R* 154 aptari: applicari *R*/quoniam: quod *R*/hanc: N *FP1*/*post* totalem *scr. et del.* circulum *O* 155 erit: licebit *R*/ad quod *inter. a. m. E*; a quo *O*/refertur: referuntur *L3C1E*; reflectantur *R* 156 punctis *om. L3E*/punctis speculis *transp. R*/speculi: speculis *E*/*post* ymago *add.* erit *R* 157 quoniam: quod *R*/*post* duplus *add.* est *OR* (*inter. O*) 158 IAB: CAB *R*/IAO: NLO *FP1*/IAG: YAG *S* 159 angulo: alico *O*/maiori: maiore *R* 160 *post* angulo *inter.* angulo *O*/NAI: NAR *P1*/OAI . . . duo[1] (161) *om. R*/ZAN: IAN *E*/quod: qui *O*/quod . . . tertio[1] (161) *mg. a. m. E*/est *om. FP1* 161 duo[1]: dico *S*/*post* duo[1] *inter.* anguli *a. m. C1*/OAI: IAO *R*/IAN: ZAN *C1*; IAM *P1*; *corr. ex* IAM *F*/*post* tertio *add.* qui est *R*/NAZ . . . IAO (162) *om. P1*/ZAN *corr. ex* NAZ *E* 162 IAO *om. F*/*post* IAO *add.* et duo NAZ IAO maiores tertio NAI *R*/*post* maiores *add.* sunt *R* 163 *post* tertio *add.* et omnes simul quattuor rectis minores *R* 164 est: licet *R*/fiat *corr. ex* fiet *a. m. C1* 165 SA erecta: SAE recta *FP1*

lis angulo IAO, angulus NAS equalis angulo NAZ. Angulus NAI manebit immotus, et fiet linea AS equalis lineis AN, AI, que omnes sunt equales.

[4.61] Et producantur linee TS, QS. Palam quoniam angulus TAS equalis angulo TAO, et duo latera duobus lateribus. Erit basis TS equalis basi TO, et triangulus triangulo, et ita angulus GTA equalis angulo STA. Similiter, angulus QAS equalis angulo QAZ, et latera lateribus. Erit triangulus equalis triangulo, et angulus MQA equalis angulo SQA.

[4.62] Dividatur angulus TAS per equalia per lineam AY; sit Y punctus in quo linea illa secabit lineam TS. Palam, cum angulus IAG sit medietas anguli IAO, erit angulus TAG equalis angulo TAY, et angulus GTA equalis angulo YTA, et unum latus commune, scilicet TA. Erit TG equalis TY, et triangulus triangulo, et erit AY equalis AG, et ita Y in superficie spere. Igitur angulus IAG equalis angulo IAY, et latera lateribus. Erit triangulus triangulo equalis, et erit AGI equalis angulo AYI. Linea IY producta erit equalis IG.

[4.63] Et producatur AY extra speram usque ad punctum P. Restabit angulus IGR equalis angulo IYP. Verum, cum TS sit equalis TO, et TY equalis TG, restat GO equalis YS. Igitur AY, YS equalia AG, GO, et basis AS equalis basi AO. Erit triangulus equalis triangulo; erit angulus AYS equalis angulo AGO. Restat angulus SYP equalis angulo OGR. Igitur duo anguli IGR, OGR equales sunt duobus angulis IYP, SYP.

[4.64] Verum linea AS secat speram. Sit punctus sectionis O. Igitur tria puncta O, Y, D sunt in superficie spere, quare linea OYD est pars circuli spere, et est linea communis superficiei spere et superficiei ITASP, quare punctus I refertur ad punctum S a puncto Y, et locus ymaginis T.

166 IAO . . . angulo^2 *mg. F*/*post* IAO *add.* et *ER*/angulus^2: angulo *S* 167 fiet: fiat *R*/ lineis: linee *R*/*post* AN *add.* vel *R* 169 producantur: producuntur *L3*/*post* TAS *add.* est *R* 170 duobus lateribus *transp. R*/erit: ER *FP1* 171 TO: TOM *FP1*/triangulus: triangulum *R*/GTA *corr. ex* GAT *E* 172 QAZ: NAZ *L3* 173 erit: et *R*/triangulus equalis: triangulum equale *R*/MQA *alter. in* MAQ *a. m. E* 175 AY: AI *P1O*/*post* AY *add.* et *FP1* 176 punctus: punctum *R*/illa *om. FP1* 178 angulo *om. R* 179 TG *corr. ex* CG *O*/TY . . . equalis^2 *mg. O*/triangulus: triangulum *R* 180 Y *inter. E*/spere: speculi *R*/igitur: erit etiam *R* 181 latera lateribus *transp. deinde corr. S*/erit: et *R*/ triangulus: triangulum IAG *R*/*post* triangulo *scr. et del.* equlis *P1*/equalis: equale *R*/*post* erit^2 *add.* angulus *R* 182 *post* AYI *add.* et *R*/IY: AY *L3*; TY *E*/erit *om. ER* 185 *post* TG *scr. et del.* reg *L3*/equalia: equalis *FP1*/AG GO: AGO *L3* 186 AO *corr. ex* A *O*/triangulus equalis: triangulum equale *R*/equalis triangulo *transp. FP1*/*post* triangulo *add.* et *C1R* (*inter. C1*)/angulus *om. P1* 187 SYP: SIP *S*; *corr. ex* SIP *O*/OGR: EGR *L3E* 188 anguli *inter. O*/sunt *om. C1* 189 secat: secabit *ER*/sit: si *S*/*ante* punctus *scr. et del.* punctus *F*/punctus: punctum *R*/O *inter. O*; E *ER* 190 puncta: punctu *O*/O: E *R*; *inter. O*/Y: I *L3*/quare: quia *FP1*/OYD: O *FP1*; EYD *R*; *corr. ex* YD *O*/est *inter. O* 191 superficiei^2: speciei *S* 192 *ante* ITASP *add.* reflexionis *R*/ITASP: REASP *F*; TSP *R*; RASP *O*; *alter. ex* RESP *in* REASP *P1*/punctus *mg. F*; punctum *R*/refertur: reflectitur *R* 193 *post* ymaginis *add.* est *R*

[4.65] Similiter, diviso angulo NAS per equalia per AZZ, probabitur predicto modo quoniam QZ equalis QM, et AZ equalis AM, et ZS equalis MZ, et duo anguli NZZ et SZZ equales duobus angulis NMU, ZMU. Et ita N refertur ad S a puncto Z, et locus ymaginis Q, et ita TQ ymago IN, quod est propositum.

[4.66] Amplius, si a puncto I ducatur perpendicularis super NA, cadat inter N et Q, non extra N, cum angulus INA acutus, quoniam equalis angulo NIA, et si caderet perpendicularis illa extra N, esset acutus maior recto. Faciet ergo perpendicularis illa angulum rectum super NQ, quem angulum respiciet linea IN, quare linea IN maior illa perpendiculari, quare perpendicularis illa minor TQ.

[4.67] Punctus linee NQ in quem cadit perpendicularis reflectitur ad punctum S, et ymago eius cadet in linea NA supra punctum Q, quia quanto remotiora sunt puncta que reflectuntur tanto loca ymaginum magis accedunt ad centrum circuli, ex decima quinti huius.

[4.68] Et quecumque linea ducatur a puncto T ad aliquod punctum NQ supra Q erit maior TQ. Igitur ymago perpendicularis erit maior ipsa perpendiculari. Eodem modo, quecumque linea ducatur a puncto I ad NQ inter hanc perpendicularem et IN, erit ymago ipsius maior ipsa.

[4.69] Verum determinentur hec certius. Punctus N refertur ad Z a puncto M, et locus ymaginis Q. Linea QM secat circulum in puncto quod sit E. Contingens ergo ducta a puncto Z ad circulum cadet super punctum aliquod arcus ME, et contingens illa cadet supra Q, quoniam punctus in quem cadet erit finis contingentie et finis ymagi-

194 per[1] *inter. O* / AZZ: AZ *FP1*; AZI Sl ATZ *L3E*; AX *R* / probabitur: probatur *O* 195 predicto modo *transp. L3ER* / quoniam: quod *R* / QZ: QX *R* / *post* QZ *add.* est *OL3* / equalis[1] *mg. a. m. C1* / *post* equalis[1] *add.* est *ER* / AZ: AX *R* / ZS: ZZS *FP1*; XS *R* 196 MZ *corr. ex* MZZ *FP1* / *post* anguli *scr. et del.* MZSS *L3* / NZZ: MZZ *L3E*; NXÆ *R* / et SZZ *mg. F* / SZZ: SXÆ *R* / equales: equalis *FSL3C1E* / NMU *corr. ex* MUN *E* 197 *ante* ZMU *add.* quam *FP1E* / ZMU: CZMU *FP1*; sed MU *L3*; *mg. a. m. E* / ita *om. L3* / refertur: reflectetur *R* / S: SZ *FP1* / Z: X *R* 198 IN: YZ *FP1*; YN *SOL3C1E* 200 cadat: cadet *OER* (*alter. in OE*) / *post* INA *add.* sit *OR* (*inter. O*) 201 N: M *FP1* 202 illa angulum *transp. deinde corr. C1* 203 quem: quoniam *FP1OE* / quem . . . IN[1] *om. L3* / angulum *corr. ex* angulus *OC1* / respiciet: respicit *R* / quare linea IN *om. S* / *post* maior *add.* est *R* / *post* illa *add.* est *mg. a. m. E* / illa *mg. a. m. E* 204 *ante* perpendiculari *scr. et del.* ri *E* / perpendiculari quare *mg. F* / perpendicularis: perpendiculari *P1* 205 punctus: punctum *R* / *post* punctus *add.* reflexionis *P1*; *add.* igitur *R* / linee *om. SOL3C1E* / quem: quod *R* 206 S: ? *L3* / et *om. L3ER* / *post* ymago *add.* vero *R* 207 quanto: quantum *FP1* 208 magis *corr. ex* MG *O* / ex . . . huius *om. R* / decima: tertia decima *O* / quinti: quarti *FP1*; quadrati *L3* / huius *om. SO* 209 ducatur: ducetur *R* 210 Q *inter. E* / TQ . . . maior (211) *rep. S* 211 ducatur: ducetur *R* 212 ipsius . . . ipsa (213) *corr. ex* maior ipsa ipsius *C1* 214 hec certius: huius tertius *FP1* / punctus: punctum *R* / *post* N *add.* quia *R* / refertur: reflectitur *R* / *post* ad *scr. et del.* C *F* / Z: S *L3E* 215 a . . . Z (216) *mg. O* / *post* ymaginis *add.* est *R* / QM: que *L3E*; ZMQ *R* 216 sit: est *R* / E: 3 *R* / *ante* ergo *scr. et del.* ergo *F* / ergo *om. P1* 217 ME: MZ *R* / *post* ME *add.* si vero caderet in punctum 3 secaret peripheriam non tangeret cadit igitur in peripheriam M3 *R* / contingens: communis *L3E*; continens *FP1SO* 218 punctus: punctum *R* / quem: quod *R* / cadet: cadit *ER* / finis contingentie et *om. FP1* / ymaginum: ymaginis *C1*

num, et puncta sub puncto illo qui est finis contingentie non poterunt reflecti; superiora poterunt.

[4.70] Igitur perpendicularis ducta a puncto I, si ceciderit supra punctum qui est finis contingentie, refertur punctus in quem cadit, et erit ymago perpendicularis maior perpendiculari. Si vero perpendicularis cadat in punctum contingentie aut infra, non refertur punctus in quem cadit, quare nulla erit ymago perpendicularis. Verumptamen, quoniam finis contingentie est infra N, erunt inter finem contingentie et N infinita puncta quorum quodlibet reflectetur, et ymago cuiuslibet supra NQ. Et cuiuslibet linee ducte a puncto I ad aliquod illorum punctorum erit ymago maior linea cuius fuerit ymago.

[4.71] Igitur accidit in hiis speculis ymaginem aliquando equalem rei vise, aliquando maiorem, quod erat explanandum. Huius autem rei explanationem nec scriptam legimus nec aliquem qui dixisset aut intellexisset audivimus

[4.72] Amplius, in hiis speculis linee recte videntur curve, ut in pluribus curvitate quidem speculum non respiciente sed ei adversa. Similiter, curve apparebunt in hiis speculis curve, et si curvitas speculum respexerit, contrario situ apparebit, et hoc quidem intelligendum non in omnibus sed in pluribus, ad cuius rei explanationem necesse est quedam antecedentia premittere, unum quorum est:

[4.73] **[PROPOSITIO 4]** Si fuerint duo puncta eiusdem longitudinis a centro speculi et inequalis longitudinis a centro visus, ymago puncti remotioris a centro visus erit remotior a centro spere quam propinquioris, et finis contingentie remotioris remotior a centro fine contingentie propinquioris, sive puncta illa sint in eadem superficie cum centro visus, sive in diversis.

219 qui: quod *R* / *post* contingentie *scr. et del.* refertur punctus in quem cadit *S* 221 I *corr. ex* Q *E; om. FP1* / *post* I *add.* super NQ *R* / ceciderit *mg. C1* 222 qui: quod *R* / *post* refertur *scr. et del.* igitur *C1* / refertur: reflectetur *R* / punctus in quem: punctum in quod *R* 224 aut infra *inter. O* / aut . . . est (226) *om. S* / refertur: reflectetur *R* / punctus: punctum *R* 225 erit *inter. a. m. C1* / perpendicularis *inter. O* 226 est *om. L3E; inter. O* / infra: intra *FP1SOC1;* inter *L3E* / erunt *corr. ex* erit *O* / inter *corr. ex* intra *S* 227 N *om. FP1; alter. in* MF *O* / infinita: infra *FP1SOL3* (*inter. O*); ? *E* / quodlibet: quolibet *S* / *post* quodlibet *scr. et del.* puncta quorum *S* / *post* et^2 *add.* erit *R* 228 supra: super *R* / NQ: PQ *C1; corr. ex* N *E* / aliquod: quodlibet *ER* 229 fuerit: fuit *C1* 231 *post* maiorem *add.* esse *R* 232 rei *corr. ex* re *O* / nec: non *FP1* / aut intellexisset (233) *om. FP1* 234 linee recte videntur *om. S* / ut: et *R* 236 similiter curve *transp. O* / curve *inter. O* / speculum: speculis *L3E* 237 apparebit: apparebunt *S* 238 *post* cuius *scr. et del.* e *S* 239 antecedentia *corr. ex* ante *O;* accidentia *S;* acerva *L3E* / unum: unus *FP1* / unum quorum *transp. FP1R* 240 *post* si *scr. et del.* vero *C1* 241 *post* longitudinis *scr. et del.* a centro speculi et inequalis longitudinis *F* 242 spere: speculi *C1* 243 remotioris: remotionis *L3* / *post* remotioris *add.* erit *R* / remotior *om. P1* / centro fine *transp. L3* / *post* centro *add.* speculi quam *R* / fine: finem *S;* finis *R* 244 contingentie: continente *P1O;* contingente *alter. in* continente *F;* continen *S* / eadem *corr. ex* eodem *L3* 245 *post* cum *add.* a *FP1*

[4.74] Probatio: Sint T, D [FIGURE 6.4.4, p. 307] duo puncta equaliter a G centro speculi remota, E centrum visus. Superficies DGT secabit speculum super circulum qui sit AB. Et sit angulus EGD equalis angulo TGZ, angulus EGT equalis angulo TGH, et sumatur in circulo punctum a quo T refertur ad Z, quod sit Q. Dico quoniam T non refertur ad H ab aliquo puncto BQ.

[4.75] Palam enim quoniam non a puncto Q. Si autem sumatur punctum quodcumque in BQ, linea ducta a puncto H ad illud punctum secabit lineam QZ. Igitur ad illud punctum sectionis refertur T a puncto sumpto in BQ, et ad idem punctum sectionis refertur a puncto Q. Igitur T refertur ad idem punctum a duobus punctis illius circuli, quod est impossibile in hiis speculis, ut in libro quinto patuit.

[4.76] Restat ut T reflectatur ad H ab aliquo puncto QA. Sit illud M, et a puncto M ducatur contingens circulum usque ad lineam GT, que sit MN. Erit N finis contingentie T respectu H.

[4.77] Et a puncto Q ducatur contingens que sit QO, que quidem necessario cadet sub MN. Producatur ZQ usque dum cadat super GT in puncto C. Erit C locus ymaginis Z. Erit igitur proportio GT ad TO sicut GC ad CO. Igitur maior erit proportio GT ad TN quam GT ad TO. Ergo multo maior GT ad TN quam GC ad CN. Sit ergo GT ad TN sicut GL ad LN. Erit GL maior GC, et L locus ymaginis H.

[4.78] Sint ergo linee HG, EG, ZG equales, GF equalis GC, GS equalis GO. Cum igitur angulus EGD sit equalis angulo TGZ, et remotio D a puncto E sicut Z a puncto T, erit ymago D respectu G tantum elevata in linea GD quantum ymago T in linea GT. Igitur ymago D in puncto F. Et similiter, finis contingentie D respectu E erit eius-

246 duo puncta *transp. C1* 247 equaliter: equalia *L3* / a G *inter. a. m. E* / *post* visus *add.* et D propinquius visui quam T *R* / *post* superficies *add.* communis sectionis *R* 248 DGT: DTG *FR;* DG *P1* / super circulum *om. P1* 249 angulus . . . TGH *mg. O* 250 refertur: reflectatur *R* / *post* refertur *scr. et del.* ad *F* / Z: S *L3E* / quoniam: quod *R* 251 refertur: reflectitur *R* / aliquo: alio *FP1SO;* a *L3E; corr. ex* alio *C1* / BQ: HQ *L3* 252 enim *om. FP1R; inter. E* / quoniam: quod *R* / non *om. FP1* 253 *post* ad *scr. et del.* al *S*/illud: illum *FP1O* 254 QZ: quia *FP1;* QS *L3E* / illud *corr. ex* aliud *P1* / refertur: reflectitur *R* / a: ab *R* 255 *ante* puncto[1] *add.* aliquo *R* / BQ: HG *S* / punctum sectionis *transp. R* / refertur: reflectitur *R* / a puncto *rep. F* 256 refertur: reflectitur *R* / punctis *corr. ex* punctum *C1* 257 est *om. SOL3C1ER* / ut *inter. a. m. E* / patuit *om. L3* 258 *post* restat *add.* ergo *R* / ab *om. S* / aliquo: alio *FP1SL3C1E* / puncto *mg. F* / illud: illum *FP1* 259 et *inter. P1* / ad: a *O* 260 N: enim *F* / *post* T *scr. et del.* respectu *F* 261 *post* Q *scr. et del.* erit *S* / QO: QA *SL3E* 262 sub: super *FP1E* / *post* MN *add.* et *R* / ZQ: SQ *L3E* / GT: GR *O* 263 C[1,2]: P *R* / Z: S *L3E* / proportio . . . erit (264) *om. P1* / TO: PG *R* 264 sicut: sunt *L3* / GC: TO *R* / CO: OP *R* / TN: CQ *FP1;* TA *SL3* / ad[3] *inter. O* 265 GT[1]: TG *L3* / quam: quoniam *FP1* / GC: GP *R* / CN: PN *R* / sit . . . TN (266) *mg. F* 266 maior: minor *E* / GC: GP *R* / *post* et *add.* erit *R* / L *inter. E* 267 sint: sicut *FP1* / linee . . . ZG: HG EG ZG linee *R* / EG *corr. ex* EH *L3* / ZG: SG *L3E* / *post* GF *scr. et del.* EG *P1* / GC: GP *R* / *post* GS *scr. et del.* G *C1* 268 GO *inter. a. m. E* / TGZ: TGS *L3E* 269 Z: S *L3E* / G: E *R* / tantum *corr. ex* deinde *E* 270 elevata: elevatum *FP1;* elevavata *S* / *post* in[1] *scr. et del.* medio *C1* / GD: G *FP1* / T: Z *FP1SOC1;* S *L3E* / *post* T *add.* respectu Z *R* / *post* GT *scr. et del.* eri *P1; add.* erit *R* 271 eiusdem *om. C1; corr. ex* eius *O* / eiusdem altitudinis (272) *transp. ER*

dem altitudinis cuius est finis contingentie Z, quare finis contingentie D in puncto S.

[4.79] Verum, quoniam angulus EGT equalis angulo TGH, et HG equalis EG, erit L ymago T respectu E, sicut est respectu H. Et N finis contingentie respectu E, quare ymago puncti remotioris ab E remotior a centro ymagine propinquioris, et finis contingentie remotioris remotior a centro fine propinquioris, quod erat propositum.

[4.80] **[PROPOSITIO 5]** Amplius, proposita linea AB [FIGURE 6.4.5, p. 307], et divisa in punctis G, D ut sit proportio AB ad BD sicut AG ad GD, si a punctis sectionis ducantur tres linee concurrentes in punctum unum, scilicet GE, DE, BE, et a puncto A ducatur linea secans illas tres lineas, dico quoniam linea illa divisa erit secundum predictam proportionem.

[4.81] Probatio: Ducatur linea AT secans tria latera GE, DE, BE in tribus punctis Z, H, T. Dico quoniam proportio AT ad TH sicut AZ ad ZH.

[4.82] Ducatur a puncto H equidistans AB, que sit HQ. Palam quoniam proportio AB ad BD constat ex proportionibus AB ad HQ et HQ ad BD. Sed quoniam QH equidistans AB, erit triangulus TQH similis triangulo BTA, et erit proportio AB ad QH sicut AT ad TH. Similiter, triangulus QEH similis triangulo BED. Igitur erit proportio QH ad BD sicut HE ad ED. Ergo proportio AB ad BD constat ex proportionibus AT ad TH et HE ad ED.

[4.83] Producatur QH usque cadat super EG in puncto M. Proportio AG ad GD constat ex proportionibus AG ad HM et HM ad GD. Sed cum angulus EMH sit equalis angulo ZGD, erit angulus HMZ

272 *post* contingentie[1] *add.* puncti T respectu *R* / Z . . . contingentie[2] *mg. F* / Z: S *L3E* / *post* Z *add.* et *P1* / *post* quare *add.* erit *R* 273 D *om. FP1* 274 *post* equalis *add.* est *R* 275 sicut . . . E[1] (276) *scr. et del. E* / est: E *L3* / *post* respectu[2] *add.* puncti *R* / N *inter. O* 276 respectu . . . contingentie (277) *om. S* / *post* E[1] *add.* sicut est respectu puncti H *R* / remotioris *corr. ex* remotionis *C1* 277 *ante* a *add.* est *R* / ymagine: ymaginis *FP1* / remotioris *corr. ex* remortioris *F* 280 ut . . . GD (281) *rep. et del. C1* / sit *inter. a. m. E* / ad *om. FP1* 281 AG: AD *L3E* / sectionis: sectionum *R* / concurrentes: currentes *L3E* 282 punctum unum *transp. E* / *ante* scilicet *scr. et del.* unum *E* / DE BE *transp. E* / *post* linea *scr. et del.* AT secans tria latera GE DE BE *S* 283 quoniam: quod *R* 284 proportionem: probationem *FP1L3* 285 probatio *om. OR* / AT: AC *R* / BE: LE *E* / tribus punctis (286) *transp. C1* 286 Z: S *L3E* / T: C *R* / quoniam: quod *R* / AT: AC *R* / TH: CH *R* / AZ: AS *L3E*; *corr. ex* AIZ *F* / ZH: SH *L3E* 287 ducatur . . . H *om. S* / palam . . . HQ (288) *mg. C1* 288 quoniam: quod *R* / proportio: probatio *E* / AB[2]: AH *FP1* / ad[2] *rep. F* 289 HQ: BQ *O* / equidistans: equidistat *P1* / *post* equidistans *add.* equalis *L3* / triangulus: triangulum *R* / TQH: IQH *L3E*; CQH *R* 290 similis: simile *R* / BTA: HTA *L3C1E*; CAB *R* / erit *om. C1* / AT: AC *R* / TH: CH *R* 291 triangulus: triangulum *R* / QEH: EQH *C1* / QEH . . . triangulo *mg. O* / similis: simile *R* / igitur erit *transp. S* 292 sicut . . . BD[2] *mg. a. m. E* / *post* ED *scr. et del.* producatur QH *C1* / AB *rep. et del. F* 293 AT: AC *R*; *corr. ex* AG *S* / TH: CH *R* 294 *post* usque *add.* dum *OR* / cadet: cadat *OE* 295 *ante* AG[1] *add.* igitur *R* / HM: ? *FP1* / *post* GD[2] *add.* sicut dictum est *S* 296 ZGD: SGD *L3E* / angulus[2]: triangulus *C1* / HMZ: HMS *L3E*

equalis angulo ZGA, et erit triangulus AZG similis triangulo HZM, et erit proportio AZ ad ZH sicut AG ad HM.

[4.84] Sed triangulus HEM similis triangulo GED. Erit proportio HM ad DG sicut HE ad ED. Igitur proportio AG ad GD constat ex proportionibus AZ ad ZH et HE ad ED, et eadem est AG ad GD que est AB ad BD. Igitur illa eadem constat ex proportionibus AT ad TH et HE ad ED, et similiter constat ex proportionibus AZ ad ZH et HE ad ED. Igitur eadem est proportio AT ad TH que est AZ ad ZH, et ita propositum.

[4.85] Eadem erit probatio quecumque linea ducatur a puncto A secans illas lineas tres concurrentes. Et si ducantur alie tres linee a tribus punctis G, D, B ad aliud punctum quam E concurrentes, et a puncto A ducatur linea quecumque secans eas, dividetur secundum predictam proportionem. Et ita quocumque modo concurrant tres linee, et si tres linee EG, ED, EB producantur ultra tria puncta B, D, G ex alia parte, et a puncto A ducantur linee secantes eas ex illa alia parte, numquam ille linee dividantur secundum predictam proportionem.

[4.86] **[PROPOSITIO 6]** Amplius, data linea AB predicto modo divisa, si a puncto A ducatur alia linea, velut AT, que dividatur iuxta eandem proportionem, et a punctis divisionum AB ducantur linee ad puncta divisionum AT, que quidem non sit equidistans, dico quoniam ille tres concurrent in uno eodem puncto.

[4.87] Probatio: Sit proportio AT ad TH sicut AZ ad ZH. BT, DH non sunt equidistantes; igitur concurrent in puncto quod sit E. Linea GZ aut concurret ad idem punctum, aut non. Si ad illud, habemus

297 ZGA: SGD *L3E;* EGO *C1* / triangulus: triangulum *R* / AZG: ASG *L3E* / similis: simile *R* / HZM: AZM *P1;* HSM *L3E;* HMZ *R* 298 AZ: AS *L3E* (*alter. ex* TIS *E*) / ZH: SH *L3E;* HZ *C1;* ZB *O* 299 triangulus: triangulum *R* / HEM: BEM *O* / similis: simile est *R* / *post* erit *add.* igitur *R* 300 DG: GD *C1R* / HE *inter. a. m. E* / ED: DE *C1* / GD: DG *FP1* 1 proportionibus: proportione *R* / AZ: AS *L3E* / ZH: SH *L3E;* ZB *O* / et HE: ZHE *S* / GD: DG *C1* 2 est *om. SOER* / AT . . . proportionibus (3) *om. R* 3 et[1] *om. FP1* / AZ: AZG *FP1;* AG *S* / ZH: SH *L3E* 4 est[1]: erit *E* / AT: AC *R* / TH: CH *R* / AZ: AE *L3E* / ad[3] *inter. O* / ZH: SH *L3E* / *post* ita *add.* est *R* 6 erit probatio *transp. C1* / probatio: proportio *FP1L3* / A: D *S; corr. ex* H *E; inter. O* 7 illas: alias *FP1* / *post* illas *scr. et del.* duas *C1* / illas lineas *transp. R* / alie tres *transp. L3* 8 B: H *S* / aliud: alium *FP1;* illud *L3* 9 secans . . . ita (10) *mg. a. m. E* / eas *inter. O* / dividetur: divideretur *FP1; corr. ex* dividatur *C1* 10 proportionem: probationem *FP1SL3E* / quocumque *corr. ex* quecumque *a. m. E* / concurrant: currant *E* 11 et . . . linee *om P1* / ED: AD *E* 12 illa *mg. a. m. C1* 13 *post* parte *scr. et del.* alius *O* / numquam: in quam *FP1SOC1* / ille *inter. O* / ille linee *transp. O* / dividantur: dividuntur *E;* dividentur *R* / proportionem: probationem *FP1SL3C1E; corr. ex* probationem *O* 16 alia *om. C1* / velut *om. P1* / AT *corr. ex* TAT *P1;* AC *R* 17 proportionem: probationem *L3E* / divisionum: divisionis *L3* / AB *corr. ex* BAB *L3* 18 AT: AC *R* / sit equidistans: sint equidistantes *R* / quoniam: quod *R* 19 *post* uno *add.* et *R* 20 probatio *om. R* / AT: AC *R* / TH: CH *R* / AZ: AS *L3E* / ZH: SH *L3E* / BT: BC *R* / DH: BH *L3* 21 *post* in *add.* aliquo *R* 22 GZ: GS *L3E* / *post* non *add.* et *FP1* / illud: illum *FP1;* idem *R*

propositum. Si non, ducatur linea EG. Secabit quidem lineam AT in alio puncto quam Z. Sit illud punctum L. Erit igitur proportio AT ad TH sicut AL ad LH, iuxta priorem probationem. Sed positum est AT ad TH sicut AZ ad ZH, et ita impossibile.

[4.88] Similiter, si ponatur quod linea GZ concurrat cum DH ad punctum E, probabitur hoc modo quod linea BT concurret ad idem. Similiter, si ponatur quod GZ, BT concurrant ad punctum E, convincetur quod DH concurret ad idem.

[4.89] **[PROPOSITIO 7]** Amplius, divisa AB secundum hanc proportionem, si fuerint linee GZ, DH, BT equidistantes, et ducatur AT dividens illas, erit AT divisa secundum hanc proportionem.

[4.90] Probatio: Cum DH sit equidistans GZ, erit proportio AZ ad ZH sicut AG ad GD, et cum BT sit equidistans DH, erit AB ad BD sicut AT ad TH. Sed AB ad BD sicut AG ad GD; erit AT ad TH sicut AZ ad ZH, et ita propositum. Hiis premissis, accedamus ad propositum.

[4.91] **[PROPOSITIO 8]** Et primum, de arcu declaretur quomodo in hiis speculis ymago eius sit curva curvitate quidem speculum non respiciente, sed centrum.

[4.92] Verbi gratia, sit AB [FIGURE 6.4.8, p. 308] arcus oppositus speculo, et sit G centrum illius arcus et similiter centrum speculi, D centrum visus. Et ducantur linee DG, AG, BG. Et sumatur E in arcu AB quocumque modo, et ducatur linea EG. Linea DG non sit in superficie ABG. Linea DG aut erit ortogonalis super superficiem ABG, aut declinata.

[4.93] Sit ortogonalis. Erunt anguli DGA, DGE, DGB equales, et latera lateribus, quare bases equales. Igitur omnia puncta arcus AB eiusdem longitudinis erunt a centro visus, quare ymagines omnium

23 *post* propositum *scr. et del.* m *O*/AT: AC *R* 24 Z: S *L3E*/illud: illum *F*/L *om. FP1*/AT: AC *R*/ad TH (25) *om. S* 25 TH: CH *R*/*post* LH *scr. et del.* et ita impossibile *C1*/iuxta . . . ZH (26) *mg. a. m. E*/priorem *om. E*/AT: AC *R* 26 TH: CH *R*/AZ: AS *L3E*/ZH: SH *L3E* 27 quod linea *transp. deinde corr. C1*/GZ: GS *L3E* 28 BT: BC *R*/concurret: concurrat *R*/*post* idem *add.* punctum *P1* 29 similiter . . . idem (30) *rep. FP1* (*mg. F*)/*post* quod *add.* linea *FP1*/GZ: GS *L3E*/BT: BC *R*/convincetur: coniuncte *FP1O*; probabitur *R* 32 proportionem: probationem *L3E*/GZ: GS *L3E*/DH: AD *L3*/BT: BC *R* 33 AT^{1} *inter. a. m. C1*; AC *R*/dividens . . . AT^{2} *om. P1*/*post* illas *scr. et del.* AT *C1*/AT^{2}: AC *R*/divisa *corr. ex* dividens *C1*/hanc *om. O*/proportionem: probationem *L3* 34 probatio *om. R*/GZ: GS *L3E*/AZ: AS *L3E* 35 ZH: SH *L3E*/BT: HT *FP1*; BC *R* 36 AT^{1,2}: AC *R*/ad^{4} *mg. F*/TH^{1,2}: CH *R* 37 AZ: AS *L3E*/ZH: SH *L3E*/*post* ita *add.* patet *R* 38 *post* propositum *add.* primum *FP1* 39 et *om. L3ER*/primum: primo *C1*; *om. O*/declaretur: declaratur *L3*; declaremus *ER* 40 eius *rep. et del. C1*/curva: concurva *FP1* 42 oppositus: opponens *C1* 44 *post* linee *scr. et del.* EG linea *C1*/AG: HD *FP1*/BG: AG *F*; AH *P1*; *corr. ex* B *O* 45 *post* linea^{2} *add.* vero *R* 46 *post* linea *add.* igitur *R*/aut erit ortogonalis *corr. ex* ortogonalis aut erit *C1* 48 *post* equales *add.* quia recti *C1* 50 eiusdem: eiudem *P1*/erunt *inter. O*/*post* omnium *add.* punctorum *R*

eiusdem longitudinis a centro.

[4.94] Sint Q, M, L ymagines A, E, B. Erit igitur GQ equalis GM et GL, quare QML erit arcus, et convexitas ipsius respectu centri non respectu speculi, sive reflexionis loci, quod est propositum.

[4.95] Si vero linea DG non fuerit perpendicularis super superficiem AGB, ducta perpendiculari a puncto D super hanc superficiem, cum illa perpendicularis sit minor omnibus lineis ductis a puncto D ad hanc superficiem, erit angulus quem respicit hec perpendicularis supra G minor quolibet angulo supra punctum G intellecto quem respiciat alia linea a puncto D ad superficiem ducta, et linea ducta a puncto D ad superficiem quanto remotior erit a perpendiculari, tanto maior erit, et respiciet maiorem angulum. Si igitur hec perpendicularis non cadat in arcu AEB, sed ex parte una, erunt omnes linee ducte a puncto D ad hunc arcum declinate in partem unam, et remotiores maiores et maiorem angulum respicientes.

[4.96] Sit ergo, et sumantur tria puncta in arcu, scilicet E, C, B. Finis contingentie puncti B sit L; finis contingentie puncti C sit M, quoniam, cum C propinquius D quam B, erit M propinquior G quam L, et ita CM maior BL.

[4.97] Q sit ymago C, T ymago B, et ducatur TQ. Et ducantur linee CB, ML, que quidem producte concurrent, si enim a puncto M duceretur equidistans CB, secaret ex GB lineam equalem CM. Concurrant in puncto O.

[4.98] Et quoniam proportio GC ad CM sicut GQ ad QM, similiter, BG ad BL sicut GT ad TL, linea QT concurret cum lineis CB, ML. Et sit concursus in puncto O.

[4.99] Finis contingentie puncti E sit N [FIGURE 6.4.8a, p. 308]. Quoniam punctus N dimissior est puncto M, erit EN maior CM.

51 eiusdem longitudinis *transp. C1* / *post* longitudinis *add.* sunt *R* / *post* centro *scr. et del.* TM quare ymagines omnium eiusdem longitudinis a centro *O* 52 sint: sicut *SL3;* sintque *R* / *post* ymagines *add.* ipsorum *R* / igitur *om. C1* / GQ *corr. ex* GL *C1* / GM: GLN *FP1* 53 et[1] *om. R* / QML: QM *E* 54 sive: fine *FP1* / reflexionis loci *transp. R* / loci: BOCI *FP1* 55 super . . . perpendiculari (56) *mg. a. m. E* 56 AGB *corr. ex* ABGB *L3* / ducta . . . D *om. P1* 57 perpendicularis: perpendiculari *SL3* / sit: sicut *L3* / D: O *S* 58 quem: quam *FP1* / respicit: continet *R* 59 supra[1,2]: versus *R* 60 respiciat: respiciet *FP1;* respicit *C1;* continet *R* / *post* ad *add.* hanc *R* 61 *post* D *add.* ducta *SE* / *post* ad *add.* hanc *R* / quanto: quando *S* / a *om. S* 62 *post* erit *scr. et del.* OE *C1* / respiciet: continebit *R* / *post* angulum *add.* versus G *R* / igitur hec *transp. L3* 63 arcu: arcum *R* / una *corr. ex* unam *P1* 64 in: ad *R* 65 respicientes: continentes versus G *R* 66 *post* B *add.* finis contingentie puncti C sit M *mg. O* 67 sit[1] *om. FP1* / finis . . . M *om. O* / C: GET *S;* GTC *L3;* GET *alter. in* GCT *E* / M: LM *S* 68 cum: igitur *R* / propinquior: propinquius *R* / C *inter. O* / propinquius *corr. ex* propinquioris *S* / D *om. FP1* 69 CM: tantum *S; corr. ex* C *O* 70 et[2] *om. S; inter. a. m. E* 71 CB: CP *FP1;* CH *SL3E* / enim *rep. et del.* F 72 ex GB *om. S; inter. a. m. E* 74 CM *corr. ex* GM *C1; corr. ex* QN *E* 75 BG: GB *SL3ER* / TL: D *S* / *post* TL *add.* ergo *R* / concurret: concurrent *SL3; corr. ex* concurrent *O* / et *om. R* 76 sit *om. O* / *post* O *scr. et del.* et quoniam proportio *C1* 77 puncti: in *S* 78 punctus N dimissior: punctum N dimissius *R*

Ductis ergo lineis EC, NM, concurrent. Sit concursus in puncto P, et ducatur linea QP, et procedat donec cadat super EG in puncto F. Et ducatur linea TQ usque ad EG, et cadat in puncto K.

[4.100] Palam quoniam K erit supra F. Verum, cum proportio GC ad CM sicut GQ ad QM, et a punctis divisionum ducantur tres linee concurrentes in aliam partem producte, secabunt lineam EG secundum predictam proportionem, quare proportio GE ad EN sicut GF ad FN. Sed N finis contingentie, quare F locus ymaginis. Igitur linea FQT erit ymago arcus ECB, et erit linea curva non recta, quoniam TQK est recta, et curvitas linee non ex parte speculi.

[4.101] Similiter, si perpendicularis a puncto D cadat ex alia parte arcus, similis erit probatio. Si vero cadat perpendicularis in medio arcus AB, linee a puncto D ex diversis partibus ad arcum ducte equaliter distantes a perpendiculari erunt equales, et equales angulos respicient supra G. Et ymagines earum equaliter a G distabunt, et finis contingentie similiter, et erit probare predicto modo de utraque parte arcus per se secundum quod dividitur a perpendiculari quod eius ymago sit linea curva modo predicto, quod est propositum.

[4.102] **[PROPOSITIO 9]** Amplius, sumatur circulus cuius non sit centrum centrum speculi; verumptamen, sit in eadem superficie cum centro speculi. Dico quoniam, si in hoc circulo exteriori sumatur arcus ex parte centri speculi, id est propinquior ei, erit eius ymago curva.

[4.103] Dato enim hoc arcu, ducatur linea a centro speculi ad centrum exterioris circuli, et producatur hec linea usque ad arcum datum. Linea ducta a centro speculi ad hunc arcum, que est pars dyametri maioris circuli, erit brevior omnibus lineis ductis ab eodem centro speculi ad illum arcum. Et a centro speculi possunt duci ad

79 EC *inter. E*/NM: MN *R*/*post* NM *scr. et del.* N *C1* 80 QP *corr. ex* Q *O* 81 puncto: punctum *R* 82 quoniam: QM *S*; quod *R* 83 punctis *corr. ex* punctus *S* 84 lineam *mg. a. m. E* 85 GE *corr. ex* EG *L3*/EN: EM *FP1*; EF *SC1E* 86 *post* N *add.* est *OR* (*inter. O*)/*post* locus *add.* est *R* 87 ECB: ACB *SL3*; QCB *E*/non recta quoniam: quoniam non recta *C1*/quoniam . . . recta (88) *om. P1* 88 *post* non *add.* est *ER*/*post* speculi *add.* S *FP1* 89 cadat: cadet *C1* 90 probatio: proportio *SL3E*/in: a *L3E*/medio: medium *R* 91 linee: lineis *S*/D: B *FP1*/ducte: deducte *O* 92 distantes: equidistantes *F*; equidistat *P1*/a *om. S*/equales[1] *corr. ex* superficies *O* 93 respicient: respicientes *FP1*; respiciant *L3*; continebunt *R*/supra: versus *R*/earum *om. R*/equaliter a G distabunt: a G equaliter distabunt *R* 94 finis: fines *OL3C1ER* (*alter. in E*)/similiter *inter. O*/erit: licebit *R* 95 quod[1] *om. L3* 96 eius: est *SL3E*/sit: super *C1* 97 amplius *om. FP1*/*post* cuius *add.* centrum *C1*/non sit centrum (98): centrum non sit *R* 98 centrum[2] *mg. F; inter. O; om. L3C1* 99 quoniam: quod *R*/exteriori: exteriore *R* 100 id est: vel *E; om. R*/ei *om. FP1*/*post* ei *add.* secundum medium eius punctum *R*/eius ymago *transp. R* 102 *post* enim *scr. et del.* dato *F*/ducatur *inter. O* 103 centrum: arcum *SL3E*/hec: B *FP1; om. OC1* 104 ducta *corr. ex* data *C1*/a centro: ad centrum *FP1*/que: qui *SL3E* 105 circuli *om. FP1*/lineis *mg. F* 106 illum arcum: illuminatum *L3*

arcum datum due equales a diversis partibus huius brevis, que quidem maiores illa brevi. Et si secundum alteram illarum fiat circulus cuius centrum centrum speculi, transibit per capita harum duarum linearum arcus excedens arcum datum.

[4.104] Et palam quod ymago huius arcus excedentis erit linea curva, secundum predicta. Et ymagines punctorum huic arcui et arcui dato communium eedem, et medius punctus arcus excedentis remotior a centro puncto arcus dati quod ipsum respicit, quare eius ymago propinquior centro quam ymago puncti arcus dati illum respicientis. Et ita cuiuslibet puncti arcus exterioris ymago propinquior centro ymagine puncti arcus dati quod illud respicit. Quare ymago arcus dati curvior quam ymago arcus exterioris, quare ymago arcus dati curva est, quod est propositum.

[4.105] **[PROPOSITIO 10]** Amplius, quod linee recte ymago in hiis speculis sit curva sic probatur.

[4.106] Sit AB [FIGURE 6.4.10, p. 309] linea visa, G centrum speculi. Ducantur linee AG, BG. Aut sunt equales, aut non. Si equales, fiat circulus cuius G centrum ad quantitatem illarum, qui sit AEB. Cadet quidem linea AB intra circulum. Palam ex predictis quoniam ymago arcus AEB erit curva. Sit ergo ymago eius ZTH. Ymago A sit Z; ymago B sit H; ymago E sit T.

[4.107] Et ducatur linea GE secans AB in puncto C. Palam quoniam E est in eadem linea cum C remotior a centro quam C. Erit eius ymago propinquior centro quam ymago C. Sit ergo M. Palam ergo quod linea ZMH est ymago linee AB, et est linea curva, quod est propositum.

[4.108] **[PROPOSITIO 11]** Si vero linee AG, BG fuerint inequales, linea AB protracta aut secabit speculum, aut non. Sit quod non secet,

107 *post* due *add.* linee *R* / equales: quales *C1* / a: ad *S* / huius: huiusmodi *FP1* 108 *post* maiores *add.* erunt *R* / si secundum alteram: in C alter utram *FP1* 109 *post* centrum[1] *add.* sit *R* / centrum[2] *om. FP1S; inter. O* / centrum speculi *transp. R* / *post* per *scr. et del.* equalia *C1* / harum *inter. O* 112 ymagines: ymaginis *SL3E* / et arcui (113) *om. OL3* 113 communium: contrarium *FP1O* / eedem: eodem *SOL3E; corr. ex* eodem *mg. C1* / et *om. O* / medius punctus: medium punctum *R* / excedentis *corr. ex* excedentes *O* 114 *ante* remotior *add.* est *OR* / remotior: remotius *R* / *post* centro *add.* speculi quam *R* / puncto: punctum *R* 115 *post* propinquior *add.* est *R* / illum: illi *E* 117 *ante* centro *add.* est *R* / illud: ipsum *R* 118 curvior . . . dati (119) *om. S; mg. a. m. E* / quare *corr. ex* qurare *F* 120 quod *inter. O* / in *rep. et del. C1* 121 sic probatur *transp. SL3ER* 123 linee: linea *FP1; om. L3* / *post* BG *add.* hee *R* 124 ad: secundum *R* / AEB: EAB *FP1* 125 linea *om. O* / intra: inter *SL3* / quoniam: quod *R* 126 sit[1]: si *S; corr. ex* si *O* / ZTH: STH *SL3E; corr. ex* ZH *C1* 127 Z: S *SL3E* 128 et: ed *FP1O* / ducatur: producatur *SE* / linea *om. R* / C: F *R* / quoniam: quod *R* 129 E *inter. O* / C[1]: F *R* / *post* C[1] *inter.* cum C sit *O* / quam *om. R* / C[2]: G *R* / *post* erit *add.* ergo *R* 130 *post* quam *add.* F *R* / C: O *S; om. R* 131 ZMH: SMH *SL3E* 134 *ante* linea *scr. et del.* in *O* / sit: si *S; corr. ex* si *O* / sit quod *corr. ex* si *a. m. C1*

et sit AG maior BG [FIGURE 6.4.11, p. 309], et fiat circulus supra G ad quantitatem AG, qui sit AEQ, et producatur AB usque cadat in circulum ex parte B. Cadat in puncto Q.

[4.109] Patet ex superioribus quoniam ymago arcus AE est curva. Punctus ymaginis A sit Z; punctus ymaginis E sit M. Erit ZM ymago arcus AE, et quoniam ymago puncti B remotior a centro quam ymago puncti E, erit ymago linee AB curva, quod per puncta media arcus AE et linee AB poterit probari, quod est propositum.

[4.110] Nota quod in priori figura, si secetur a linea AB ex parte A pars quedam, et ex parte B secetur pars ei equalis, residuum linee habebit ymaginem curvam, et erit eadem probatio que est de linea AB. Et in hac figura secta alia parte linee AB ex parte B, de residuo erit eadem probatio que est de linea AB.

[4.111] **[PROPOSITIO 12]** Si vero linea AB tangit speculum, aut secabit, aut continget. Sit contingens [FIGURE 6.4.12, p. 309]. G sit centrum speculi, et ducantur linee AG, BG. Superficies ABG secat speculum supra circulum communem, qui sit SEZ. Palam quoniam linea AB continget speculum in hoc circulo. Contingat in puncto E. Protrahatur ergo AB usque ad E. D sit centrum visus. Superficies in qua sunt linee DG, AG secat speculum supra circulum communem superficiei et speculo. Et sit arcus illius circuli ZP. Similiter, linea communis superficiei in qua sunt DG, BG et circulus arcus illius circuli sit HP.

[4.112] Palam quoniam B refertur ad D ab aliquo puncto arcus HP. Si a puncto illo ducatur contingens, secabit lineam BG, et punctus sectionis erit finis contingentie. Sit punctus ille M.

[4.113] Palam etiam quod, si a puncto M ducatur contingens super circulum SEH, cadet contingens illa citra E, quoniam AB est

135 BG: GB *R*/supra: super *R* 136 AB: AH *FP1*/usque: quousque *R*/cadat: cada *P1* 137 puncto Q: punctum E *R* 138 patet: patebit *FP1O*/quoniam: quod *R*/arcus AE *transp. deinde corr. C1* 139 punctus[1]: punctis *S*; punctum *R*/punctus[2]: punctum *R*/erit: sit *L3*/ZM: MZ *C1* 140 *post* AE *scr. et del.* et quoniam ymago arcus AE *FC1*/*post* remotior *add.* est *OR* (*inter. O*) 141 *post* quod *add.* etiam *R*/AE *corr. ex* EA *O* 142 *post* AB *add.* faciliter *R* 143 nota . . . AB (147) *om. R*/si *om. E* 144 *post* secetur *scr. et del.* a linea AB ex parte A pars equedam *S*/pars[2] *om. L3* 145 probatio: proportio *SL3* 146 et . . . AB[2] *om. L3*/secta: recta *O* 147 que: quod *L3* 148 si . . . AB *om. P1* 149 sit contingens: tangat primo et *R*/*post* G *scr. et del.* ce *P1* 150 secat: secans *FP1*; secabit *SR*; *corr. ex* secabit *a. m. E* 151 supra: super *FP1R*/communem *corr. ex* commeunem *F*/SEZ *corr. ex* SZ *C1*; EHZ *R*/quoniam: quod *R* 152 continget: contingit *S* 153 *post* ergo *scr. et del.* AD *F* 154 secat: secabit *SR*; *corr. ex* secabit *a. m. E*/supra: super *R* 155 *post* superficiei *add.* reflexionis *R*/speculo: speculi *R*/et[2] *om. SL3C1ER*/ZP: SP *E*; speculi *L3*/ZP . . . circuli (156) *om. S* 156 *post* superficiei *add.* reflexionis et speculi *R*/et: est *O*/circulus *om. R* 158 quoniam: quod *R*/B: HP *SL3E*/refertur: reflectitur *R*/aliquo: alio *SL3E* 159 punctus: punctum *R* 160 erit finis *transp. C1*/*post* erit *add.* IT *FP1*/punctus ille: punctum illud *R*/ille *inter. O* 161 si *om. FP1*/ducatur: ducantur *O* 162 super *om. FP1*/super circulum: arcum circuli *R*/SEH: EH *R*/citra: circa *SL3*/est contingens (163): contingit *R*

contingens in puncto E, et punctus B est altior puncto M. Cadet ergo in puncto F, que contingens producta secabit lineam AE. Secet in puncto T. Ex alia parte secabit lineam AG. Secet in puncto C.

[4.114] Fiat angulus BGS equalis angulo BGD, et producatur GS usque ad punctum L ad equalitatem linee DG. Erit ergo arcus HS equalis arcui HP, et sicut refertur B ad D a puncto arcus HP, refertur ad L a puncto arcus HS. Et erit reflexio a puncto F sicut in arcu HP est reflexio a puncto a quo ducatur contingens ad punctum M, et illa duo puncta a puncto M eiusdem longitudinis. Ducantur ergo linee BF, LF.

[4.115] A refertur ad D ab aliquo puncto arcus ZP. Verum in triangulo HZP duo arcus HZ, HP sunt maiores tertio, scilicet ZP. Sed HP est equalis HS. Igitur ZP est minor ZS. Rescindatur ZS ad equalitatem in puncto Y, et ducatur linea GY, que producta ad equalitatem GD secabit necessario lineam FL. Secet in puncto X, et sit GXK equalis GD.

[4.116] Palam quoniam sicut A refertur ad D ab aliquo puncto arcus ZP, similiter, refertur ad K ab aliquo puncto arcus ZY. Dico quoniam non refertur ad ipsum nisi a puncto quod est citra F ex parte Z.

[4.117] Si enim dicatur quod potest a puncto F vel aliquo puncto arcus FY, linea ducta a puncto A ad punctum reflexionis secabit lineam BF. Ad illud punctum sectionis reflectitur punctus K, et ad idem punctum refertur punctus L, et ita duo puncta in hiis speculis reflectuntur ad idem punctum ex eadem parte, quod est impossibile. Restat ut punctus A refertur ad K ab aliquo puncto arcus ZF.

[4.118] Si ab illo puncto ducatur contingens, secabit lineam AZ, et cadet inter C et Z, quoniam punctus F dimissior quolibet puncto

163 punctus: punctum *R*/altior: altius *R*/cadet: cadit *O*; cadit *alter. in* cadat *C1*; cadat *R* 164 puncto: punctum *R*/que: Q *L3*/contingens: continens *FP1O*/producta: predicta *FP1* 166 angulus: circulus *P1O* 167 usque *om. C1*/HS: hiis *P1* 168 refertur[1, 2]: reflectitur *R*/ad: at *FP1*/a: ab aliquo *R*/*post* HP *add.* sic *R* 169 ad *om. L3*/a: ab aliquo *R*/HS: BS *E* 170 ducatur: ducitur *SOC1R*/ad punctum: a puncto *S* 171 *post* puncta *add.* sunt *R*/a . . . longitudinis: eiusdem longitudinis a puncto M *R*/longitudinis: lineis *S* 173 *ante* A *add.* item *R*/refertur: reflectatur *R*/aliquo: alio *SL3E*; angulo *O* 174 HP: SP *S*/sunt maiores *transp. R*/scilicet *om. SL3ER* 175 rescindatur: recindatur *FP1*/rescindatur ZS *mg. C1* 177 secabit necessario *transp. R*/FL: BL *FP1O* /GXK: GTK *S*; GX LI *P1* 179 quoniam: quod *R*/quoniam sicut *transp. deinde corr. O*/refertur: reflectitur *R*/puncto . . . axis (144) *om. S* 180 *post* ZP *add.* et *FP1*; *scr. et del.* et *O*/*post* similiter *inter.* S *E*/refertur *inter. E*; reflectitur *R*/K: KAB *P1*/K ab: KAB *F*/quoniam: quod *R* 181 non refertur *transp. FP1*/ refertur: reflectetur *R*/citra: circa *L3*; *corr. ex* circa *a. m. E*/F *om. FP1* 182 dicatur quod *rep. P1*/quod *inter. O*/potest: possit *R*/aliquo: alio *L3C1ER* 183 FY: SY *F* 184 *post* BF *add.* et *ER*/ad illud: aliud idem *E*/illud: idem *R*/reflectitur: reflectetur *L3C1ER*/punctus: punctum *R*/punctus . . . et (185) *mg. a. m. E* 185 refertur: reflectetur *R*/punctus *om. FP1*; punctum *R*/L: B *ER*/reflectuntur: reflectetur *L3*; reflectentur *ER* 186 restat *corr. ex* restabit *a. m. E* 187 punctus: punctum *R*/refertur: reflectatur *R*/aliquo: alio *O*/ZF: ZB *O* 188 illo *corr. ex* alio *a. m. E* 189 C et Z: Z et C *R*/punctus F dimissior: punctum F dimissius est *R*

arcus ZF, et ita contingens a puncto F altior aliis a punctis arcus ZF ductis. Cadat ergo contingens illa in puncto N, et ducatur linea NM, que quidem linea, cum transeat per acumen trianguli BMT et producta dividat angulum, necessario secabit BT. Secet in puncto Q, et ducatur linea GQ.

[4.119] Sit autem I ymago puncti A; O sit ymago puncti B; U sit ymago puncti Q. Palam, cum B sit propinquior puncto G quam A, erit O remotior a puncto G quam I. Ducatur ergo linea IO. Palam etiam quod proportio AG ad AN sicut GI ad IN, et proportio BG ad BM sicut GO ad OM. Cum ergo linee AG, BG dividantur secundum hanc proportionem utraque in tribus punctis, et a punctis divisionum ducantur linee quarum due, scilicet AB, MN, concurrant ad idem punctum, scilicet ad idem punctum Q, tertia necessario concurret ad idem illud punctum.

[4.120] Igitur IO producta cadet supra Q, quare IOQ recta linea. Igitur IOU non erit recta. Sed IOU est ymago linee AQ, quare ymago linee AQ erit curva. Posito autem puncto B loco puncti Q, et aliquo puncto linee AB posito loco puncti B, erit eodem modo penitus probare quoniam ymago linee AB est curva, et hoc est propositum.

[4.121] **[PROPOSITIO 13]** Si vero AB [FIGURE 6.4.13, p. 310] secat circulum, secet in puncto E, M finis contingentie linee BG. B refertur ad D ab aliquo puncto arcus HP. Arcus ab illo puncto reflexionis usque ad H aut est equalis arcui HE, aut maior, aut minor.

[4.122] Si equalis (sed palam quoniam arcus ille est equalis arcui HQ), sit Q punctus circuli in quem cadat contingens ducta a puncto M ex parte E. Igitur AE transit per punctum Q, et ita MQ secat AE per punctum E.

[4.123] Si vero arcus ille minor est arcu HE, secabit MQ lineam AE ultra punctum Q, ut efficiatur triangulus EQT.

190 contingens: continens *O* / F: R *P1* / aliis: illis *C1* / a^{2} *inter. E* / *post* punctis *scr. et del.* ZF *E* 191 contingens: continens *FP1O* / puncto: punctum *R* / NM: MN *R* 193 *post* dividat *scr. et del.* punctum *C1* 195 *post* puncti^{1} *scr. et del.* O *L3* / U: R *R* 196 propinquior: propinquius *R* 197 I: C *R* 200 hanc *mg. F* / proportionem: probationem *L3E* / tribus: duobus *R* / et *om. F* / a punctis *mg. F* 202 ad idem punctum *om. R* / idem *om. O* / necessario *om. C1* 203 idem *om. L3C1E* / illud: illum *E*; *om. R* / *post* punctum *add.* necessario *C1* 204 supra: super *R* / *post* IOQ *add.* est *R* 205 IOU^{1}: IOY *P1*; IOR *R* / non: N *L3* / IOU^{2}: IOR *R* / quare . . . AQ (206) *inter. a. m. E* / quare: quia *E* 206 *post* autem *add.* a *L3* / *post* puncti *add.* P *FP1* / Q *inter. O* / aliquo puncto (207) *transp. P1* 207 puncto *inter. F* / modo penitus *transp. OL3ER* 208 quoniam: quod *R* 209 vero *inter. O* / secat: secet *ER* 210 *post* linee *add.* contingentis circulum EHZ a puncto F producte ad lineam *R* / *post* B *add.* igitur *R* / refertur: reflectitur *R* 211 aliquo: alio *OL3C1E* / illo *corr. ex* alio *a. m. E* / puncto^{2} *mg. a. m. C1* 213 sed *om. R* / quoniam: quod *R* / est equalis *transp. P1* 214 HQ: HF *R* / Q *om. FP1* / punctus: punctum *R* / quem: quod *FR*; quo *P1* / cadat: cadit *R* 217 *post* arcus *scr. et del.* linee *P1* / *post* secabit *add.* quidem *R* / lineam: linea *OL3E* / AE: HE *P1* 218 *post* Q *add.* secet in T *R* / ut . . . Q (220) *mg. C1* / triangulus: triangulum *R*

[4.124] Si vero arcus ille fuerit maior arcu HE, secabit quidem linea MQ lineam AE citra punctum Q.

[4.125] Sive hoc sive illud fuerit, iteretur predicta probatio, et eodem penitus modo probetur quoniam ymago linee AB est curva, quod est propositum.

[4.126] **[PROPOSITIO 14]** Amplius, si in superficie in qua sunt linea visa et centrum spere fuerit centrum visus—superiora enim dicta sunt visu non existente in illa superficie—linea ergo visa recta aut concurret cum circulo communi illi superficiei et speculo, aut non concurret.

[4.127] Si concurret, aut erit perpendicularis super speculum aut declinata. Si perpendicularis, angulus illarum linearum cadet supra centrum speculi, que quidem linea videbitur recta, ymago enim cuiuslibet puncti illius linee apparebit in ipsa linea, et ita ymago illius linee recta.

[4.128] Si vero linea proposita declinata fuerit, aut erit declinatio ex parte visus, aut ex alia parte. Si ex alia parte, sumatur punctus circuli a quo reflectatur aliquid ad visum, et sumatur linea reflexionis. Aliqua linearum declinatarum cadet forsitan super hanc lineam reflexionis, quod si fuerit, non videbitur quidem hec linea declinationis.

[4.129] Protracta a centro visus ad centrum speculi linea, si sumatur in arcu circuli citra hanc lineam punctus a quo refertur ad visum aliquis punctus linee declinationis. Sed ille punctus refertur a puncto prius assignato, qui est terminus linee reflexionis, cum linea declinationis sit supra lineam reflexionis, et ita ille punctus linee declinationis refertur ad visum a duobus punctis arcus, quod est impossibile.

[4.130] Licet autem reflectatur punctus ille a puncto primum sumpto, non tamen videtur, cum sit in linea reflexionis, quoniam oc-

219 fuerit maior *transp. C1* / quidem *om. C1* 220 lineam *om. FP1; rep. C1;* linee AM *L3* / citra: circa *L3* 221 predicta *om. ER* 222 probetur quoniam: probabitur quod *R* 224 *post* si *add.* etiam P1; *scr. et del.* etiam *F* 225 et *inter. C1; corr. ex* ad *O* / centrum[2] *om. ER* 226 dicta *corr. ex* ducta *O* / visu non: visuum *FP1* / visu non existente: non existente visu *L3ER* / ergo *om. R* 227 aut: autem *FP1* / communi: cum *P1* 229 concurret: concurreret *L3;* concurrat *R* / aut[1] . . . perpendicularis (230) *mg. C1; om. FP1OL3ER* 230 angulus: alia *O* / illarum: earum *L3* / supra: super *R* 232 apparebit: apparet *L3ER* 233 *post* linee *add.* est *R* 234 proposita: posita *FP1* 235 si . . . parte[3] *om. P1* / alia parte *transp. O* / punctus: punctum *R* 236 ad *om. R* 237 aliqua *rep. ER* / *post* linearum *scr. et del.* declinata *F* / cadet *corr. ex* cadat *E* / *post* hanc *scr. et del.* superficiem *C1* 238 videbitur: videtur *C1* / declinationis: declinata *R* 239 *post* declinationis *add.* nisi secundum unum punctum *R* 240 *post* protracta *add.* igitur *R* / si *om. R* 241 citra *corr. ex* circa *O* / punctus: punctum *R* / refertur: reflectatur *R* 242 aliquis punctus: aliquod punctum *R* / declinationis: declinate *R* / ille punctus: illud punctum *R* / refertur: reflectitur *R* / a *corr. ex* ad *C1* 243 qui: quod *R* / terminus *corr. ex* tres *O* / declinationis: declinata *R* 244 supra: super *L3* / ille punctus: illud punctum *R* / linee . . . visum (245) *rep. P1* / declinationis: declinata *R* 245 refertur: reflectitur *R* 246 punctus ille: punctum illud *R* 247 tamen: cum *FP1* / in *inter. P1* / quoniam: que *R*

cultatur per precedentia puncta, et ita linea adiacens linee reflexionis non videtur.

[4.131] Si vero sumatur linea declinationis cuius declinatio non ex parte visus iacens quidem sub linea reflexionis et secans ipsam in puncto circuli, dico quod nullus punctus illius linee videbitur.

[4.132] Sumpto enim puncto, si dicatur quod punctus ille potest reflecti ab aliquo puncto arcus interiacentis lineam reflexionis et lineam a centro visus ad centrum speculi ductam, et ducatur linea ab illo puncto ad punctum arcus sumptum, hec secabit lineam reflexionis, et punctus sectionis reflectitur ad visum a duobus punctis arcus, quod est impossibile.

[4.133] Si vero dicatur quod punctus sumptus in linea refertur a puncto arcus circuli qui est sub ipsa linea, erit impossibile, quia ille totus arcus occultatur a linea.

[4.134] Si vero linea sumpta non attingit circulum, poterit quidem videri, sed modicum est. Si vero sumatur linea declinationis predicte inter lineam reflexionis et lineam per punctum reflexionis primo sumptum transeuntem ad centrum, poterit quidem videri hec linea, et minuetur curvitas ymaginis huius linee secundum quod magis accesserit ad lineam transeuntem ad centrum per punctum reflexionis.

[4.135] Si vero sumantur linee inter lineam ad centrum transeuntem per punctum reflexionis, videbuntur quidem sive declinatio earum sit ex parte visus, sive non. Et modus visus earum simili modo visus linearum inter lineam reflexionis et lineam ad centrum transeuntem. Et hec quidem intelligenda sunt de lineis concurrentibus in arcu circuli qui apparet visui, id est in arcu qui interiacet duas contingentes ductas a centro visus ad circulum.

[4.136] Linearum autem concurrentium cum circulo in parte circuli occulta visui aliqua erit equidistans linee reflexionis. Illa quidem non videbitur. Similiter, conterminabilis equidistanti que est sub

248 puncta *scr. et del. E* 249 *post* videtur *scr. et del.* eum *C1* 250 declinationis: declinata *R*/non: sit *R*/*post* non *inter.* est *O* 251 iacens: adiacens *L3; rep. et del. F*/quidem *om. ER* 252 nullus punctus: nullum punctum *R*/illius *om. P1* 253 punctus ille: punctum illud *R*/potest: possit *R* 254 aliquo: alio *FP1OL3E; corr. ex* alio *C1* 255 speculi ductam *transp. O*/*post* ab *scr. et del.* alio *O* 256 illo puncto *transp. O* 257 punctus: punctum *R*/sectionis *om. L3*/reflectitur: reflectetur *R*/*post* arcus *add.* speculi *R* 259 punctus sumptus: punctum sumptum *R*/refertur: reflectatur *R* 260 *post* puncto *scr. et del.* a circulis *F*/erit impossibile *transp. FP1*/quia: quod *L3; corr. ex* quod *a. m. E* 261 totus *alter. in* punctus *O* 262 attingit: attingunt *P1;* attingat *R*/poterit . . . modicum (263) *mg. O* 263 *post* sed *scr. et del.* pdocum *O*/*post* modicum *add.* linea E sub ipsa linea erit impossibile *O*/est *om. FP1O*/declinationis: declinata *R*/predicte: predicta *R* 265 videri: videre *FP1* 266 minuetur: imminuetur *R* 267 reflexionis *corr. ex* ionis *a. m. E* 268 si . . . reflexionis (269) *mg. a. m. E* 269 per: in *FP1*/sive: sine *L3* 270 *ante* earum *scr. et del.* sit *C1*/sit *inter. O*/modus *corr. ex* modo *C1*/simili: similis *L3C1E* 271 inter *corr. ex* in *O*/inter . . . reflexionis *corr. ex* reflexionis inter lineam *O* 273 qui[1]: quia *L3*/id est *mg. C1* 274 contingentes: continens *FP1L3E;* continentes *O* 276 aliqua *om. L3*/*post* reflexionis *add.* et *R*/*post* quidem *inter.* equidistans *O* 277 conterminabilis: conterminalis *OR*

equidistanti occultabitur, sed conterminabilis equidistanti supra ipsam existens poterit videri.

[4.137] Si vero sumatur linea inter equidistantes non conterminabilis aliqui earum, si fuerit eius declinatio ex parte visus, videbitur. Si ex alia parte, aliquando videbitur, aliquando non, quoniam, si a termino eius producatur equidistans linee reflexionis, si fuerit linea illa sub equidistantem, non videbitur; si supra eam videri poterit.

[4.138] Si vero linee non concurrant cum circulo, aut secabunt lineam ductam a centro visus ad centrum speculi, aut equidistabunt ei. Si secet aliquam earum, linea illa aut secabit eam ex parte visus, id est inter visum et speculum, aut ultra speculum. Si ultra, occultabitur linea illa, sed forsan apparebunt eius capita. Si vero secet lineam visualem ex parte visus, apparebit quidem similiter. Si fuerit equidistans linee visuali, poterit videri. Omnium autem harum linearum ymagines curve.

[4.139] Visu autem existente in eadem superficie cum centro speculi et lineis visis, diminuta est apparentia, et que sic manifestius apparet est illa que declinata est maxima declinatione et illa visum respiciente. Pari modo, arcuum in hiis speculis apparentium et in eadem superficie cum centro speculi et visu existentium, ymagines quidem curve curvitate speculum respiciente.

[4.140] Hec autem intelligenda sunt duplici visu existente in eadem superficie cum centro speculi et re visa. Si enim alter visus modicum declinetur quoad ipsum, alio modo res visa comprehendetur. Et visu existente extra superficiem rei vise et centri speculi, certior erit ipsius rei comprehensio quam existente in ea.

[4.141] **[PROPOSITIO 15]** Quod autem ymago rei vise sit curva, visu existente in superficie centri speculi et rei vise, probabitur.

[4.142] Sit D [FIGURE 6.4.15, p. 311] centrum visus, G centrum speculi. HE sit linea visa, que quidem HE non concurrat cum circulo,

278 equidistanti[1]: equidistante *R*/conterminabilis: conterminalis *R*/*post* conterminabilis *add.* quidem *FP1* 279 existens *corr. ex* exterus *O*/existens poterit *transp. C1*/videri *om. C1* 280 sumatur linea *transp. C1*/linea *inter. O*/conterminabilis: terminabilis *E*; conterminalis *R* 281 aliqui: alium *L3*; alicui *E* 282 alia parte *transp. L3*/quoniam si *inter. O* 283 producatur: ducatur *R* 284 illa *om. R*/equidistantem: equidistanti *E*/videbitur: videtur *L3E*/si *inter. a. m. C1*/eam: eum *E* 287 aliquam: alia *OL3E*; *corr. ex* alia *C1*; aliqua *R*/earum: illarum *P1*/linea illa *transp. P1C1*; lineam illam *O*/eam: illam *L3ER* 288 occultabitur: concultabitur *FP1O* 289 eius capita *transp. C1*/*post* secet *scr. et del.* i *C1* 291 visuali: visualis *O*/autem *om. L3* 292 ymagines *corr. ex* ymaginis *O* 293 in *inter. C1* 294 et[2] *om. FP1*/sic: sit *ER*/*ante* manifestius *inter.* que *ER* 295 declinata: declarata *FP1OL3E*/declinatione: declaratione *FP1L3E* 296 hiis *om. FP1* 297 et *inter. O*/*post* ymagines *scr. et del.* circa *F* 298 quidem *om. FP1*/*post* curve *add.* sunt *R* 299 autem: aut *FP1E* 300 et *inter. O* 1 comprehendetur *corr. ex* comprehensa *O* 2 centri: centrum *L3ER* 4 autem *om. O* 7 HE *corr. ex* E *E*/concurrat: currat *E*/cum *om. P1*/*post* circulo *add.* speculi *R*

sed sit equidistans linee DG, vel secet eam ex parte D. Sumatur superficies in qua sunt linea DG et linea HE; circulus communis huic superficiei et speculo sit AB.

[4.143] Producatur linea HG. Z sit ymago H, punctus circuli a quo refertur H ad D sit B, et a puncto B ducatur contingens, que secet lineam HG super punctum T. Erit T finis contingentie.

[4.144] Ducatur linea GB, que producta necessario concurret cum HE, si enim HE fuerit equidistans DG, concurret quidem. Si vero DG concurrat cum HE, multo fortius GB concurret cum eadem. Concursus ille aut erit in linea HE, aut ultra hanc lineam.

[4.145] Sit ultra. Concurrat in puncto M; ymago puncti M sit Q; finis contingentie sit S. Et ducatur linea ZQ, similiter linea TS, et producatur a puncto A contingens AU. Palam quoniam AB est minor quarta, quare D videat ex circulo minus medietate, quare angulus AGB est acutus, et angulus UAG est rectus. Igitur AU concurret cum GB. Concurrat in puncto U. Dico quoniam punctus U cadat supra punctum S.

[4.146] Cum enim punctus M reflectatur ab aliquo puncto arcus AB, et A sit dimissior illo puncto, erit finis contingentie A altior fine contingentie illius puncti. Et ita S dimissior puncto U. Procedat ergo TS donec concurrat cum linea AU, et sit concursus in puncto K.

[4.147] Et ducatur linea GK, que producta concurrat cum HM in puncto C. Punctus C refertur ad D ab aliquo puncto arcus AB. Sit ille punctus F, a quo ducatur linea contingens usque ad GC, que quidem dimissior linea AK, et erit punctus O dimissior puncto K.

[4.148] Sit O finis contingentie. Ducatur linea DF usque cadat super GC. Sit casus in puncto R. Et producatur ZQ usque ad lineam GC, et cadat in puncto L. Dico quoniam L est supra R.

8 sit *inter. O*/sumatur *om. R* 9 *ante* in *add.* incidentie sit *R*/sunt: sint *R*/linea^{1}: linee *R*/et linea *om. R*/*post* huic *scr. et del.* S equalis *O* 10 AB: AD *L3* 11 *post* HG *add.* et punctum in ipsa *R*/Z *inter. P1*/H *om. P1*/punctus: punctum *R* 12 refertur: reflectitur *R*/a *inter. a. m. E*/*post* ducatur *add.* linea *R*/contingens: communis *FP1L3E; corr. ex* communis *O*/que: qui *L3* 14 GB: BG *R; om. P1*/que . . . DG (15) *mg. O*/necessario *om. P1O*/*post* cum *add* linea *O* 15 quidem: equidem *L3* 16 concurrat: concurret *FP1OL3E; corr. ex* concurret *a. m. C1*/cum^{1} *inter. O*/concurret *om. OL3E*/concurret cum eadem: cum eadem concurret *R* 18 sit^{1}: si *C1*/*post* ultra *inter.* et *O* 19 *post* TS *add.* et DG secet circulum in A *R*/producatur: ducatur *R* 20 contingens: continens *O*/AU: aut *FP1*/quoniam: quod *R* 21 *post* quarta *add.* circuli *R*/quare^{1}: quarum *L3;* cum *R; alter. in* cum *O*/*post* quare^{1} *add.* D cum *deinde del.* D *C1*/medietate: mediante *L3E*/angulus *inter. O* 22 AGB: ABG *C1E*/UAG *corr. ex* AND *O* 23 GB: BG *C1R*/concurrat *corr. ex* concurret *a. m. E*/quoniam: quod *R*/punctus: punctum *R*/cadat: cadet *C1R;* cadit *E* 25 punctus *inter. a. m. E; om. R*/ab: a *R*/aliquo: alio *OL3; alter. ex* illo *in* alio *a. m. E*/aliquo puncto *transp. R*/arcus . . . puncto (26) *mg. a. m. E* 26 dimissior: demissius *R*/A^{2}. . . contingentie (27) *om. P1* 27 dimissior: demissius *R* 28 TS: OS *F;* DS *P1*/cum *om. FP1L3; inter. E*/K *inter. E* 29 GK: GTK *O* 30 C^{1}: O *L3E*/punctus: punctum *R*/punctus C *inter. O*/C^{2}: O *L3*/refertur: reflectitur *R*/aliquo: alio *OL3E*/ille punctus (31): illud punctum *R* 31 ad *inter. a. m. C1*/*post* ad *inter.* lineam *a. m. C1*/GC *om. L3*/*post* quidem *add.* erit *R* 32 punctus O dimissior: punctum O demissius *R* 33 usque: quousque *R* 34 *post* GC *scr. et del.* sit punctus *C1*/sit casus: cadat *R*/casus: cadens *L3C1E*/puncto: punctum *R*/ZQ *corr. ex* Q *O* 35 puncto: punctum *R*/dico quoniam *transp. deinde corr. O*/quoniam: quod *R*/L: LF *L3E; corr. ex* LF *C1*

[4.149] Linee enim HC, TK, ZL aut sunt equidistantes, aut concurrent. Sint equidistantes. Cum ergo hee equidistantes, secent lineam CG super tria puncta C, K, L, et secent utramque linearum MG, HG. Et proportio HG ad HT sicut GZ ad ZT; similiter, MG ad MS sicut GQ ad QS. Erit proportio eadem GC ad CK sicut LG ad LK.

[4.150] Sed palam quoniam R est ymago C, linea enim DF linea reflexionis concurrens cum CG in puncto R, et O finis contingentie, quare proportio GC ad CO sicut GR ad RO. Sed maior GC ad CK quam GC ad CO, et ita maior GL ad LK quam GR ad RO. Ergo maior OR ad RG quam KL ad LG, et ita maior OG ad RG quam KG ad LG. Sed KG maior OG, quare LG maior RG. Igitur R dimissior puncto L. Sed ZQL est linea recta. Igitur ZQR est linea curva, et ita ymago linee HC est curva. Posito ergo aliquo puncto linee HC loco puncti M et puncto E loco puncti C, erit probare quod ymago HE est curva.

[4.151] Si vero linee HC, TS, ZQ concurrant, aut erit concursus ex parte D, aut ex parte HG. Sit ex parte D [FIGURE 6.4.15a, p. 311], et sit concursus in puncto C. Erit ZQC linea recta, quare ZQR erit curva, et ita ymago linee HE curva, quod est propositum.

[4.152] Si vero proponatur arcus extra speculum, erit de eo probare quod ymago sit curva sicut probatum est visu non existente in eadem superficie cum arcu et centro speculi, et hoc est propositum.

[4.153] Igitur in hiis speculis linee recte apparent curve, et curve similiter apparent curve. Si autem proponatur visui in hiis speculis corpus curvum sed longum, modicum habens latitudinis, apparebit quidem illius corporis curvitas manifeste, cum ipsa discerni possit per ea que supra corpus aut intra. Non enim plane discernitur curvitas nisi magna, ubi occulte fuerint extremitates longitudinis et latitudinis, unde proposito visui corpore convexitatis modice et quantitatis

36 concurrent: concurrunt *R* 37 sint: sunt *FP1* / equidistantes^{2} *inter. O* / *post* equidistantes^{2} *add.* hec *FP1* / secent: secant *FP1* 38 CG: GC *R* / super: sunt *FP1O* 39 ZT *corr. ex* ZF *a. m. E* 41 quoniam: quod *R* / C: O *FP1* 42 concurrens: concurrit *R* / CG: OG *E* / O *om. FP1* 43 quare: qua *L3* / *post* GC^{1} *scr. et del.* sicut *O* / CO *corr. ex* CK *O* / *post* maior *add.* est proportio *R* 44 quam^{1}: quoniam *FP1O* / Gl *corr. ex* GCL *E* / *post* ad^{2} *scr. et del.* C *C1* / *post* maior^{2} *add.* est proportio *R* 45 KL: LK *ER* / LG: BG *FP1* / et . . . LG^{2} *om. L3; scr. et del. O* / *post* maior *add.* est proportio *R* 46 KG: CG *C1* / *post* maior^{1} *add.* est *R* / OG: EG *FP1;* CG *OL3E;* KG *C1* / *ante* quare *scr. et del.* igitur *O* / R *inter. O* / dimissior: demissius est *R* / L: H *FP1* 48 est . . . HC^{2} *mg. a. m. E* / aliquo: alio *FP1O* / linee *om. FP1L3* / HC^{2}: HE *R* 49 HE: E *O* 50 si . . . propositum (53) *mg. a. m. O* 51 HG: HDG *L3E; corr. ex* HDG *C1* / *post* HG *scr. et del.* sicut *F* / sit: si *FP1* / D^{2} *om. P1* 52 ZQC: ZQT *R* 53 linee *om. P1* / *post* curva *add.* est *L3* 54 proponatur *corr. ex* proponantur *E* / de eo probare: probare de eo *R* 55 ymago sit *transp. L3* / sit: est *L3* 57 et . . . curve (58) *om. L3; mg. a. m. O* / *post* et *add.* similiter *E* / curve similiter (58) *transp. R* 58 similiter *rep. F* / *post* apparent *add.* similiter *R* / proponatur: proportionatur *L3E* 59 curvum: ? *E* / modicum: modicam *R* / latitudinis: longitudinis *L3E* 60 illius corporis *transp. R* 61 corpus *inter. O* / *post* corpus *add.* sunt *R* / intra: infra *L3C1E* / plane *alter. in* plene *O* 62 *post* occulte *scr. et del.* f *F* / et latitudinis *om. P1* 63 corpore *inter. O* / convexitatis *corr. ex* convectans *a. m. E* / *post* quantitatis *scr. et del.* m *F*

magne, non planum discernitur eius convexitatis, licet ymago ipsius sit convexa, cum non appareant termini corporis in longitudine vel latitudine.

[4.154] Amplius, errores in speculis planis accidentes omnes accidunt et in hiis, et preter illos accidit ymagines linearum rectarum esse curvas, quod a speculis planis est remotum.

[PARS QUINTA
In speculis columpnaribus exterioribus]

[5.1] Amplius, in speculis columpnaribus exterioribus errores accidunt idem qui in speculis spericis exterioribus, linee enim recte videntur curve et diminuta apparet rei vise quantitas, ut in hiis, sed longe fortius quam in eis, quoniam in spericis res magna apparebit quidem minor, sed non multum parva, sed in hiis res etiam maxima videbitur minima. Similiter, linea recta apparebit curva in spericis speculis, sed si modice curvitatis in columpnaribus maxime, unde multiplicantur errores columpnaris speculi super errores sperici.

[5.2] Verum in columpnaribus aliquando fit reflexio a linea recta, scilicet a longitudine speculi, aliquando a circulo, aliquando a sectione. Quando linea visa fuerit equidistans longitudini speculi, fiet reflexio a linea longitudinis, et linea visa apparebit recta modice curvitatis. Et hec quidem probabuntur, ad quorum probationem necesse est quiddam premitti, quod hoc est:

[5.3] **[PROPOSITIO 16]** Sumpta columpnari sectione, et sumpto in ea puncto qui non sit punctus reflexionis, si ab illo puncto ducatur linea ad perpendicularem que est a puncto reflexionis ad axem—et linea illa faciat angulum acutum cum perpendiculari—si ducatur a puncto sumpto linea que sit ortogonalis super contingentem illius

64 non *scr. et del. C1* / planum: plane *ER; alter. in* plana *C1; alter. in* plene *O* / convexitatis: convexitas *OR* (*alter. in O*) / ymago ipsius *transp. FP1* 65 non: enim *L3E* / corporis *inter. O* / vel *inter. O* 67 errores: erroris *FP1* / accidentes omnes *transp. FP1* / accidunt *corr. ex* accedunt *O* 68 et *om. L3C1E* 69 planis *inter. a. m. E* 70 *ante* amplius *add.* de erroribus qui accidunt in speculis columpnaribus convexis capitulum quintum *R* / errores: exteriores *FP1* 71 idem: hidem *C1* / enim *om. P1* 72 diminuta *corr. ex* diminutea *F* / vise *om. R* / quantitas *corr. ex* quantitatis *C1* / ut in hiis *om. R* 73 *post* fortius *add.* in his *R* / in^2 ... magna: magna res in spericis *C1* 74 quidem *om. FP1* / multum parva: multo minor *R* 75 recta *om. P1* / curva *mg. a. m. E* / spericis *om. P1* / spericis speculis (76) *transp. ER* 76 sed: et *O* / si *om. OL3C1ER* / curvitatis: curvitas *P1; corr. ex* curvitas *E; corr. ex* curvitates *L3* / *post* maxime *add.* curvitatis *ER* (*inter. a. m. E*) / unde *om. O* 77 sperici: spericis *FP1* 81 a linea: aliena *C1* 82 probabuntur: probantur *E* 83 est *om. OR; inter. E* / quiddam *corr. ex* quidam *C1* / premitti: puncti *L3* / hoc: huiusmodi *R* 85 qui: quod *R* / punctus: punctum *R* 87 linea illa *transp. C1* / *post* linea *scr. et del.* recta *F* / faciat *corr. ex* faciet *O* / si: sicut *L3* 88 ortogonalis: ortogonaliter *FP1* / contingentem: continentem *O* / illius puncti (89): illud punctum *R*

puncti, hec linea concurret cum perpendiculari sub axe et sub concursu prioris linee cum perpendiculari.

[5.4] Verbi gratia sit AEB [FIGURE 6.5.16, p. 312] sectio, E punctus datus, N punctus visus, B punctus reflexionis, BD perpendicularis, EDB angulus acutus, QEL contingens.

[5.5] Supra B fiat circulus columpne equidistans basi, scilicet BTO, et ducatur a puncto E linea longitudinis columpne, scilicet ET. Ducatur axis DH, et ducatur linea DC perpendicularis supra BD.

[5.6] Palam quod superficies HDC est ortogonalis super superficiem circuli. Superficies vero contingens columpnam in puncto B erit equidistans huic superficiei, quoniam linea longitudinis ducta a puncto B erit equidistans axi, et contingens supra B erit equidistans CD. Igitur superficies in qua sunt linee LE, ET non est equidistans superficiei HDC. Igitur concurret cum ea. Concurrat in linea LC, et ducatur linea TC, que quidem erit contingens, cum superficies LET sit contingens. Ducta autem linea TD, erit angulus CTD rectus, quoniam TD dyameter.

[5.7] Fiat autem supra E circulus columpne equidistans basi, scilicet ESP. Punctus axis in hoc circulo sit K, et ducatur linea KE. Ducatur etiam linea DL, que quidem secabit superficiem circuli ESP. Secet in puncto F, ubicumque sit punctus extra circulum vel intra, et ducantur linee KF, EF. Et a puncto F ducatur perpendicularis super superficiem circuli BTO que sit FM, et ducatur linea TM.

[5.8] Palam quoniam KD equidistans et equalis FM, et ita KF equidistans et equalis DM. Similiter KD equidistans et equalis ET, et KE equidistans et equalis DT. Erit ergo TE equidistans et equalis FM, et ita EF equidistans et equalis TM.

89 concurret: concurrit *FP1* / *post* perpendiculari *scr. et del.* cu *P1* / sub[1] *corr. ex* cum *a. m. E* / sub[1]. . . perpendiculari (90) *rep. FP1O* (*inter. O*) 90 prioris: minoris *E* 91 sit *om. L3E* / *post* AEB *scr. et del.* et B *O* / punctus: punctum *R* 92 datus: datum *R*; *om. O* / punctus[1]. . . punctus[2] *mg. O*; punctum visum B punctum *R* 93 EDB: EBD *C1* / contingens: continens *O* 94 supra: super *R* / *post* fiat *scr. et del.* a *E* / columpne . . . basi: equidistans basi columpne *R* / equidistans: quidem *L3E* 95 ET *inter. O*; ? *E* 96 *post* ducatur[2] *add.* a *FP1* / DC: DG *R* / supra: super lineam *R* 97 HDT: HDG *R* 98 contingens: continens *P1O* 99 linea longitudinis *transp. E* 100 erit[1, 2]: est *R* / contingens: continens *O* / *post* contingens *add.* circulum *R* / supra: super *R* / *post* equidistans[2] *add.* supra *L3* 101 TD: DG *R* / LE *om. P1* / LE EC *inter. O* 102 HDT: HDG *R* / igitur concurret *transp. R* / concurret: concurrit *FP1* / concurret cum ea: cum ea concurret *C1* / concurrat *corr. ex* concurret *C1* / in linea *rep. P1* / LT: LG *R* 103 TC: TG *R* / LET *corr. ex* LE *C1* 104 sit *corr. ex* si *O* / TCD: GTD *R* 106 autem: quando *FP1* / supra: super *R* / E *mg. C1* / columpne . . . basi: equidistans basi columpne *R* / basi: basis *O* 107 ESP *corr. ex* EZP *O* / punctus: punctum *R* / *post* KE *add.* et *C1* 108 linea DL *transp. FP1* / que *om. FP1; inter. E* 109 punctus: punctum *R* / circulum: circumferentiam *R* / intra: ultra *L3* 110 F *om. FP1; inter. O* 111 BTO: BRO *L3* 112 quoniam: quod *R* / *post* equidistans *add.* est *R* / FM: CFM *E* 113 KD: FM *R* / ET *inter. O* / et[3] *om. FP1* 114 equidistans et equalis: equalis et equidistans *R* / et equalis DT *inter. a. m. E* / erit . . . FM *om. R* 115 EF: F *O* / *post* EF *add.* erit *L3ER*

[5.9] Verum superficies KDL est ortogonalis super superficiem sectionis BEO, et est ortogonalis super superficiem circuli ESP. Ergo est ortogonalis super lineam communem sectioni et circulo que est EF. Igitur angulus EFK rectus. Similiter angulus TMD rectus.

[5.10] Cum ergo angulus DTC sit rectus, multiplicatio DM in MC sicut TM in FE, sed quoniam FM equidistans CL, erit proportio DF ad FL sicut DM ad MC. Sed DF maior DM; igitur FL maior MC. Igitur maior est multiplicatio DF in FL quam DM in MC, quare, cum TM sit equalis EF, erit multiplicatio DF in FL maior ductu linee EF in FE, quare angulus LED maior recto, si enim esset rectus, cum linea EF sit perpendicularis super LD, esset ductus DF in FL equalis quadrato EF. Restat ergo ut angulus DEQ sit acutus. Ergo ortogonalis ducta a puncto E, ortogonalis in quam super contingentem QL, cadet sub lineam ED et concurret cum perpendiculari BD sub puncto D, quod est propositum.

[5.11] Hiis premissis, accedendum est ad propositum.

[5.12] **[PROPOSITIO 17]** Proponatur columpna [FIGURE 6.5.17, p. 313]; linea equidistans axi sit TH. Erit quidem TH equidistans linee longitudinis columpne.

[5.13] Si ergo visus fuerit in eadem superficie cum axe et linea TH, poterit quidem reflecti linea, et erit reflexio a linea longitudinis columpne, que linea est communis superficiei in qua sunt visus et axis et superficiei columpne, sicut ostensum est in libro quinto. Sic videbitur linea TH linea recta, quoniam quelibet perpendicularis ducta a puncto linee TH erit in eadem superficie cum visu et axe, et probabitur ymaginem linee TH esse rectam, sicut probatur in speculis planis de visis lineis.

[5.14] Sit autem visus extra superficiem linee TH et axis, et TH equidistans axi, qui axis sit ZK. Fiat superficies per visum transiens

116 *post* verum *add.* etiam *C1* / ortogonalis *corr. ex* cor *O* 117 BEO: BET *FP1L3E;* BEA *C1* 118 ortogonalis: perpendicularis *R* 120 DTC: DMT *R* / *post* rectus *add.* et GTD rectus *R* / MC: MG *R* 121 *ante* sicut *add.* erit *R* / equidistans: equidistat *ER* / CL: D *FP1;* TB *L3;* GL *R* / ad FL (122) *om. FP1* / ad . . . DF (122) *mg. a. m. E* 122 $MC^{1,2}$: MG *R* / sed: si *P1* 123 est *inter. O* / quam . . . FL (124) *mg. a. m. E* / MC: MG *R* / *ante* quare *add.* ergo maior quam TM in FE *R* / TM: OM *O* 124 FL: FB *L3E* / EF: ES *L3* 125 esset rectus *transp. ER* 126 *post* esset *scr. et del.* D *O* / ductus *corr. ex* ductis *E* / DF *alter. ex* DI *in* DS *O* / FL: LF *C1* 128 sub: supra *FP1L3E;* super *OC1* 129 lineam: linea *R* 133 quidem: equidem *FP1* / TH^2 *om. R* 134 longitudinis: E lineis *FP1* 135 superficie *om. C1* 136 et *inter. C1* 137 *post* columpne *scr. et del.* si ergo visus fuerit in eadem superficie *C1* / linea est *transp. ER* / est communis *transp. C1* / superficiei *corr. ex* superficie *E* 138 sicut *corr. ex* sint *O* / *post* sic *add.* igitur *R* 139 TH: TB *L3E* / linea *om. L3E* / quoniam: quando *L3* 141 sicut: sic *E* / probatur: probabitur *L3C1E;* probatum est *R* / speculis *corr. ex* speculo *F* 142 de visis: tenis *L3* / visis: divisis *FP1;* rectis *R* 143 sit: si *R* / *post* visus *add.* sit *R* / et^1. . . TH^2 *scr. et del. E* / et^2 *om. L3E; inter. C1* 144 equidistans: equidistat *P1;* equidistet *R* / per visum transiens: transiens per visum *C1*

secans superficiem columpne equidistantem basi. Secabit quidem super circulum. Sit circulus ille BF. Aliquis punctus linee HT refertur ad visum ab aliquo puncto huius circuli. Sit a puncto B, et visus sit E.

[5.15] Punctus ille linee TH sit Q, et ducantur linee EB, QB, et ducatur a puncto B linea longitudinis, que sit ABG, et ducatur a puncto B perpendicularis cadens super axem in puncto L, que sit ML. Et ducatur a puncto E linea equidistans ML, que sit EO, et ducatur QB usque dum concurrat. Sit concursus in puncto O.

[5.16] Palam quoniam angulus QBM equalis est angulo EBM, sed angulus QBM equalis angulo BOE, quia LM equidistans OE. Similiter angulus MBE equalis angulo BEO, quia coalternus. Igitur angulus BOE equalis est angulo BEO, quare BO, BE equalia.

[5.17] Sumatur autem alius punctus in linea TH, qui punctus sit T, et ducatur linea TO. Palam quoniam linea TH equidistans linee longitudinis, que est AG. Ergo sunt in eadem superficie, et in illa superficie est linea QBO, quare in eadem erit linea TO. Secabit ergo lineam AG. Secet in puncto G, et ducatur linea EG.

[5.18] Palam etiam quoniam linea AG est perpendicularis super superficiem circuli BF sicut axis cui equidistat, et superficies illius superficies EOBF secans scilicet columpnam equidistantem basi. Igitur angulus GBO rectus, et angulus GBE rectus. Ergo quadratum linee GO valet quadratum linee GB et quadratum linee BO. Similiter quadratum GE valet quadrata GB et BE, et quoniam BE, BO equalia et GB communis, erit GO equalis GE. Igitur angulus GOE equalis angulo GEO.

145 equidistantem: equidistanter *R* 146 super: secundum *R; om. SL3E*/super circulum *transp. deinde corr. C1*/aliquis punctus: aliquod igitur punctum *R*/HT: HS *L3*/refertur: reflectitur *R* 147 aliquo: alio *SOL3E*/a *om. R*/puncto: punctum *R* 148 E *om. FP1* 149 punctus ille: punctum illud *R*/ille linee *transp. deinde corr. C1*/QB: QL *FP1*/*post* QB *add.* QE *R* 150 ducatur[1]: ducantur *L3; corr. ex* ducantur *C1*/linea ... B (151) *om. S; mg. a. m. E*/ABG *om. FP1*/ducatur[2]: ducantur *L3* 152 *post* linea *add.* longitudinis *P1*/ML: LM *R* 153 QB: BQ *C1*/usque: quousque *R*/dum *om. R*/*post* concurrat *inter.* cum eo *O* 154 quoniam: quod *R*/equalis est *transp. SER*/est *om. L3* 155 *post* equalis *add.* est *R*/OE: OZ *FP1;* EO *C1* 156 angulus: angulis *F* 157 est *inter. a. m. E*/*post* quare *add.* latera *R* 158 alius punctus: aliud punctum *R*/qui: quod *R*/punctus[2] *om. R*/qui ... TH (159) *om. S* 159 T: TG *FP1*/quoniam: quod *R*/equidistans: equidistat *ER* 160 illa: eadem *C1* 161 linea[2] *om. FP1*/TO: TQ *R* 162 et *om. SL3ER*/et ducatur *corr. ex* educatur *O*/linea *corr. ex* lineam *S* 163 quoniam: quod *FP1R*/AG *rep. et del. F*/est *corr. ex* et *O* 164 cui equidistat *transp. S*/equidistat: equidistabit *S; alter. ex* equidistabit *in* equidistabat *E; corr. ex* equidistant *C1*/superficies[1]: superficiei *L3*/*post* illius *add.* circuli *R* 165 equidistantem: equidistanter *SC1ER*/basi: bas *S;* basis *L3* 166 GBO *corr. ex* BGO *C1*/*post* GBO *add.* est *R*/*post* GBE *add.* est *R*/et angulus *om. P1*/GBE: GEBE *S*/*post* GBE *inter.* est *E*/rectus[2] *om. S; inter. E* 167 GB: BG *ER;* BO *S*/et ... BO *om. FP1S; mg. a. m. E* 168 quadrata: quadratum *L3E*/*post* BE[2] *add.* et *R*/equalia: sunt equales *R*/et[2] *om. C1*/*post* BE[2] *add.* et *SL3E*/et[3] *om. O* 169 communis: communi *O*

[5.19] Ducta autem perpendiculari ZGN, erit equidistans EO, cum sit equidistans MBL. Igitur angulus TGN equalis angulo GOE, et angulus NGE equalis angulo GEO, quare angulus TGN equalis angulo NGE. Cum autem E, O, N, G, Z sint in eadem superficie, et in illa sit G, E, G, T erunt in eadem superficie, et ita in eadem superficie sunt linee EG, NG, TG. Igitur T refertur ad E a puncto G.

[5.20] Sumpto autem in linea TH puncto H eiusdem longitudinis a puncto Q cuius est punctus T, et ducta linea HO, transibit quidem per punctum linee AG. Transeat per punctum A. Ducta perpendiculari DA et lineis EA, HAO, erit sicut prius probare quod duo anguli ABO, ABE recti, et duo latera AO, AE equalia, et duo anguli HAZ EAZ equales. Et ita H refertur ad E a puncto A. Similiter sumpto quocumque puncto linee TH, erit probare quod refertur ad E ab alio puncto linee AG, quare linea TH refertur a linea longitudinis que est AG.

[5.21] **[PROPOSITIO 18]** Restat probare ymaginem linee TH esse curvam. Palam ex predictis quoniam Q refertur ad E a puncto B, qui est punctus circuli. Sed cum sic refertur a circulo, si ducatur linea a puncto Q ad centrum illius circuli, concurret cum perpendiculari ducta a puncto B, et erit concursus in puncto axis. Ducatur ergo QL concurrens cum ML in puncto axis qui est L, et est centrum circuli FB. Et producatur EB usque concurrat cum QL. Sit concursus in puncto C. Erit C ymago Q, et est C in superficie in qua sunt linee QH, et axis, et linea longitudinis AG.

[5.22] Palam etiam quod T refertur ad E a puncto sectionis columpne, scilicet a puncto G. Est autem a puncto T lineam ducere perpendicularem super lineam contingentem in puncto alio sectio-

171 *post* perpendiculari *add.* super axem *R* / erit equidistans *transp. ER* 172 TGN: ZGN *FP1* / *post* TGN *add.* est *P1* / GOE *alter. ex* GNE *in* NGE *E* 173 et . . . NGE (174) *scr. et del. E* / angulo[2] *om. R* 174 E *om. FP1* / EO *inter. E* / E . . . Z: TGO NGZ *R* / O *om. S* / et . . . E (175): in qua G ergo puncta O *R* / et . . . superficie[2] (175) *om. L3* 175 G[1] *om. O* / *post* E *add.* O *C1* / erunt: erit *O* / *post* superficie[1] *scr. et del.* et in illa *S*; *add.* ET *SE*; *add.* cum ET *O* / ita *inter. O* 176 NG: OG *R* / refertur: reflectitur *R* 177 *post* linea *add.* TG *P1* 178 Q *om. FP1* / punctus: punctum *R* / et *inter. O* / ducta linea *transp. SL3ER* / HO: HD *L3* / transibit . . . HAO (180) *om. FP1* 179 transeat: transiet *SE* / ducta: ducte *S* / *post* ducta *add.* que a puncto A super axem *R* / perpendiculari: perpendicularis *S* 180 DA: DD *S* / lineis: linea *R*; *corr. ex* longitudinis *O* / HAO: HAC *L3E* (*alter. in E*); *om. R* 181 ABO ABE *transp. C1* / AO: AC *FP1* / HAZ: HAR *R* 182 EAZ: EAR *R* / refertur: reflectetur *R* 183 quocumque *corr. ex* quoque *O* / *post* erit *add.* probare quod refertur ad E a puncto A similiter sumpto quocumque puncto linee TH erit *FP1* / refertur: reflectatur *R* / ad E *om. R* / alio: aliquo *FP1C1R* 184 linea[1] *om. C1* / refertur: reflectetur *R* 187 esse: ee *S* / quoniam Q refertur: quod Q reflectitur *R* / a *om. S* 188 qui *corr. ex* que *F* / qui est punctus: quod est punctum *R* / sic: sit *L3E* / refertur: reflectatur *R* 189 a *om. F* / Q: B *L3* / circuli *om. C1* 191 qui: quod *R* 192 usque: quousque *R* 193 *post* C[1] *scr. et del.* erunt *P1* 194 et *inter. O* 195 refertur: reflectitur *R* 196 columpne: columpnaris *R* / lineam: unam *OL3C1ER* 197 contingentem *om. L3* / puncto alio *transp. R* / alio: aliquo *FP1C1ER* / sectionis: sectionem *R*

nis, que quidem concurret cum perpendiculari ducta a puncto G, que est NGZ, sub axe, id est sub puncto Z, qui est concursus perpendicularis NZ et axis, quoniam ducta linea TZ, erit angulus TZN acutus. Ducatur ergo TX concurrens cum NZ in puncto X, et producatur EG donec concurrat cum TX in puncto I. Erit I ymago puncti T.

[5.23] Similiter ducta a puncto H linea, que sit ortogonalis super punctum sectionis a qua refertur, concurret cum perpendiculari DAZ sub puncto D, que est punctus axis. Concurrat in puncto P, et producatur EA donec concurrat cum HP in puncto S. Erit ymago puncti H punctus S. Ducatur autem linea SI.

[5.24] Palam cum linea TI concurrat cum perpendiculari NZ, que est equidistans linee EO, concurret cum linea EO. Similiter linea HS; quoniam concurrit cum perpendiculari DAZ, que est equidistans EO, concurret cum EO. Sed quoniam situs T respectu puncti E idem est cum situ H et eadem longitudo, similiter situs puncti T et puncti H ad punctum O idem, et punctorum I, S respectu O etiam est idem. Erit idem situs linearum TI, HS respectu linee EO.

[5.25] Igitur linee TI, HS concurrent super idem punctum linee EO. Concurrant in puncto U. Erit TUH triangulus, et in superficie huius trianguli erit linea IS. Axis autem non est in hac superficie.

[5.26] Verum TH est in eadem superficie cum axe; ergo superficies illa secat superficiem trianguli super lineam communem que est TH, non super aliam. Cum ergo punctus C sit in superficie linee TH et axis, et non sit in linea TH, non est in superficie trianguli TUH, et duo puncta I, S sunt in superficie illius trianguli, quare linea ICS est curva, et ymago linee TH erit curva, quod est propositum.

[5.27] Sed eius curvitas est modica, quia perpendicularis ducta a puncto C ad superficiem circuli est valde parva, et quanto maior fue-

198 G: igitur *L3E* / que: qui *L3* 199 est[1] *om. L3* / qui: quod *R* 200 NZ: ZNZ *S* / *post* TZ *add.* et *FP1* / TZN: TNZ *E* / *post* acutus *add.* producatur NZ ultra Z in X *R* 201 TX: IX *E* / NZ *corr. ex* NY *a. m. E* / X *corr. ex* I *S* 202 TX: IX *C1* / *post* erit *scr. et del.* Y *F* / *post* I *add.* Y *O* 203 *ante* similiter *add.* et *C1* / a puncto *om. FP1* / *post* super *add.* lineam contingentem in speculum in puncto aliquo *R* 204 punctum *om. R* / qua: quo *R* / *post* qua *add.* sectione *C1*; *add.* H *R* / refertur: reflectitur ad E *R* / cum *inter. O* / DAZ: DAR *R* 205 D: O *L3* / que: qui *SOL3C1E*; quod *R* / punctus: punctum *R* 206 H: B *E* 207 punctus: punctum *R* / *post* autem *scr. et del.* est *C1* / SI: sed *FP1*; ST *R* 208 TI: Q *L3* / NZ *corr. ex* ZN *S* / que: qui *E* 209 concurret *inter. O* / cum linea *mg. C1* / *post* EO[2] *add.* sit concursus in U *R* 210 concurrit: concurret *E* / DAZ: DAR *R* 211 concurret *corr. ex* concurrat *O* / T *inter. O*; est *P1* / est *mg. a. m. C1* 212 situs *corr. ex* secundus *C1* / situs puncti *transp. C1* 213 O[1]: Q *R* / etiam *corr. ex* vel *O*; et *L3E* 214 idem *om. FP1* / TI *inter. E*; N *S*; U *L3* 215 *post* igitur *scr. et del.* linee EO quod *O* / TI: N *S*; TS *O* / HS: HC *P1* 216 concurrant: concurrent *FP1*; *corr. ex* concurrent *C1* / *post* erit *add.* ergo *R* / triangulus *corr. ex* visus *a. m. E*; triangulum *R* / et *om. L3* / in[2] *inter. a. m. C1* 217 huius *inter. O* / trianguli: anguli *L3E* / *post* erit *add.* in *C1* / axis *rep. S* / hac: eadem *R* 218 verum . . . superficie *om. FP1* / ergo *inter. O* 220 *post* super *scr. et del.* line *F* / punctus: punctum *R* / C: S *L3* / *post* superficie *scr. et del.* trianguli *L3* / et axis (221) *inter. O* 221 et[1] *inter. a. m. E* / in[1] *inter. O* 222 IS *corr. ex* IO *a. m. E* / illius *mg. a. m. E* / *post* trianguli *add.* illius *C1* / IES: NS *SE*; US *L3* 223 *post* et *inter.* sic *O* / erit: est *FP1* / erit curva *transp. C1* 224 est *om. SOL3C1E* 225 *post* ad *add.* punctum sectionis linee IS et *R* / superficiem: superficiei *R* / et *rep. E*

rit linea visa equidistans linee longitudinis speculi, tanto ymago eius erit minus curva, quanto minor magis.

[5.28] **[PROPOSITIO 19]** Amplius, si linea TH [FIGURE 6.5.19, p. 314] secet superficiem in qua sunt centrum visus et axis, et sit ortogonalis super eam, visus aut erit in illa superficie linee TH secante ortogonaliter superficiem axis et visus, aut extra.

[5.29] Si fuerit in illa superficie, aut erit supra lineam TH, aut infra. Si supra, cum illa linea sit corporalis, occultabit visui speculum, et ita non reflectetur, sed forsan capita eius apparebunt et reflectentur a circulo columpne qui communis est superficiei linee TH secanti columpnam et columpne. Et erit horum capitum ymago sicut in spericis exterioribus.

[5.30] Similiter, si visus fuerit sub linea TH, occultabitur pars eius propter capud in quo est visus. Pars autem linee visa refertur a circulo eodem penitus modo quo in exterioribus spericis.

[5.31] Si vero visus fuerit extra superficiem linee TH ortogonaliter secantem superficiem visus et axis, sit E visus, et XZG columpna. Refertur H ad E ab aliquo puncto columpne. Sit a B. Sit T eiusdem longitudinis a puncto E. Dico quod T refertur ad E ab alio puncto columpne, et cum puncta H, T sint eiusdem situs et eiusdem longitudinis a puncto E, erunt similiter puncta reflexionum, scilicet B, G, eiusdem longitudinis et eiusdem situs a puncto E. Igitur duo puncta B, G erunt in circulo.

[5.32] Sit circulus BZG, eius centrum D. Ducantur linee HB, BE, TG, GE, et a centro ducantur perpendiculares supra contingentes B, G, scilicet DBO, DGS. Et ducatur linea ED, et producantur HB, TG usque concurrant cum linea ED.

226 longitudinis: US *FP1* 227 *post* curva *add.* et *R* / minor: maior *FP1* / *post* minor *add.* tanto *R* / minor magis *mg. O* 228 si linea *om. FP1* 230 *post* erit *add.* visa *L3* / in illa *mg. F* 231 ortogonaliter: ortogonalis *L3*; ortogonalem *E* / *post* ortogonaliter *add.* super *FP1O* 232 si *inter. O* / illa superficie *transp. L3ER* / erit *om. ER* / infra: intra *S* 233 illa linea *transp. FP1* / sit *om. E* / occultabit: occultabitur *C1* 234 non *om. S* / reflectentur: reflectetur *L3C1E* 235 a: in *FP1* / est *inter. C1E* (*a. m. C1*) / secanti *alter. ex* secant *in* secantis *O* 236 columpne: columpnas *L3* / *post* columpne *add.* erit *S* / et^2 *inter. C1* 237 *ante* spericis *scr. et del.* speculis *E* 239 est *inter. O* / visa: vise *R* / refertur: reflectitur *R* 240 *post* eodem *scr. et del.* puncto *F* / in *inter. a. m. E* / spericis *inter. a. m. E* 241 ortogonaliter: ortogonalis *L3E* 242 visus et^1 *om. L3* / *post* axis *add.* et *L3* / sit E visus: visus sit E *L3* / quod: et *SL3E* / XZG: XZX *OL3C1E*; BGX *R* 243 refertur: reflectetur *R* / aliquo: alio *SL3E* / *post* a *add.* puncto *R* / *post* B *add.* et *R* 244 *post* E^1 *add.* cuius est H *R* / refertur: reflectetur *R* / alio: aliquo *FP1C1R* 245 columpne et *om. C1* / eiusdem: eius *FP1* 246 scilicet *inter. O* / B: L *FP1* 247 eiusdem *corr. ex* eius *O* 248 B: L *FP1* 249 BZG: BRG *S* / eius *om. FP1* / *post* D *add.* et *R* / BE: LE *FP1* 250 ducantur perpendiculares: ducatur perpendicularis *FP1* / supra: super *OL3ER* / *post* contingentes *add.* circulum in punctis *R* 251 scilicet: SI *L3E* / DGS: GDS *FP1* / ducatur: ducantur *L3* / et^2. . . ED (252) *om. R* 252 usque: usquoque *O*

[5.33] Cum puncta H, T sint eiusdem situs et longitudinis respectu E et respectu D, et similiter puncta B, G eiusdem situs respectu D et respectu E, habebunt linee HB, TG eundem situm respectu linee ED, et ita concurrent in idem punctum illius linee. Sit in puncto L.

[5.34] Fiat linea longitudinis columpne in qua punctus Z, et sit hec linea in superficie visus et axis, que sit AZ, et ducantur LZN, DZC. Q sit punctus linee TH, punctus scilicet qui est in superficie visus et axis, et a puncto Q ducatur equidistans linee DZC. Cadet quidem hec linea super axem, et LZN cadet in hanc lineam supra punctum Q. Cadat in puncto N.

[5.35] Palam ex predictis quod angulus HBO equalis angulo OBE. Sed angulus HBO equalis angulo LBD per contrapositionem, et angulus OBE equalis duobus angulis BED, BDE, quia extrinsecus. Ergo angulus LBD equalis duobus angulis BED, BDE. Fiat ergo angulus MBD equalis angulo BDE. Remanet angulus MBL equalis angulo BEL, quare ductus EM in ML equalis quadrato BM.

[5.36] Ducatur linea MZ. Quoniam angulus BDM maior angulo ZDM, et duo latera ZD, DM equalia duobus lateribus BD, DM, erit MB maior MZ, quare ductus EM in ML maior quadrato MZ. Sit ductus EM in MI equalis quadrato MZ, et ducantur linee IB, IZ. Erit ergo angulus MZI equalis angulo ZEI, quare MZL maior angulo ZED.

[5.37] Sed quoniam angulus MBD positus est equalis angulo BDM, erit linea MD equalis linee MB. Sed MB maior MZ, quare MD maior MZ. Igitur angulus MZD maior angulo MDZ; igitur angulus DZL maior duobus angulis ZDE, ZED. Sed angulus DZL equalis angulo NZC, et angulus CZE equalis duobus angulis ZDE, ZED, quare angulus NZC maior angulo CZE.

[5.38] Secetur ad equalitatem per lineam FZ, que quidem concurret cum linea NQ supra punctum N. Cum ergo angulus FZC equalis

253 T: E *R* / sint *corr. ex* sicut *C1* / longitudinis: lis *FP1* 254 B G *inter. E* / D[2]: E *R* 255 E: D *R* 256 concurrent: concurrant *FP1SC1E; corr. ex* concurrant *O* / in[1] *om. S* / idem: eundem *L3* / *post* sit *add.* concursus *R* 257 columpne *corr. ex* ? *O* / *post* qua *inter.* sit *O* / punctus: punctum *R* 258 que . . . axis (260) *om. S* / *post* ducantur *inter.* linee *OR* / DZC *corr. ex* DCZ *C1* 259 punctus[1,2]: punctum *R* / scilicet: S *L3* / qui: quod *R* / *post* visus *add.* et visus *E* / et *inter. E* 260 et a puncto *rep. P1* / a *om. S* / Q: que *L3* / *post* ducatur *add.* linea *R* / DZC: DZI *S* / quidem: equidistans *C1* 261 et BZN: ZL ZN *L3E*; et LZN *SC1* 263 *post* equalis *inter.* est *OR* / angulo *om. L3ER* / angulo . . . equalis (264) *om. P1* 264 *post* equalis *add.* est *R* / contrapositionem: circa positionem *C1* 265 *post* equalis *add.* est *R* / BDE: HDG *L3*; BDG *E; inter. O* 266 LBD: BBD *FP1* / *post* equalis *add.* est *R* / BED: DEB *C1* / BDE *inter. O* 267 remanet: remaneat *FP1O* / equalis . . . BEL (268) *mg. a. m. O* 269 *post* quoniam *add.* igitur *R* / *post* maior *add.* est *R* / angulo *inter. O* 270 ZDM . . . latera *om. P1* / DM[1] *corr. ex* M *P1* 271 *post* maior *add.* est *R* / MZ[2]: ZM *R* 272 EM in MI: in MI EM *S* / in MI *inter. O* / MI: M *L3*; IN *E* / et . . . MZI *mg. a. m. E* 273 MZI: MZL *L3* 274 MBD: MHD *O* 275 MB[1]: MD *R* / quare . . . MZ (276) *om. L3; mg. a. m. E* 276 MZD: MZ *FP1* / angulus[2] *om. R* 277 DZL[1] *corr. ex* L / ZED *mg. C1* / *post* maior *add.* est *R* 278 NZC: NZS *L3* / et . . . CZE *mg. a. m. O* / CZE: EZC *R* 279 NZC *corr. ex* NZO *O*; BZT *L3*; BZC *E* / *post* equalis *add.* est *R* / CZE: GZE *E*; EZC *R* 280 per *inter. a. m. E* / concurret: concurrent *C1* 281 supra: super *FP1* / *post* NQ *add.* concurrat *R* / supra: super *R* / N: F *R* / FZC: FCZ *L3* / *post* FZC *add.* est *L3*; *add.* sit *R* / *post* equalis *inter.* est *O*

angulo CZE, refertur F ad E a puncto Z. Q refertur ad E a puncto linee longitudinis que transit per Z, a puncto que est AZ, scilicet ultra Z. Si enim a puncto citra Z, id est propinquiori E, linea ducta a puncto Q ad punctum illud reflexionis secabit lineam FZ, et ita punctus sectionis refertur ad E a duobus punctis, quod est impossibile.

[5.39] Sumatur ergo ultra Z punctus K a quo refertur Q ad E, et ducatur linea EK donec concurrat cum linea NQ in puncto P. Erit P ymago Q. Sed H refertur ad E a puncto sectionis columpne. Si ergo a puncto H ducatur perpendicularis super contingentem sectionem in aliquo puncto, perpendicularis illa concurret cum perpendiculari CZD sub axe. Concurrat in puncto U.

[5.40] Similiter a puncto T est ducere unam perpendicularem super sectionem a cuius puncto refertur ad E. Et quoniam puncta H, T sunt eiusdem situs respectu linee CZD, et puncta sectionis similiter per que transeunt perpendiculares, igitur ille due perpendiculares concurrent in idem punctum linee CZD. Concurrant ergo in puncto U.

[5.41] Linea EB concurret cum linea HU. Sit concursus in puncto R. Similiter EG concurrat cum TU in puncto Y, et ducatur linea RY. Palam quod R est ymago H, Y est ymago T, et habemus triangulum ERY. Extra superficiem huius trianguli est punctum Z, et ita superficies huius trianguli altior est linea EP, et ita P extra. Quare linea RPY erit curva, et illa est ymago linee TH, et est quidem hec ymago curvitatis non modice, quod est propositum.

[5.42] Palam ergo quod in hiis speculis, si linea recta visa equidistans fuerit linee longitudinis columpne, erit ymago eius aut recta aut accedens ad rectitudinem. Si vero linea visa recta equidistans fuerit latitudini columpne, erit ymago eius curva curvitate non modica.

282 refertur[1, 2]: reflectetur *R* / *post* Q *add.* vero *R* / E[2]: ZE *S* 283 a puncto *scr. et del. O* / que[2]: qui *L3*; quod *R* / AZ *corr. ex* AI *O* / AZ scilicet *om. R* 284 citra: circa *L3* / id est *mg. a. m. C1* / propinquiori: propinqiore *R*; *corr. ex* propinquior *O* / *post* a[2] *add.* puncto A *S* 285 punctum illud *transp. C1* / illud: illum *FP1* / FZ: F *FP1* / punctus: punctum *R* 286 refertur: reflectetur *R* / quod est *inter. O* 287 *post* sumatur *add.* punctus *P1* / *post* ultra *add.* punctum *R* / punctus: punctum *R* / a quo refertur *om. FP1* / refertur: reflectatur *R* / Q: AQ *FP1* 289 sed: verum *O* / H *inter. O* / H refertur *transp. O* / refertur: reflectitur *R* / *post* columpne *add.* quo refertur *FP1* / ergo *inter. O* 290 H . . . sub (292) *mg. a. m. O* / *post* super *add.* lineam *R* 291 aliquo: alio *FP1SL3E* (*alter. in E*) / cum *mg. a. m. C1* 292 CZD: ZD *O* 293 T: L *R* 294 refertur: reflectatur *R*; *om. S* / *post* refertur *add.* T *R* / et *inter. O* / T: D *FP1L3E*; *corr. ex* D *O* 295 CZD: ED *R* 296 transeunt: transeant *S* / *post* perpendiculares[1] *add.* ab ipsis ducte *R* / igitur *inter. O* / ille due *transp. C1* / due perpendiculares *om. S* 297 CZD: ED *R* 298 *ante* linea[1] *add.* et quia *R* / linea[1]: linee *FP1SL3E* / EB: EDZ *P1*; *alter. ex* DZ *in* EDZ *F* / concurret: concurrit *R*; concurrent *L3E*; *corr. ex* concurrent *O* / linea[2] *om. R* 299 R: RE *FP1* / TU: TRI *FP1*; FD *C1*; IN *L3E* 300 H Y est ymago *rep. S* / *post* H *add.* et *R* 1 ita superficies: in superficie *R* 2 altior *corr. ex* altius *O* / *post* P *add.* est *R* / *post* extra *add.* P *SO* / quare: quoniam *FP1* 3 curvitatis: curvitas *L3* 4 *post* non *scr. et del.* est *C1* 5 ergo *om. L3* / *post* visa *add.* est *L3* 6 linee longitudinis *transp. E* / *post* eius *scr. et del.* a *S* / aut[1] *om. L3C1E* 7 accedens: accidens *L3* / visa recta *transp. SER* / *post* fuerit *scr. et del.* la *F* 8 latitudini *om. R* / *post* latitudini *scr. et del.* n *O* / non *inter. O*

[5.43] Linee autem inter has duas site, que magis accedunt ad situm linee equidistans longitudini columpne erunt ymagines earum rectitudini magis vicine, et ymagines earum que propinquiores sunt situi equidistantium latitudini erunt magis curve. Et minuetur vel augmentabitur curvitas ymaginum secundum accessum vel elongationem linearum ad alterum horum situum, et hoc est propositum.

[PARS SEXTA
In speculis piramidalibus exterioribus]

[6.1] Amplius in speculis piramidalibus exterioribus idem errores accidunt qui in columpnaribus exterioribus eveniunt, linee enim vise equidistantes longitudinis piramidis aut recte videntur, aut forte equidistantes latitudini curve, et intermedie augmentant vel diminuunt curvitatem secundum propinquitatem harum vel harum remotionem, et hoc quidem probabitur. Quoddam tamen premittendum proponamus et est:

[6.2] **[PROPOSITIO 20]** Si sumatur in superficie piramidis punctus reflexionis et fiat sectio transiens per punctum illud, et in sectione sumatur punctus remotior ab acumine piramidis puncto reflexionis et a puncto sumpto ducatur perpendicularis super contingentem sectionem, hec perpendicularis concurret cum perpendiculari ducta a puncto reflexionis sub axe.

[6.3] Verbi gratia, sit ABGZ [FIGURE 6.6.20, p. 315] piramis erecta super bases suas, A acumen piramidis, BFZ sectio, E punctus reflexionis, Z punctus sectionis remotior a puncto A quam E. Supra punctum Z sit superficies secans piramidem equidistans basi. Secabit quidem

9 inter: intra *O*/duas *om. P1*/site: si de *F;* si DO *P1*/accedunt: accidunt *L3; alter. in* accedint *O* 10 equidistans: equidistantis *C1E*/equidistans . . . vicine (11): equidistantis respectu columpne habebunt ymagines suas rectitudini magis vicinas *R*/longitudini: respectu *E*/ymagines . . . et (11) *om. FP1* 11 et *om. O*/que *inter. P1* 14 horum: eorum *C1* 15 *ante* amplius *add.* de erroribus qui accidunt in speculis piramidalibus convexis capitulum sextum *R* 16 accidunt *corr. ex* accedunt *O*/columpnaribus: spericis *FP1SOL3C1ER* 17 equidistantes: equidistant *S*/longitudinis: R *FP1;* scilicet *L3;* respectu *ER*/forte: fere *C1* 18 equidistantes: quidem *S*/latitudini *corr. ex* linea *O* 19 secundum propinquitatem *om. FP1*/harum[1]: earum *R*/harum[2] *om. R*/*post* harum[2] *add.* vel *FP1SOL3C1*/remotionem: removentur *FP1; corr. ex* removentur *O* 20 quidem *om. OR*/quoddam: quiddam *FP1R*/tamen: cum *L3*/*post* tamen *add.* proponendum *P1* 22 piramidis: piramidali *L3*/punctus: punctum *R* 23 illud: illum *FP1* 24 punctus remotior: punctum remotius *R*/ab acumine: a vertice *R*/reflexionis: rationis *S* 25 a *corr. ex* in *O*/contingentem: continentem *O* 26 concurret cum perpendiculari *inter. O*/*post* perpendiculari *add.* super contingentem sectionem *R* 28 ABGZ: ABGT *FP1*/piramis: piramidis *O*/erecta *corr. ex* recta *OC1* (*a. m. C1*) 29 bases suas: basim suam *R*/A: et *L3*/acumen: vertex *R*/BFZ: FZ *O*/E: est *S*/punctus: punctum *R* 30 Z: et *S*/punctus: punctum *R*/remotior: remotius *R*/a *om. E*/supra: super *R* 31 Z: E *FP1SL3E; corr. ex* E *OC1*/sit: fiat *R*/equidistans: equidistantem *FP1L3C1; alter. ex* equidistantem *in* equidistanter *O;* equidistanter *R*/quidem: equidem *FP1*

supra circulum communem. Sit circulus ille GBRZ, et ducantur linee AZ, AE, et producatur AE donec sit equalis AZ. Veniet quidem ad circulum. Cadat ergo in puncto eius O.

[6.4] C sit centrum circuli, et ducatur axis AC, et a puncto E ducatur perpendicularis super superficiem contingentem piramidem. Concurret quidem cum axe circa centrum circuli quod est C. Sit in puncto D, et ducatur linea DZ.

[6.5] Et a puncto O ducatur perpendicularis concurrens cum axe in puncto K, et ducantur linee DZ, KZ. Et supra punctum Z ducatur contingens sectionem, que sit TQ, et alia contingens circulum BGZ, que sit ZY.

[6.6] Et ducatur linea BCZ, et a puncto C ducatur perpendicularis super lineam BCZ, que sit CR. Erit quidem perpendicularis super axem, cum axis sit perpendicularis super superficiem circuli, quare CR est perpendicularis super superficiem ACZ. Et erit equidistans contingenti ZY, quare ZY est perpendicularis super superficiem ACZ, quare TQ non est perpendicularis super eandem superficiem.

[6.7] Verum quoniam K est polus ad circulum BRZ, cum linee KO, KZ sint equales, et axis AK communis, erit angulus AOK equalis angulo AZK, et ita angulus AZK rectus. Cum ergo linea KZ sit perpendicularis super AZ, que est linea longitudinis, erit perpendicularis super superficiem contingentem piramidem super hanc lineam longitudinis. Sed TQ est in superficie contingenti, quia est communis superficiei contingenti et sectioni. Igitur KZ est perpendicularis super TQ.

[6.8] Ducatur autem HZ in superficie sectionis perpendicularis super lineam TQ. Cum autem linea KZ sit extra superficiem sectionis, secabit lineam HZ, nec erit una linea cum illa. Superficies ergo

32 supra: super *R*/GBRZ: GBIZ *FP1E;* BGRZ *C1*/et *om. FP1* 33 et . . . AE[2] *mg. a. m. E*/ veniet: venit *FP1* 34 puncto: punctum *R*/*post* puncto *scr. et del.* i *P1*/eius *corr. ex* illo *O*/O: OC *S;* ECO *L3E* 35 *ante* C *add.* et *R*/C: OC *L3E*/circuli *inter. a. m. E* 37 circa: citra *SOL3C1ER* 38 educatur: et ducatur *OE* (*alter. in O*)/DZ: DZI *L3*/*post* DZ *add.* continens angulum acutum cum perpendiculari ED *R* 39 *post* perpendicularis *add.* super lineam AO *R* 40 ducantur linee: ducatur linea *R*/DZ *om. R*/KZ: HZ *FP1;* KT *L3E; corr. ex* HC *O*/*post* KZ *add.* HZ *C1*/supra: super *R* 41 contingens[1]: continens *SO*/sectionem: sectioni *P1S*/TQ *corr. ex* TA *O*/contingens[2]: continens *OE* 42 ZY *corr. ex* ZI *O; corr. ex* ZQ *L3* 43 BCZ: BOZ *L3*/et[2]. . . BCZ (44) *rep. S; scr. et del. E* 44 BCZ: BOZ *E; corr. ex* BZ *C1*/que sit CR *mg. a. m. E* 47 contingenti ZY *transp. R*/quare ZY *om. FP1* 48 quare *om. L3*/TQ: DQ *FP1* 49 K: KZ *FP1; corr. ex* Q *C1*/est *om. FP1*/ad circulum: circuli *R*/BRZ *corr. ex* BLZ *E*/*post* BRZ *add.* palam *R* 50 equales *corr. ex* quales *O*/AK: AQ *S*/*post* communis *add.* et AO equalis AZ quod *R*/AOK: ACK *FP1SOL3E* 51 KZ: U *FP1;* KI *L3E*/KZ . . . linea (52) *mg. O* 54 est[1]: erit *O*/*post* contingenti *scr. et del.* circuli cum ergo linea KZ sit perpendicularis super AZ que E *O*/quia: quod *S*/*post* communis *scr. et del.* perpendicu *E; add.* sectio *R* 55 *post* KZ *add.* est perpendicularis super TK *L3* 57 HZ: BZ *L3; corr. ex* KZ *O* 58 KZ *corr. ex* TZ *L3*/sit *corr. ex* sint *O* 59 *post* secabit *scr. et del.* unam *O*/HZ: KZ *L3*/nec: non *FP1*/cum: quare *R*/ergo *om. R*

KZH secat superficiem sectionis super lineam communem HZ, et secat lineam TQ super punctum Z. Et superficies AZK secat superficiem AZH super lineam communem KZ.

[6.9] Verum DZ est in superficie sectionis, et secatur a linea KZ in puncto Z, et punctus T supra superficiem KZH, punctus Q infra. Et ita superficies KZH secat superficiem DZQ super lineam communem, et illa linea communis est perpendicularis super lineam TQ, quia linea illa est in superficie HZK super quam est perpendicularis TQ. Et quoniam superficies HZK secat superficiem DZQ, et declinatio superficiei HZK sit ex parte ZE, erit linea communis sectioni illarum superficierum inter lineas QZ, DZ, et ita concurret cum perpendiculari ED sub axe. Et quod necessario concurrat cum ea probatum est in libro quinto, figura 19, et ita propositum.

[6.10] **[PROPOSITIO 21]** Sit ergo piramis cuius acumen A [FIGURE 6.6.21, p. 316], axis AH, linea longitudinis AZ, et a puncto Z ducatur perpendicularis supra superficiem contingentem piramidem in linea AZ, que necessario concurret cum axe. Sit linea TZH.

[6.11] Ducatur a puncto A linea extra piramidem supra superficiem contingentem piramidem in linea AZ faciens angulum acutum cum axe et cum linea longitudinis AZ, que sit AN. Et in superficie AHN a puncto H ducatur linea cum axe faciens angulum equalem angulo AHZ, que linea necessario concurrat cum linea AN, que sit HO. Et facto supra punctum Z circulo equidistans basi, transibit HO per circulum sicut HZ transit per ipsum.

[6.12] Ducatur autem linea OZ, et producatur usque ad punctum F. Quoniam linea OZ est supra superficiem contingentem piramidem

60 secat *corr. ex* secabit *a. m. E* / *post* sectionis *scr. et del.* sectioni *F* / communem HZ *transp. R* / HZ . . . KZ (62) *mg. O* / et . . . KZ (62) *rep. L3E* (secat[2] [61]: secabit / AZH [62]: AZKB *E*); *rep. et del. C1* (TQ [61]: CO; AZH [62]: AZ KB) 61 secat[1] *corr. ex* secabit *E* / lineam: linea *L3* / TQ: CO *L3E* / Z *om. S* / AZK: AKZ *C1*; HZK *R* / *post* AZK *add.* et *L3E* / *post* secat *add.* super *C1* 62 AZH: AZKO *FP1*; AZBH *S*; AZKL *O*; AZKB *L3*; AZKD *E*; DZK *R* / KZ: HZ *L3E* / *post* KZ *scr. et del.* K est *O* 63 DZ *alter. in* DE *a. m. E* 64 punctus[1, 2]: punctum *R* / *post* T *add.* est *R* / *post* KZH *scr. et del.* secat *S* / Q: quod *FP1* 65 KZH *corr. ex* LTZH *O* / secat: secabit *ER* / superficiem DZQ *rep. S* 66 illa: ita *L3C1E* / super lineam *om. S* / quia . . . TQ (67) *om. S* 69 *post* HZK *add.* a superficie sectionis *R* / ZE: ZC *R* / sectioni *corr. ex* sectionis *S* 70 lineas: lineam *FP1SOL3C1E* / QZ: AZ *FP1* 71 ED *om. R* / cum ea *om. R* / ea: eo *FP1* / est *inter. O* 72 quinto figura 19 *corr. ex* quomodo S in 9[a] *a. m. O* / figura 19 *om. R* / 19: 14 *E* / *post* ita *add.* est *R* / propositum: proponimus *L3* 73 acumen: vertex *R* 74 AH *corr. ex* AZ *O* / AZ: AHZ *L3* 75 ducatur *corr. ex* dicucatur *F* / supra: super *FP1R* / piramidem: piramide *L3* 76 necessario *corr. ex* non *O* 77 supra: super *FP1*; ultra *R* / supra superficiem *mg. a. m. O* 78 contingentem: continentem *O* 79 *post* superficie *scr. et del.* a *C1* 80 AHN *corr. ex* ANH *a. m. E* / *post* AHN *scr. et del.* que linea vero concurrat cum linea *C1* / *post* faciens *scr. et del.* lineam *C1* / angulum equalem *transp. C1* 81 necessario: non *FP1*; nec *S*; vero *L3C1E*; *corr. ex* non *O* / concurrat: currat *F*; concurret *R* 82 HO[1]: BO *O* / supra: super *R* / *post* Z *add.* et *P1*; *scr. et del.* et *F* / equidistans: equidistante *OC1R*; equidistanti *E* / HO[2] *corr. ex* O *O* 83 transit: transibit *C1E*; *corr. ex* transibit *O* 84 autem *om. E* / OZ: DZ *FP1O* / usque *om. ER* 85 est supra: secat *R*

in linea AZ, cum linea HZ sit perpendicularis supra illam superficiem, erit angulus OZH maior recto. Igitur angulus FZH acutus.

[6.13] A puncto Z ducatur contingens supra circulum, que sit ZM, et a puncto F ducatur perpendicularis supra AZ cadens in puncto eius E, que producta concurret cum AO, quoniam angulus OAZ est acutus. Concurrat ergo in puncto N, et a puncto E ducatur equidistans linee TH, et sit QE.

[6.14] Et a puncto E ducatur equidistans linee MZ, que sit LE. Palam quoniam MZ est perpendicularis supra AE, quoniam est perpendicularis supra TH et supra dyametrum circuli, cuius est contingens. Igitur LE est perpendicularis super AE.

[6.15] Fiat autem superficies LQD secans piramidem. Erit quidem sectio piramidalis. Cum ergo AE sit perpendicularis super FN, et super QD, et super LE, erit FN in superficie illa secante piramidem. Fiat ergo CF equidistans QE. Erit quidem equidistans TZ.

[6.16] Verum cum angulus OZT sit acutus, angulus TZF sit obtusus. Ducatur a puncto Z linea faciens cum TZ angulum equalem angulo OZT, que quidem linea necessario secabit FC. Secet in puncto C, et ducatur linea EC. Cum ergo CZ, OZ sint in eadem superficie, et angulus OZT equalis angulo TZC, punctus O refertur ad C a puncto Z.

[6.17] Verum quoniam angulus OZT equalis angulo ZFC, et angulus OZT equalis angulo ZCF, erunt latera ZC, ZF equalia, et quia angulus FEZ rectus, quadratum FZ valet quadrata EZ,EF, et quadratum CZ valet quadrata EZ, EC. Igitur CE, FE equalia, et ita anguli ECF, EFC equales, quare anguli NEQ QEC equales. Et cum in eadem superficie sint C, E, N, refertur N ad C a puncto E.

86 *post* AZ *add.* et *C1*/cum . . . HZ *om. P1*/supra: super *FP1ER* 87 *post* recto *add.* quia AZH rectus est *R* 88 contingens: continens *O*/supra *om. R*/ZM: MZ *ER* 89 et *om. C1*/supra: super *R*/puncto[2]: punctum *R* 90 E que: eque *S*/producta *corr. ex* puncta *O*/concurret: concurrent *FP1*; concurrat *R* 92 TH . . . linee (93) *mg. O*/QE: QD *O* 93 linee *om. R*/LE: EL *R* 94 quoniam[1]: quod *R*/supra: super *R*; *om. L3*/quoniam[2]: que *L3*/*post* quoniam[2] *add.* AH *R* 95 supra[1]. . . contingens: super circulum per Z transeuntem et MZ super dyametrum illius circuli quia contingit *R*/contingens: continens *O* 96 *ante* igitur *scr. et del.* ig *S*/super: supra *S*/*post* AE *add.* et producatur QE ultra E hec concurret quidem cum axe concurrat in D *R* 97 LQD: LDQ *OE*; LE DQ *R* 98 piramidalis: piramidis *S*; piramidali *L3* 99 LE *corr. ex* L *a. m. E*/illa *om. S* 100 *post* ergo *add.* in illa superficie *R*/CF: PF *R*/QE: QB *C1*/quidem *om. R* 101 OZT: EZT *FP1L3E*; *corr. ex* EZT *O*; FZH *R*/*post* acutus *add.* erit *R*/sit *om. R* 102 equalem *om. O* 103 necessario: vere *S*; non *L3E*; *corr. ex* non *O*; *om. FP1*/FC: et *L3*; FP *R*/secet *om. L3*/C *inter. E*; P *R* 104 EC: PE *R*/CZ: CT *FP1SOL3C1E*; PZ *R*/OZ: Z *C1*/superficie: superficiei *O*/et . . . TZC (105) *scr. et del. E* 105 equalis: erit *E*/TZC: ZCF *E*; TZP *R*; *corr. ex* TZF *P1*/TZC . . . angulo (106) *om. L3*/punctus *om. R*/punctus . . . ZFC (106) *om. E*/O refertur: reflectetur O *R*/C: P *R*/*post* puncto *add.* speculi *R* 106 verum quoniam: et quia *R*/*post* equalis *add.* est *R*/ZFC: ZFP *R*/et . . . ZCF (107) *mg. C1*; *om. R* 107 OZT: ZT *FP1*; TZC *OC1*/ZC: ZP *R*/ZF: LZF *FP1* 108 rectus *rep. E*/FZ *scr. et del. E*/quadrata: quadratum *L3E*/EF: erit *S*/et *inter. C1* 109 CZ: PZ *R*/quadrata: quadratum *L3E*/EC: et *L3*; EP *R*/CE: PE *R*/anguli *om. R* 110 ECF: EST *S*; EOF *O*; EPF *R*/*post* ECF *scr. et del.* equ *S*/EFC: EFP *R*; EST *S*; *om. P1L3*/*post* EFC *add.* anguli erunt *R*/quare: quarum *L3*/anguli: angulus *L3E*/NEQ *corr. ex* NEC *P1*/QEC: QES *L3*; QEP *R*; *corr. ex* EC *O* 111 *post* sint *add.* que est *R*/C[1, 2]: P *R*/E[1]: O *F*/refertur: reflectetur *R*/C[2] *corr. ex* E *O*

[6.18] Similiter ducatur a puncto F quecumque linea ad aliquod punctum linee ZE, et producatur usque ad ON. Probabitur de puncto linee ON in quam cadit quod refertur ad C a puncto ZE, quoniam secat illa linea. Simili modo et omnium huiusmodi linearum probatio sumet initium a perpendiculari, que est FE, et a parte linee EZ, que erit terminus, et ita quodlibet punctum linee ON refertur ad C ab aliquo puncto linee EZ.

[6.19] **[PROPOSITIO 22]** Hoc ergo declarato, dicamus: cum visus comprehenderit lineas rectas transeuntes per caput speculi piramidalis convexi recti obliquas super axem speculi, in hoc speculo tunc forme earum erunt parum convexe.

[6.20] Sit ergo speculum piramidale erectum ABG [FIGURE 6.6.22, p. 316], cuius caput sit A, et cuius axis sit AD, et extrahamus in superficie eius lineam AZ, quocumque modo sit, in qua signetur punctum Z, quocumque modo sit. Et transeat per Z superficies equidistans basi piramidis, et faciat circulum ZU. Et extrahamus ex Z perpendicularem ZH super AZ. Hec ergo linea concurret cum axe piramidis, et concurrat ergo in H.

[6.21] Et extrahamus ex Z lineam contingentem circulum, et sit ZM, et extrahamus ex A lineam continentem cum utraque linea AZ, AH angulum acutum, et sit extra superficiem contingentem piramidem transeuntem per lineam AZ, et hoc possibile. Sit ergo AO, et extrahamus ex puncto H lineam in superficie in qua sunt AO, AH continentem cum AH angulum equalem angulo ZHA. Hec ergo linea concurret cum AO, nam duo anguli A, H sunt acuti. Concurrant ergo in O.

112 *post* similiter *add.* si *R*/a puncto F *mg. a. m. E*/a . . . linea: quecumque linea a puncto F *R*/aliquod: aliquem *S* 113 linee *om. R*/ZE *corr. ex* EZ *P1*/et . . . linee (114) *mg. a. m. E*/ON *corr. ex* EN *O*/probabitur: probabiliter *P1;* probababiliter *F* 114 *post* linee *scr. et del.* ZE et producatur *S*/ON: OM *S*/quam: quem *OE;* quod *R*/refertur: reflectetur *R*/C: P *R*/*post* puncto *add.* linee *R*/quoniam: quem *SC1;* quod *R* 115 huiusmodi: huius *L3* 116 sumet: sit MET *FP1*/initium: iniqum *L3* 117 terminus: communis *FP1;* communis omnibus illis triangulis *R*/quodlibet: licet *FP1*/*post* ON *scr. et del.* in quam cadit quod *C1*/refertur: reflectetur *R*/C: P *R* 118 aliquo: alio *SOL3E*/linee *inter. O*/*post* EZ *add.* et hoc est quod voluimus *C1; add.* que erit communis *E* 119 ergo *corr. ex* modo *E; om. R*/*post* dicamus *inter.* quod *O*/cum visus (120) *om. FP1* 120 caput: verticem *R* 121 recti *alter. in* erecti *a. m. C1*/super axem *rep. et del. E*/in hoc speculo *om. R* 123 piramidale: piramidalis *S*/ABG: ABC *R* 124 caput: vertex *R* 125 quocumque: quomodocumque *O*/in². . . sit (126) *om. P1*/quocumque modo: quomodocumque *FOC1* 128 linea *om. L3* 129 et *om. C1R; scr. et del. E* 130 circulum . . . contingentem (131) *rep. L3E* (ex A lineam [131]: lineam ex A *L3*/ZM² [131] *corr. ex* MZ *E*) 131 ZM: MZ *FP1E*/A: HA *FP1* 132 AH: ad *P1*/sit *mg. C1*/contingentem: continentem *O* 133 *post* hoc *add.* est *R*/AO: AN *R; corr. ex* BA *S*/*post* AO *inter.* vel O *O*/et *mg. F*/extrahamus: extrahemus *L3* 134 ex: a *SC1*/H: B *E*/AO: AN *R*/continentem: continent *FP1;* contingentem *SL3* 135 angulo *om. C1*/ZHA: ZAH *L3;* AHZ *ER* 136 AO: AC *FP1;* HO *C1*/*post* anguli *add.* ad *R* 137 in . . . ergo (138) *om. S*

[6.22] Linea ergo HO concurret cum circumferentia circuli ZU, nam angulus AHO est equalis angulo AHZ. Concurrat ergo in U, et extrahamus AU recte. Et extrahamus perpendicularem HZ ad T, et continuemus OZ, et extrahatur recte ad F, et extrahatur AZ ad E. Angulus ergo FZH erit acutus, quia linea OZ secat superficiem contingentem piramidem transeuntem per AZ. Linea ergo FZ est sub differentia communi inter superficiem OZH et superficiem contingentem, et hec differentia continet cum linea HZ angulum rectum. Angulus ergo OZH est obtusus; ergo angulus FZH est acutus.

[6.23] Ponatur ergo in ZF punctus F a quo extrahatur perpendicularis FE super AE, et extrahatur recte. Concurret ergo cum linea AO, nam angulus OAE est acutus. Concurrat ergo in N, et extrahatur ex E linea ED equidistans ZH linee. Erit ergo ED perpendicularis super superficiem contingentem piramidem transeuntem per AE.

[6.24] Et extrahatur ex E linea equidistans linee ZM, et sit EL, et extrahatur superficies in qua sunt LE, ED. Secabit ergo superficiem piramidis et faciet sectorem, nam hec superficies est obliqua super axem AD.

[6.25] Sit ergo sector BEG. Et MZ est perpendicularis super superficiem AZH, et hoc declaratum est in predictis. Ergo linea LE est perpendicularis super superficiem AED; ergo angulus AEL est rectus, et angulus AEN est rectus, et similiter angulus AED est rectus. Ergo linee LE, NE, DE sunt in eadem superficie. Ergo linea FEN est in superficie sectoris.

[6.26] Et extrahatur ex F linea equidistans linee DE, et sit FR. Hec ergo linea equidistabit linee HZ. Et extrahatur ex Z in superficie OZH linea continens cum ZT angulum equalem angulo OZT. Hec ergo linea concurret cum FR, quia secabit ZH equidistantem FR, et est in superficie eius, quia ZF est in superficie eius. Concurrat ergo in R.

[6.27] Ergo duo anguli qui sunt apud R, F sunt equales, sunt enim equales duobus angulis qui sunt apud Z. Due ergo linee RZ, FZ sunt

139 nam: linea *L3E* / AHO *corr. ex* HO *O* 140 T et (141) *om. S* 141 et[1] *inter. C1* / extrahatur[1]: extrahamus *R* 143 FZ: AZ *R* 144 OZH: OZA *S* 146 OZH: EZH *L3ER* / est[1] *om. L3ER* / est[2] *om. ER* 147 in ZF: MZF *P1* / punctus: punctum *SOL3C1ER* 148 AE: A *L3* / *post* recte *add.* ON *FP1* / concurret: concurrat *L3C1E* / concurret . . . acutus (149) *om. S* 149 OAE: AOE *FP1* 150 super *mg. a. m. E* 152 *post* extrahatur *scr. et del.* superficies in qua sunt LE ED secabit ergo superficiem *S* / EL: EB *L3E* 153 *post* sunt *add.* linee *R* 154 piramidis: piramidem *S* / faciet *corr. ex* faciat *a. m. E* / sectorem: sectionem *R* 156 sector: sectio *R* / BEG: REG *S*; DEC *R* / est *om. S* 158 AED: ADE *C1* 159 et[1] . . . rectus[1] *om. P1R* / et[1] . . . rectus[2] *mg. a. m. E* / AEN: LEN *SL3*; AEL *E* / est rectus[2] *transp. R* / *ante* ergo *add.* et AEN similiter rectus *R* 161 sectoris: sectionis *R* 162 FR: FFE *S* 163 equidistabit: equidistat *R* / OZH: EZH *FP1* 164 hec ergo *transp. C1* 165 quia *alter. in* et *a. m. E* / secabit: secat *R* / equidistantem: equidistanter *E* / FR: FRA *FP1*; FA *S*; FK *L3* 166 quia . . . eius[2] *om. P1*; *mg. C1* / ZF: FZ *C1* / R: FE *S*; K *L3* 167 *post* sunt[1] *scr. et del.* ZF *C1* / R F: SEF *S* / sunt[2] . . . equales (169) *mg. O* 168 *post* sunt[1] *scr. et del.* i *P1* / RZ: KZ *E*; *corr. ex* RI *L3*

equales. Et declaratum est quod linea FEN est in superficie sectoris, et linea FR est equidistans linee ED. Est ergo in superficie sectoris.

[6.28] Et continuemus RE. Erit ergo in superficie sectoris. Et extrahatur DE ad K, et declaratum est quod EA est perpendicularis super superficiem sectoris. Uterque ergo angulus AER, AEF est rectus, et due linee FZ, RZ sunt equales. Ergo due linee RE, FE sunt equales; ergo duo anguli ERF, EFR sunt equales.

[6.29] Ergo forma N convertetur ad R ex E, et forma O convertetur ad R ex Z. Et omnis linea extracta ex F ad aliquod punctum linee AN secabit AE. Et patet quod illa linea erit equalis linee extracte ex R, nam AE est perpendicularis super superficiem in qua sunt linee RE, FE, nam hec superficies est superficies sectoris, et due linee RE, FE sunt equales. Ergo omnes due linee extracte ex R, F ad aliquod punctum linee AE sunt equales.

[6.30] Patet ergo quod forma puncti quod est in AN convertetur ad R ex illo puncto quod est in AE. Et similiter de omni puncto posito in AN ultra N, si copulatum fuerit cum F, et per lineam rectam, illa linea secabit AE ultra E. Et patet quod forma puncti quod est in AN convertetur ad R ex puncto in AE. Patet ergo ex hoc quod forma linee AN, et quicquid continuatur cum ipsa, convertetur ad R a superficie piramidis ABG ex linea recta, et similiter omnis linea extracta ex A oblique super axem piramidis.

[6.31] Et continuemus ND. Secabit ergo circumferentiam sectoris, nam duo puncta N, D sunt in superficie sectoris, et N est extra sectorem, et D est intra sectorem. Secet igitur circumferentiam sectoris in C, et quia triangulus AOH est in eadem superficie, erit ND in superficie trianguli AOH.

169 quod: quia *C1* / FEN: FN *L3* / sectoris: sectionis *R* 170 linea: linee *S* / linee *om. R* / sectoris: sectionis *R* 171 et[1] *om. FP1* / et[1]. . . sectoris *om. L3* / continuemus: continueremus *FP1* / RE: SE *S;* KE *E* / sectoris: sectionis *R* 172 quod: quia *C1* / EA *om. FP1* 173 sectoris: sectionis *R* / uterque: utrique *L3E* / angulus: angulo *L3E;* angulorum *R* / *post* AEF *inter.* vel D *O* / est rectus *transp. R* 174 FZ RZ *transp. O* / RZ: IZ *FP1;* RBZ *S* / ergo . . . equales[2] *om. FP1; mg. a. m. E* 175 ergo . . . equales *om. P1* / EFR: EFK *E* 176 N: M *E* / convertetur[1, 2]: reflectetur *R* / O: D *O* 177 R: FI *FP1* / ex[2]: a *L3* / ad[2] *inter. O* / aliquod: O ad *FP1; mg. a. m. E* 178 AN: ON *R* / quod: quia *C1* / illa linea *transp. L3ER* 179 R *corr. ex* ER *FP1;* F *O;* K *L3E* / *post* R *add.* ad idem punctum scilicet AE ad quod extrahitur linea a puncto F *C1; add.* ad idem punctum *R* / nam: namque *E* / super *inter. a. m. E* 180 *post* FE *scr. et del.* sunt equales *C1* / est superficies *mg. F* / sectoris: sectionis *R* / RE[2]. . . linee (181) *mg. O* 181 linee: ? *O* / *post* ad *add.* unum *R* 183 quod[1]: quia *C1* / AN: ON *R* / convertetur: reflectetur *R* 184 R: N *L3;* K *E* / est in AE: secatur in ZE *R* / posito *om. P1* 185 N: QN *FP1;* quod *O;* TI *L3* / et *om. R* 186 et . . . AE (187) *om. R* 188 AN *om. L3* / ipsa *inter. E* / convertetur: reflectetur *R* / superficie *corr. ex* superficicie *F* 189 ABG: AB *FP1;* ABH *OL3E* (*deinde inter.* vel G in arabico *O*); ABHT *S* / *post* ex[2] *scr. et del.* ea *F* / A *corr. ex* ea *P1* 190 oblique: obliqua *R* / piramidis *om. L3ER* 191 et: si *FP1* / continuemus: continuemuemus *S* / sectoris: sectionis *R* 192 N D: Z E *FP1SL3C1E;* Z D *O; transp. R* / sectoris: sectionis *R* / et: Z *S* / N: enim *P1* / *post* extra *add.* circumferentiam *R* / sectorem: sectionis *R* 193 et: SZ *FP1* / sectorem: sectionem *R* / sectoris: sectionis *R* 194 triangulus: triangulum *R* 195 AOH: ACH *L3* / *post* AOH *scr. et del.* et duo puncta A U sunt *S*

[6.32] C ergo est in superficie trianguli AOH, et duo puncta A, N sunt in superficie huius trianguli. Ergo puncta A, N, C sunt in superficie trianguli AOH. Sed puncta A, U, C sunt in superficie piramidis. Ergo puncta A, U, C sunt in differentia communi superficiei piramidis et superficiei AND. Sed hec differentia est linea recta. Ergo puncta A, U, C sunt in linea recta.

[6.33] Extrahatur ergo AU recte ad C, et extrahatur RZ recte. Secabit ergo OH. Secet ergo in P. P ergo est in superficie trianguli AOH. Continuetur ergo AP, et transeat recte. Secabit ergo ND in G, et quia F est sub superficie contingente piramidem transeunte per lineam AZE, erit angulus FED acutus, et angulus DEN est obtusus. Ergo angulus ENC est acutus.

[6.34] Et sit linea CZ contingens sectorem. Patet ergo, ut in figura predicta, quod angulus DCZ est obtusus et quod perpendicularis extracta ex C super CZ secat angulum DCZ, et concurret cum ED sub D. Hec ergo perpendicularis secabit ED in S.

[6.35] Perpendicularis ergo extracta ex N super lineam contingentem sectorem secabit sectorem ultra C, scilicet, remotius ex E quam C, nam iste perpendiculares concurrent ultra circumferentiam sectoris. Perpendicularis ergo extracta ex N super lineam contingentem sectorem non secabit angulum DCZ. Erit ergo remotior ex NE quam CD, et hec perpendicularis secat ED sub D.

[6.36] Sit ergo perpendicularis extracta ex N super lineam contingentem sectorem linea NQ. Et RE secat EN, et secat circumferentiam sectoris, et est in superficie eius, et NQ est in superficie sectoris. Si ergo RE extrahatur recte, secabit NQ. Secet ergo in Y.

196 C: S *FP1L3* / AOH: ACH *FP1* / N: O *O*; U *R* 197 huius trianguli *transp. L3ER* / *post* trianguli *add.* AOH *R* / ergo . . . superficie[2] *om. E* / ergo . . . AOH (198) *om. R* 198 AOH: ACH *FP1S* / sed . . . C *mg. a. m. E* 199 ergo . . . piramidis *mg. a. m. E* 200 AND: AUD *ER* (*alter. in E*) / *post* hec *scr. et del.* est *E* 201 U C *corr. ex* N T *a. m. E* 202 *post* extrahatur *scr. et del.* ergo AN recte *S* / RZ: X *FP1*; RR *deinde inter.* vel Z in arabico *O*; TZ *L3*; KZ *E* 203 OH: H *L3E* / ergo[2] *om. O* / *post* in[1] *add.* puncto *ER* / P[2] *inter. O*; *om. L3E* / P ergo est: est ergo P *R* / AOH *om. S*; ACH *L3E* 204 continuetur *corr. ex* continetur *C1* / ND: AND *O* / G: H *OL3E* 205 est sub: non est in *R* / sub: in *L3* / *post* superficie *scr. et del.* trianguli *P1* / contingente: continente *P1* / contingente piramidem *transp. R* / transeunte: transeuntem *E* 206 AZE: AZ *R*; *corr. ex* AE *C1* 207 ENC: EQC *O*; END *R* 208 CZ: Z *FP1*; CX *R* / sectorem: sectionem in puncto C *R* / in *inter. C1* / figura predicta (209) *transp. R* 209 DCZ: DCX *R* / est obtusus *mg. C1* / quod *om. FP1* 210 CZ: CX *R*; *corr. ex* Z *O* / secat: secabit *ER*; *corr. ex* secet *C1* / DCZ: DCX *R* / ED: AD *FP1SL3C1E* / *post* ED *add.* et *L3* / D *corr. ex* DH *E* 211 hec *om. FP1*; *inter. a. m. E* / hec ergo *transp. ER* / secabit: secet *R* / ED: AD *FP1SOL3E*; *corr. ex* AD *C1* / S: Z *FP1*; C *O* 212 N *corr. ex* A *S* / contingentem: continentem *O* 213 sectorem[1, 2]: sectionem *R* / secabit *rep. FP1* / sectorem[2] *mg. F* / C: S *FP1SL3C1ER* / scilicet *om. S*; sed *ER* / ex E: a D *R* / E: eo *S* 214 C: S *FP1SL3C1ER* / sectoris: sectionis *R* 215 perpendicularis: perpendiculares *L3* / *post* ex *add.* puncto *R* / contingentem: continentem *OE* 216 sectorem: sectionem *R* / DCZ: CZG *FP1O*; CZH *SL3C1E*; DCX *R* / ex: ab *R* / *post* quam *add.* sit *R* 217 CD: ND *R* / et: ergo *R* / ED: AD *FP1SL3ER*; *corr. ex* AD *C1* / sub: supra *R* 218 *post* super *scr. et del.* perpendicularem *C1* / contingentem: continentem *O* 219 sectorem: sectionem *R* / RE: KE *E* / secat[1] *alter. in* secet *E* / EN: EQ *O* 220 sectoris[1, 2]: sectionis *R* / eius *inter. O* / et[2] *inter. a. m. E* 221 RE *corr. ex* EY *a. m. E* / ergo *om. S*

[6.37] Et superficies AND secat superficiem sectoris. Quia punctum E est extra superficiem AND, nam superficies AND non est superficies sectoris, A enim est extra superficiem sectoris, quia AE est perpendicularis super superficiem sectoris, et E est in circumferentia illius, ergo ND est differentia communis superficiei AND et superficiei sectoris, et NQ concurret cum sectore ultra C. Ergo NQ est ultra superficiem AND. Y ergo est ultra lineam APG.

[6.38] Si ergo visus fuerit in R, et linea AON fuerit in aliquo visibili, tunc P erit ymago O, et Y erit ymago N, et A videbitur in suo loco, quia est in capite piramidis. Et erit ymago linee AON linea transiens per puncta A, P, Y, sed hec linea est convexa, quia est ultra APG.

[6.39] Sit ergo illa linea APY, et patuit iam quod forme omnium punctorum que sunt in AN convertuntur ad R ex AE. Linee ergo radiales per quas convertuntur ille forme sunt in superficie trianguli RZE; omnes ergo ymagines linee AN sunt in hac superficie.

[6.40] Ergo linea APY convexa est in hac superficie, et P est propinquius ad R quam Y, et erit convexitas huius ymaginis ex parte visus, et erit convexitas parva. Et dyameter huius ymaginis erit minor ipsa linea modica quantitate. Ymagines ergo linearum rectarum que extrahuntur ex capite piramidali oblique super axem comprehenduntur a visu in tali speculo convexe, et forme harum linearum convertuntur a lineis rectis ex lineis extensis in longitudine piramidis, et hoc est quod voluimus declarare.

[6.41] Forme vero linearum equidistantium latitudini speculi piramidalis convexi convertuntur a lineis convexis in superficie speculi, et convexitas harum linearum patet ut in columpnali speculo con-

222 et: in *S* / superficies *corr. ex* superficiess *F* / secat: secabit *ER*; *corr. ex* secet *C1* / sectoris: sectionis *R* / *ante* quia *add.* item *R* 223 est[1] *om. S* / *post* extra *scr. et del.* civitatem *S* / AND[1] *corr. ex* AMD *P1* 224 sectoris[1, 2]: sectionis *R* / *ante* A *add.* quia punctum *R* / A . . . sectoris[2] *inter. a. m. E* / enim *om. R* / est[1] *om. P1E* / *post* superficiem *add.* corporis *P1* / *ante* quia *add.* et *R* 225 sectoris: sectionis *R* / et: Z *L3E* / in *inter. C1* 226 ND: NCD *R* 227 sectoris: sectionis *R* / NQ: NRQ *FP1*; LQ *L3E*; *corr. ex* Q *a. m. C1* / concurret: concurrit *R* / sectore: sectione *R* 228 Y *om. P1* / *post* ultra *add.* C ergo NQ *F* / APG: APH *FP1SOL3E* 229 linea . . . visibili: forma alicuius visibilis reflectatur a linea longitudinis *R* / AON: longitudinis *FP1L3C1E*; TO *S*; LON *O* 230 O *om. FP1SOL3E*; *inter. a. m. C1* / A *om. FP1SOL3E*; *inter. a. m. C1* / suo loco *transp. C1* / *post* loco *scr. et del.* lo *C1* 231 in *inter. O* / capite: vertice *R* 232 *post* ultra *add.* lineam *R* / APG: APH *FP1SOL3C1E* 233 illa linea *transp. C1* / iam quod *transp. P1* 234 convertuntur: concurruntur *O*; reflectantur *R* / R: K *L3E* 235 convertuntur: reflectuntur *R* 236 RZE: KZE *L3E*; RAE *R* / ergo *inter. O* 237 ergo . . . superficie *om. S* / et: Z *FP1* / propinquius: propinqus *P1* 238 ad *om. R* / R: K *L3E* / ex . . . ymaginis (239) *rep. F* / huius ymaginis *transp. R* 239 *post* ymaginis *add.* ex parte visus *P1S* 241 capite piramidali: parte piramidis *O*; vertice piramidis *R* 242 convexe *inter. a. m. E* / convertuntur: reflectuntur *R* 243 a lineis: alienienis *S* / rectis *inter. a. m. E* / ex lineis *om. R* / piramidis: piramidalis *O* 244 voluimus: volumus *P1L3C1* / declarare: dicere *FP1* 245 equidistantium *om. FP1* / latitudini: iam *L3* 246 convexi *corr. ex* convexitas *S* / convertuntur: reflectuntur *R* / a lineis: alienis *SE* 247 *post* convexitas *add.* formarum *FP1* / ut *om. S* / columpnali: columpnari *C1R* / columpnali speculo *transp. R* / convexo *om. P1*

vexo, et per illam eandem viam, et patebit similiter quod ymagines harum linearum erunt nimium convexe et manifeste sensui. Et erit centrum visus extra superficies in quibus est convexitas formarum harum linearum, et erunt dyametri ymaginum harum linearum multum minores ipsis lineis.

[6.42] De lineis vero obliquis existentibus inter hos duos modos, que appropinquant in suo situ lineis extensis in longitudine piramidis habent formas parum convexas, que vero appropinquant lineis equidistantibus latitudini piramidis habent formas manifeste convexas.

[6.43] Sed tamen linee tortuose que appropinquant capiti piramidis habent formas minores, et strictiores, et convexiores, que vero appropinquant basi piramidis habent formas ampliores propter illud quod declaratum fuit in speculis spericis convexis—scilicet quod quanto minus fuerit speculum tanto minores erunt circuli qui cadunt in superficie eius—et sic ymagines erunt propinquiores centro, ideo ergo erunt minores.

[6.44] Et similiter sectores qui cadunt in speculo piramidali qui sunt ex parte capitis piramidis sunt strictiores et minores, et sic ymago erit propinquior puncto in quo concurrunt perpendiculares exeuntes a linea visibili perpendiculariter super lineas contingentes sectores que sunt differentie communes, et ideo iste ymagines erunt minores.

[6.45] Sectores vero qui sunt ex parte basis piramidis econverso, unde accidit quod forma comprehensa in speculo piramidali convexo erit piramidata, quod scilicet fuerit ex parte capitis speculi erit strictius, et quod ex parte basis erit amplius, et convexitas latitudinis forme erit manifesta.

[6.46] Et accidit etiam in hiis speculis quod quanto magis res visa appropinquaverit speculo videbitur maior, et quanto magis erit remota videbitur minor.

248 illam eandem *transp. P1* / viam *om. P1* / et: etiam *L3ER* / patebit *om. E* / patebit similiter *transp. R* 249 nimium *corr. ex* in unum *E* 250 formarum harum (251) *transp. O* 251 et . . . linearum^2 *mg. a. m. E* / multum: multo *R* 253 lineis: linei *L3* / lineis vero obliquis: obliquis vero lineis *C1* / *post* existentibus *add.* et *O* 254 suo situ *transp. L3* / situ: motu *R*; *corr. ex* motu *a. m. E* 255 habent: habens *FP1* / habens . . . piramidis (256) *om. L3* 256 *post* equidistantibus *scr. et del.* inter hos duos modos *C1* / habent: habens *P1S* 258 capiti: vertici *R* 259 vero *om. FP1* 261 quod^2 *om. L3* / speculis: speculus *R* 263 superficie: superficiem *R* / *post* propinquiores *scr. et del.* circulo *C1* / ideo: idcirco *R* 264 ergo *om. R* / ergo erunt *transp. C1* / erunt *inter. a. m. E* 265 sectores: sectiones *R* / qui^{1,2}: que *R* / speculo piramidali: speculum piramidale *R* 266 capitis: verticis *R* / piramidis: piramidalis *L3* 268 exeuntes: exeunte *L3* 269 sectores: sectiones *R* / sectores . . . minores (270) *rep. et del. F* 271 sectores: sectiones *R* / qui: que *R* / piramidis: piramidalis *L3* / *post* piramidis *inter.* erunt *O* / econverso: econtrario *R* 272 unde: non *E* / quod: ut *R* / piramidali: piramidalis *SL3* 273 erit^1: sit *R* / capitis: verticis *R* / erit^2 *inter. a. m. E* 274 strictius *corr. ex* strictus *C1* 276 etiam *om. O* / quanto: quando *C1* 277 *post* speculo *add.* tanto *R* / et . . . minor (278) *rep. et del. E* / *post* magis *scr. et del.* res visa *S* 278 *ante* videbitur *add.* tanto *R*

[6.47] Fallacie ergo que accidunt in huiusmodi speculis sunt similes in omnibus dispositionibus illis que accidunt in speculis columpnalibus convexis preter quam in piramidatione forme. Et omnino forma rei vise que comprehenditur per conversionem semper assimulabitur forme superficiei speculi a qua convertitur forma, et huius causa est quod semper locus ymaginis constituitur ex forma superficiei speculi et ex loco concursus perpendicularium, ideo semper superficies speculi habet aliquam dignitatem in forma rei vise que comprehenditur in speculo. Fallacie vero composite in hoc speculo similes sunt fallaciis in speculis predictis.

CAPITULUM SEPTIMUM

De fallaciis que accidunt in speculis spericis concavis

[7.1] In hiis vero plures accidunt quam in omnibus speculis convexis et superficialibus, accidit enim in eis que in illis accidunt—scilicet debilitas lucis et coloris et diversitas situs et remotionis—nam causa huius est tantum conversio, non forma speculi. Accidit etiam in hiis speculis ex diversitate quantitatis plus quam in speculis convexis, nam in convexis, in maiori parte, res comprehendetur minor, in concavis vero quandoque comprehendetur maior, quandoque minor, quandoque secundum quod est, et hoc secundum diversitatem positionum eius ex speculo et ex visu, prout nos declarabimus in hoc capitulo.

[7.2] Accidit etiam in hiis speculis quod unum visibile videatur duo, et tria, et quattuor, et non est ita in speculis superficialibus et convexis, unum enim visibile non comprehenditur in illis nisi unum, in concavis vero non.

279 huiusmodi: hiis *L3ER* 280 columpnalibus: columpnaribus *L3C1ER* 281 convexis: convexa *L3*/preter: per *S* 282 comprehenditur: comprehenderunt *P1*/conversionem: reflexionem *R*/assimulabitur: assimilabitur *SER* 283 convertitur: reflectitur *R* 284 constituitur: est *R* 285 concursus: conversus *L3E*/perpendicularium *corr. ex* perpendiculariter *F* 286 speculi *om. C1* 288 in . . . predictis: predictis in speculis *L3E*/speculis predictis *transp. SR* 289 capitulum . . . concavis (290) *om. FP1S; mg. a. m. E;* de erroribus qui accidunt in speculis spericis concavis capitulum septimum *R* 291 *post* vero *add.* lineis *L3*/*post* plures *add.* errores *R* 292 superficialibus: superficiebus *L3E*/enim *om. L3E*/que: qui *O*/in^{2} *om. O*/illis: illum *O* 293 remotionis: remotius *SL3* 294 conversio: reflexio *R*/etiam *inter. O* 295 *post* plus *add.* erroris *R* 296 nam in convexis *om. L3*/convexis *corr. ex* vexis *O; corr. ex* concavis *C1*/maiori: maiore *R*/parte *corr. ex* partes *C1*/comprehendetur: comprehendere *FP1;* comprehenderetur *S;* comprehenditur *R* 297 *post* concavis *add.* q *P1*/quandoque: quando *L3*/comprehendetur: comprehenditur *R*/*post* comprehendetur *scr. et del.* ma *F* 298 quod *rep. et del. F*/*post* est *scr. et del`.* secundita *S; add.* quandoque *FP1*/et hoc *om. SO* 299 eius *om. R*/nos *om. FP1*/declarabimus: declaravimus *O* 1 videatur: videtur *ER* 2 est *om. FP1*/et^{4} *om. L3* 3 comprehenditur: comprehendetur *E*

[7.3] Item ordinatio partium rei vise comprehenditur in speculis convexis et superficialibus secundum quod est; in spericis vero concavis in pluribus sitibus alio modo, et hec duo: scilicet comprehensio unius ut unius et comprehensio ordinationis partium secundum quod est, non habent aliquam deceptionem in speculis convexis spericis, et cum in hiis accidit deceptio in speculis spericis concavis, patet quod nichil comprehenditur in huiusmodi speculis nisi cum fallacia, aut semper, aut in aliqua hora secundum diversitatem positionis.

[7.4] Debilitas vero lucis et coloris et diversitas positionis et distantia accidunt in hiis speculis sicut in aliis semper, et in omni positione. Quantitas vero, et forma, et numerus habent deceptiones in hiis speculis in aliquibus sitibus, prout declarabimus.

[7.5] De numero vero declaratum est in capitulo de ymagine quod unum visum in speculis spericis concavis habet unam ymaginem, et duas, et tres, et quattuor, et quod forma rei vise semper comprehenditur in loco ymaginis. Verum unum visum comprehensum in speculis spericis et concavis forte comprehendetur unum, et forte duo, et forte tria, et forte quattuor, quod non accidit in speculis convexis et superficialibus.

[7.6] De ordinatione vero partium rei vise dictum est etiam in capitulo de ymagine quod forma unius puncti convertitur ex circumferentia circuli, et quod visibilia quorum ymagines sunt retro vel post visum, et ante, et in centro visus apparent dubia, non certificata, et quod est huiusmodi non habet ordinationem partium sicut ipsa res visa habet. Et hoc etiam est in hiis speculis aliter quam sit in speculis convexis et superficialibus. Cause autem huius rei declarate sunt in capitulo de ymagine.

[7.7] Restat ergo declarare quod illud quod comprehenditur in hiis speculis forte comprehendetur maius, et forte minus, et forte equale,

5 *post* speculis *scr. et del.* spericis *S* 6 *post* convexis *scr. et del.* i *F*/*post* et *add.* in *FP1*/ superficialibus *corr. ex* superficiebus *E*/spericis: speculis *L3ER*/vero *om. C1* 7 scilicet *inter. E* 8 ut *corr. ex* non *S*/unius²: unis *L3*; unum *ER* 9 habent: habet *ER*/convexis spericis *transp. R* 10 in¹ *inter. E*/accidit . . . concavis: speculis spericis concavis accidit deceptio *R*/patet . . . speculis (11) *rep. P1* 11 huiusmodi: huius *L3*/*post* speculis *add.* concavis *P1* 12 semper: super *S*/in *om. L3ER*/diversitatem *om. FP1*/positionis: reflexionis *F*; reflexionem *P1* 13 lucis *corr. ex* luci *C1* 15 et² *om. L3*/deceptiones: deceptionem *L3ER* 16 sitibus *om. L3* 17 in *om. P1* 18 spericis *om. ER*/habet: habent *S* 19 quod *inter. O* 20 *post* verum *inter.* vel unde *O*/comprehensum . . . concavis (21) *rep. P1* 21 et¹ *om. OR*/*post* concavis *add.* et *E*; *add.* etiam *R*/comprehendetur: comprehenditur *R*/unum *inter. O* 22 *post* forte *scr. et del.* accidit *C1*/*post* speculis *add.* spericis *R*/et² *mg. a. m. E* 24 vero *om. L3*/*post* vise *add.* ut *L3E*; *scr. et del.* ut *C1*/etiam *om. FP1OR* 25 quod: et *S*/puncti *rep. et del. F*/convertitur: reflectitur *R* 26 *ante* circuli *add.* unius *R*/*post* quorum *scr. et del.* or *C1*/ *post* retro *inter.* vel *O*/post *inter. O* 28 huiusmodi: huius *FP1L3* 29 aliter . . . speculis *mg. a. m. E*/*post* speculis *scr. et del.* concavis *F* 30 huius: huiusmodi *C1* 32 declarare: determinare *P1* 33 comprehendetur: comprehenditur *ER*/maius *corr. ex* maior *P1*

et quod in quibusdam positionibus comprehendetur conversum, et in quibusdam erectum, et quod rectum in huiusmodi speculis comprehenditur concavum et convexum et rectum, et quod convexum et concavum comprehenduntur etiam aliter quam sint. Et hec etiam sunt ex diversitate ordinationis partium rei vise, et nos declarabimus hoc hoc modo.

[7.8] **[PROPOSITIO 23]** Sit ergo speculum spericum concavum in centro A [FIGURE 6.7.23, p. 317], et secetur superficie equali transeunte per centrum, et faciat circulum BG, et extrahatur in ipsa linea, quocumque modo sit, et dividatur in duo equalia in O.

[7.9] Et ponatur A centrum, et in distantia AO faciamus circulum, et sit EZ. Et ponatur in linea OU punctum T casualiter, quocumque modo sit, et ex T extrahantur linee TN, TM recte super lineam AU. Et extrahantur ex T linee TE, TZ tangentes circulum EZ, et continuemus AE, AZ, et transeant ad B, G. Et continuemus TB, TG, et extrahamus BM equidistantem ad AT, et GN etiam equidistantem AT, et continuemus AM, AN et extrahantur recte.

[7.10] Quia ergo AO est sicut OU, erit AE sicut EB, et AZ sicut ZG, et quia TE tangit circulum EZ, erit TE perpendicularis super BA, et similiter TZ perpendicularis super AG. Linea ergo BT est sicut TA, et TG sicut TA, et angulus TBA sicut angulus TAB, et angulus TGA sicut angulus TAG. Et quia BM est equidistans AT, erit angulus MBA sicut angulus BAT. Ergo angulus MBA est sicut angulus ABT, et similiter angulus TGA est sicut angulus AGN.

[7.11] Cum ergo visus fuerit in T, et M, N fuerit in aliquo visibili, tunc forma M extendetur per lineam MB et convertetur per BT, et

34 comprehendetur: comprehendentur *OL3; corr. ex* comprehenderetur *C1* 35 rectum: erectum *ER*/huiusmodi: huius *OL3*/comprehenditur: comprehendetur *ER* 36 convexum[1]: conversum *L3*/et rectum *om. P1*/et . . . convexum[2] *mg. O* 37 *post* et[1] *add.* quod *FP1*/aliter: alter *P1* 38 sunt *corr. ex* sint *S* 39 hoc[1]: hec *C1R* 40 ergo: vero *C1; om. R*/concavum *inter. O* 41 in centro: cuius centrum *R*/equali: plana *R*/*post* equali *add.* et *P1* 42 faciat: fiat *P1*/et[2] *om. R*/in ipsa: ab ipsius centro *R* 43 quocumque modo: quomodocumque *O*/modo sit *om. S*/in[1]: inter *P1*/in O *om. R* 44 in *om. FP1L3*/AO: AD *L3* 45 EZ: EX *FP1*/OU: TU *FP1S;* TN *L3C1E* (*inter. E*)/quocumque: quandoque *FP1;* quandocumque *S;* quomodocumque *O* 46 modo *om. FP1SOC1*/*post* ex *scr. et del.* TE *S*/TN: IN *S*/TN . . . linee (47) *inter. O*/TM: tantum *FP1;* IM *S* 47 *post* ex *scr. et del.* TE *S*/TE *corr. ex* T *F; om. L3*/TZ: ZZ *FP1; corr. ex* DZ *E*/tangentes: tanges *S*/*post* tangentes *scr. et del.* lineam *C1* 48 *post* continuemus *scr. et del.* in figura cave G et H sunt similia in arabico [sunt *inter.*] *O*/TG: BG *R*/extrahamus: protrahamus *R* 49 equidistantem[1]. . . etiam *om. S*/AT[1, 2]: AU *R*/etiam: et *FP1C1*/equidistantem[2]: equidistans *S* 50 AM AN *transp. R* 51 ergo *om. FP1*/AO: OA *FP1*/OU: OF *O; corr. ex* OB *E*/AE: AB *S*/sicut[3]: sunt *L3; rep. et del. E* 52 BA: AB *R* 53 est *om. P1*/et TG (54) *rep. P1* 54 TA: TQ *O*/TGA *corr. ex* TAG *E* 55 angulus[1] *om. FP1*/TAG: TAH *deinde inter.* G in arabico *O*/AT: AU *R*/angulus[2] *om. R* 57 angulus[1] *inter. a. m. E*/TGA: TGAG *F; alter. ex* TGIAG *in* TGAG *P1;* THA *deinde inter.* G in arabico *O; corr. ex* TAG *E*/est *om. R*/AGN: AGNG *FP1;* AHN *deinde inter.* G in arabico *O* 58 N: B *FP1SOL3ER*/fuerit . . . MB (59) *om. S*/in . . . visibili: aliquod visibile *R* 59 extendetur *corr. ex* extenditur *a. m. E*/convertetur: reflectetur ad visum *R*/*post* per *add.* lineam *R*/et[2]. . . GT (60) *scr. et del. E*

forma N extendetur per NG et convertetur per GT. Visus ergo T comprehendet puncta M, N ex punctis B, G, et lineam MN ex arcu BG.

[7.12] Et quia TE est perpendicularis super AB, erit angulus ABT acutus. Sed angulus MBA est sicut angulus ABT. Ergo TB est maior BM; ergo AT est maior BM, et sunt equidistantes. Ergo TB concurret cum AM. Concurrant ergo in F. F ergo est ymago M, et sic declarabitur quod TG concurret cum AN. Concurrat ergo in Q. Q ergo erit ymago N.

[7.13] Et continuemus FQ, que est dyameter ymaginis MN, et quia TE, TZ sunt equales, erunt anguli TAB, TAZ equales, et erunt linee TB, TG equales, et linee BM, GN equales, et linee AM, AN equales. Et proportio AF ad FM sicut proportio AT ad BM, et proportio AF ad FM est sicut proportio AT ad GN, et proportio AT ad BM; ergo proportio AF ad FM est sicut proportio AQ ad QN, et AM est sicut AN. Ergo AF est sicut AQ; ergo FQ equidistat MN. Ergo FQ est maior MN. Sed FQ est dyameter ymaginis MN. Ergo si visus fuerit in T et MN fuerit in aliquo visibili, tunc visus comprehendet formam maiorem quam sit.

[7.14] **[PROPOSITIO 24]** Item iteremus circulum BG [FIGURE 6.7.24, p. 317], et lineam AT, et lineas AB, AG, TB. Et super punctum T sit perpendicularis super superficiem circuli BG, et sit TK, et continuemus KA, KB, KG. Superficies ergo KBA, KGA sunt secantes speram super suum centrum perpendiculariter super superficies tangentes ipsam. Ex ipsis ergo convertitur forma, et due differentie communes inter has duas superficies et speram sunt circuli magni a quorum circumferentia convertuntur forme.

60 N: B *FP1SOL3E*; *corr. ex* B *C1* / *post* per[1] *add.* lineam *R* / NG: ZH *deinde inter.* G in arabico *O* / convertetur: reflectetur *R* / GT: HT *deinde inter.* G in arabico *O* 61 *post* N *add.* et *L3* / *post* B *add.* H *FP1S* / G *corr. ex* H *O* / BG: BHG *S*; *corr. ex* BH *O* 62 TE: TG *S*; MB *O*; MT *R* / AB: AT *OR* / ABT: MTB *R* 63 *post* acutus *add.* ergo angulus BLT est obtusus *O* / sed: et quia *R* / MBA *corr. ex* MAB *E*; BMT *R* / ABT: MTU *R* 64 *post* BM[1] *add.* et linea TB est equalis linee AT *R* / ergo[1]... BM[2] *om. P1E* / *post* ergo[1] *add.* linea *R* / *post* maior *add.* linea *R* / ergo[2]: OG *S* 65 concurrant: concurrent *FP1*; concurram *L3* / F[2] *inter. O* 66 quod *om. L3* / TG: THG *FP1S*; *corr. ex* TH *O*; *corr. ex* GT *E* / concurret: concurrat *L3* / AN: AZN *FP1*; AZH *S*; AZ *O* / ergo[1] *om. E* / *post* in *add.* puncto *P1* / *post* Q[1] *add.* L *O* / Q[2] *om. L3E* 67 ymago *rep. et del. P1* 68 continuemus *corr. ex* continemus *C1* / MN: MB *R* 69 TZ: EZ *FP1*; TS *L3E* / TAB: TAE *R* 70 TG: TH *FP1* / et[1]... equales[2] *om. FP1* / *post* GN *inter.* H in arabico *O* 71 proportio[1]... sicut *om. S* / AT ... proportio[2] (72) *om. L3* / BM: MB *R* / AF[2]: AQ *R* / FM[2]: QN *R* 72 GN: NG *R* / *post* GN *inter.* H in arabico *O* / et ... BM *om. ER* / BM: BN *FP1SOL3* 73 FM: FQ *L3C1E*; *alter. ex* FQ *in* MQ *O* / proportio *om. L3* / est[2] *om. P1* 74 ergo[2]... MN[2] *mg. O* / FQ[2]: FA *E* / *post* MN[2] *add.* quia AF ad AM sicut FQ ad MN sed AF maior est AM *R* 75 *post* et *add.* linea *R* 76 tunc: nunc *L3* 78 AT: AU *R* / TB: FB *O* / *post* TB *add.* TG *R* 79 T *om. L3E*; *inter. O* / et[2] *om. R* 80 KA: KL *O* / KG *mg. F*; KH *deinde inter.* G in arabico *O* / superficies *om. P1* / KBA: KB *O* / *post* KBA *add.* AK *FP1SO* / sunt secantes: secant *R* 81 suum centrum *transp. ER* / super[2]: et *R* 82 tangentes *corr. ex* contingentes *P1* / *post* tangentes *scr. et del.* eam *F* / convertitur: reflectitur *R* 83 inter: in *E* / *ante* has *add.* et *E* 84 convertuntur: reflectuntur *R*

[7.15] Et extrahamus BM in superficie BKA equidistantem AK, et sit minor quam AK. Et continuemus AM, et extrahatur recte, et extrahatur KB donec concurrant in F. Et extrahatur NG in superficie KGA, et sit equidistans AK, et ponatur equalis BM. Et continuemus AN, extrahatur recte, et extrahatur KG recte donec concurrant in Q. Et continuemus MN, FQ.

[7.16] Quia ergo BT est sicut TA, erit BK sicut KA, et GK sicut KA. Ergo BK est sicut GK, et angulus KBA est sicut angulus KGA, et angulus KAB est sicut angulus KBA. Et similiter angulus KGA est sicut angulus KAG; ergo angulus ABM est sicut angulus ABK, et angulus AGN est sicut angulus AGK, et erit angulus ABM sicut angulus AGN. Et linea BM erit sicut linea GN. Tunc linea AM erit sicut linea AN; tunc AF erit sicut linea AQ. Tunc due linee FQ, MN erunt equidistantes; tunc FQ erit maior linea MN.

[7.17] Tunc quando fuerit visus super punctum K, et fuerit linea MN in aliquo visibili, tunc forma M extendetur super lineam MB et convertetur per lineam BK in superficie circuli transeuntis per puncta B, A, K, et forma puncti N extendetur super lineam NG et convertetur super lineam GK in superficie circuli transeuntis per puncta G, A, K.

[7.18] Et erit punctum F ymago puncti M, et punctum Q erit ymago puncti N, et erit linea FQ dyameter ymaginis MN. Et iam declaravimus quod linea FQ est maior linea MN; tunc quando fuerit visus super punctum K, et fuerit linea MN in aliquo visibili, tunc visus apprehendet formam linee MN super lineam FQ. Tunc comprehendet formam maiorem re visa.

85 BM: BN *L3* / BKA: BA *FP1* / *post* BKA *scr. et del.* et continuemus *E* / equidistantem: equidistans *S*; equidistanter *L3C1* 86 AK: AB *FP1E* 87 concurrant: concurrat *OR* / *post* concurrant *add.* cum AM *R* / et: sed *FP1* / NG: MG *FP1*; HZ *deinde inter.* ZG in arabico *O* 88 KGA *corr. ex* KHA *O* / AK: TK *O* 89 *post* AN *add.* et *OC1E* / extrahatur recte *om. SL3* / et extrahatur *om. C1* / et . . . recte[2] *om. R* / KG: BG *E*; KH *deinde inter.* G in arabico *O* / KG recte *transp. L3* / concurrant: concurrat *R* / Q: que *P1* 91 BK: BQ *FP1* / KA: K *S* / et . . . KA (92) *mg. F* / GK *corr. ex* HK *O* 92 GK *corr. ex* HK *O* / et . . . KGA *om. R* / KGA *corr. ex* KHA *O* / KGA et angulus (93) *mg. F* 93 KAB . . . angulus[2] *om. P1* / est[1]. . . KGA *om. L3* / KGA *corr. ex* KHA *O* 94 KAG: KGA *FP1O* / ABM: AMB *FP1L3* / et . . . AGN (96) *mg. a. m. E* 95 *post* AGN *inter.* HZ in arabico *O* / AGK: ABK *FP1*; KBA *O* / *post* AGK *add.* ergo *R* / erit *om. O* / angulus[2] *om. SC1* / angulus ABM sicut *om. P1* / *post* ABM *scr. et del.* est *S*; *add.* erit *O* 96 AGN: ANG *L3E*; AGZ *deinde inter.* N in arabico *O* / linea[2] *inter. E* / GN: GM *FP1* / *post* GN *inter.* Z in arabico *O* / AM *om. S* / erit[2]: est *L3* 97 AN: AZN *FO* (*alter. in O*); GAZN *P1*; ANZ *S*; AQ *E* / tunc[1] . . . AQ *scr. et del. E*; *om. R* 99 fuerit visus *transp. R* 100 *post* visibili *add.* inferiore *R* / extendetur *corr. ex* existetur *P1* 101 convertetur: reflectetur *R* / per[1] *inter. O* / BK: KK *P1* / *post* circuli *add.* qui transit per puncta B A K et forma puncti N extendetur super lineam NG et convertetur super lineam GK in superficiei circuli *mg. O* 102 B *corr. ex* G *O* / A *inter. L3* / et[1]. . . K (103) *om. O* / convertetur: reflectetur *R* 103 GK *corr. ex* NGK *F* / *post* per *add.* tria *R* 104 punctum F ymago: ymago puncti F *FP1SL3C1ER* (F: M *C1*) / puncti: punctum *FP1L3C1ER*; puncto *S* / M: N *L3*; F *C1* 105 *post* puncti *scr. et del.* F *F* 106 fuerit visus *transp. R* 107 apprehendet: comprehendet *L3* 108 linee . . . formam (109) *om. R*

[7.19] Et sic, si revolverimus item totam figuram in circuitu linee AU, ipsa immobili, tunc punctus K faciet circulum perpendicularem super lineam AU, et sic omne punctum ultra illum punctum illius circuli habebit situm respectu linee comparis linee MN sicut est situs K respectu MN.

[7.20] Si ergo visus fuerit in aliquo puncto circumferentie huius circuli et linea compar linee MN fuerit in superficie alicuius rei vise, tunc visus comprehendet formam illius linee maiorem. Et similiter si extraxerimus TK recte et posuerimus in ipsa aliquod punctum preter K, et extraxerimus semper ab illo puncto, quod est quasi punctum K, erit modus eius sicut modus puncti K.

[7.21] Ex hiis ergo duabus figuris patet quod in spericis speculis concavis multa et ex multis sitibus comprehenduntur maiora.

[7.22] **[PROPOSITIO 25]** Item sit speculum spericum concavum AB circa centrum E [FIGURE 6.7.25, p. 318], et extrahamus superficiem transeuntem per E, et faciat circulum AB. Et extrahamus ex E lineam EZ, quocumque modo fuerit, usque ad G, et ex G extrahamus GD perpendicularem super superficiem circuli AB, et in ipsa signemus punctum D, quocumque modo fuerit. Et continuemus DE, et extrahamus ipsam usque ad O, et extrahamus EB ita quod contineat cum ED angulum obtusum, et extrahamus EA ita quod contineat cum ED angulum equalem angulo DEB. Et continuemus DA, DB. Sic ergo superficies duorum triangulorum DAE, DBE secant se super lineam DE, et duo anguli acuti DBE, DAE erunt equales.

[7.23] Et extrahamus ex B lineam in superficie trianguli DBE continentem cum EB angulum equalem angulo DBE. Hec ergo linea con-

110 si revolverimus *corr. ex* scire voluerimus *a. m. E* / item *om. OC1R* 111 punctus: punctum *R* / perpendicularem *om. O* 112 AU: AY *FP1* / *post* punctum[1] *inter.* quod est *O* / ultra illum punctum *om. R; inter. O* / punctum[2]: circulum *FP1SOL3C1E* 113 comparis: operis *S*; corporis *L3* / linee[2] *om. R* / MN: MH *FP1*; MZ *O*; MQ *L3E*; unius *C1* / sicut . . . MN (114) *om. P1* 114 MN: MK *F*; MZ *O*; MQ *L3E* 115 ergo: vero *O* / fuerit in aliquo: in aliquo fuerit *FP1* 116 et *mg. a. m. E* / MN: MK *FP1*; MZ *O*; MQ *L3E* 117 comprehendet: comprehendit *C1* / *post* maiorem *scr. et del.* et quando extraxerimus semper ab illo puncto quod est Q punctum R erit modus eius sicut modus puncti R *O* / et *om. FP1* 118 extraxerimus: extra erimus *S*; extrahamus *L3ER*; extraherimus *C1* / TK: KT *FP1* 119 et . . . K (120) *om. O* / extraxerimus: extra erimus *S*; extraherimus *L3C1*; *corr. ex* extraherimus *E* / *post* extraxerimus *add.* lineas *R* / quasi: Q *SL3E* / K[2] *om. S* 120 sicut: si *FP1* 122 *post* concavis *add.* et *P1R* 123 sit: si *L3* / concavum *om. L3C1ER* 124 circa: citra *FP1* / extrahamus: extramus *S* / superficiem . . . extrahamus (125) *om. S* 125 transeuntem *mg. F* / *post* per *scr. et del.* O *F* / faciat: faciet *FP1* / ex *om. L3* 126 *post* G[2] *add.* et *L3E* 127 GD: ergo *O* / perpendicularem: perpendiculariter *O* / signemus: signavus *S* 128 quocumque: quomodocumque *O* / DE: D *O*; TE *L3E* / et[2] *om. S* 129 O *corr. ex* A *P1* / EB: EA *S* / quod: ut *R* / contineat: concurrat *L3* 130 angulum . . . ED (131) *om. S* / quod: ut *R* 131 angulo: modo *L3* / et *rep. C1* 132 secant: secent *FP1* 133 acuti *om. P1* 134 et *om. L3ER* / *post* extrahamus *add.* ergo *L3ER* / DBE: DEB *ER* / continentem: contingentem *FSL3C1*; contingente *P1* 135 *post* cum *add.* FP *FP1* / EB: ED *L3* / DBE *corr. ex* DEB *a. m. E* / concurrit: concurret *R*

currit cum linea DE, quia angulus BEO est acutus, et angulus qui est apud B est acutus. Concurrat ergo in O.

[7.24] Et extrahamus etiam ex A lineam in superficie trianguli DAE continentem cum AE angulum equalem angulo DAE. Concurrat ergo cum DE in O, quia duo anguli AEO, BEO sunt equales, et anguli qui sunt apud duo puncta A, B sunt equales.

[7.25] Et extrahamus ET ita quod contineat cum EB angulum rectum, et extrahamus TE in parte E, et BO in parte O, et concurrant in H, et erit TE equalis EH. Et similiter extrahamus EK ita quod contineat cum EA angulum rectum, et extrahamus illam in parte E, et extrahamus AO, et concurrant in L. Sic ergo KE erit equalis EL.

[7.26] Et continuemus TK, LH. Erunt ergo equales. Si ergo visus fuerit in D et LH fuerit in aliquo visibili, tunc D comprehendet LH in speculo AB, et erit T ymago H, et K ymago L, et sic TK erit dyameter ymaginis LH, et est ei equalis.

[7.27] Si ergo revolverimus totam figuram, HL immobili, tunc D faciet circulum, et si visus fuerit in aliquo puncto circumferentie illius, poterit comprehendere aliquod visibile compar linee LH, et erit ymago eius equalis ei. Et similiter si visus fuerit in O, et res visa fuerit TK, erit ymago equalis rei vise.

[7.28] Sed tamen cum res visa fuerit LH, et visus fuerit D, et fuerit ymago TK, ymago erit conversa; si H fuerit in dextro, erit T in sinistro, et si H fuerit in sinistro, T erit in dextro, et si H fuerit supra lineam, erit T infra lineam, et similiter L.

[7.29] Et si res visa fuerit TK, et visus fuerit O, et ymago fuerit LH, forma erit recta, nam ymago LH erit retro post visum, et comprehen-

136 *post* DE *scr. et del.* qua *S*/BEO: BED *L3R; alter. ex* DEB *in* BED *E*/acutus: obtusus *R*/et . . . acutus (137) *om. P1* 137 B *corr. ex* BF *O*/concurrat: currat *FP1*; concurrant *R*/ergo *om. R*/O *corr. ex* E *S* 138 *post* ex *inter.* A *O* 139 continentem: contingentem *SL3C1; corr. ex* contingentem *E*/concurrat: concurret *R* 140 in O *om. L3*/*post* O *add.* quod *FP1S*/quia *om. S*/AEO: AED *FP1* 141 duo puncta *om. R* 142 et *om. FP1*/ET *inter. a. m. E*/*post* ET *scr. et del.* et *C1*/quod: ut *R*/contineat *corr. ex* continuet *P1*; concurrat *S*/*post* angulum *scr. et del.* acutum *P1*/rectum *inter. a. m. E* 143 et . . . H (144) *rep.* [extrahamus: erit] *FP1*/TE: DE *FP1O*/in[1,2]: ex *R*/BO: HO *E*/concurrant: concurrat *FP1O* 144 TE: ET *R*/*post* EH *add.* et BT equalis BH *R*/extrahamus . . . et[1] (145) *om. S*/*post* EK *add.* et *P1*/quod: ut *R* 145 angulum: angulus *F*/et[1]. . . E *rep. S*/in: ex *R* 146 AO: AD *SL3*/EL: LE *O*; KL *FP1SL3E*/*post* KL *add.* et KA equalis AL et TE equalis EH *R* 148 comprehendet: comprehendit *S*/*post* in *add.* hoc *L3E* 149 *post* K *add.* etiam *E*/et[3] *om. ER*/*post* sic *add.* igitur *R*/dyameter: dyametrum *FP1* 150 ymaginis *om. FP1*/equalis *corr. ex* equales *L3* 151 revolverimus: volverimus *L3; corr. ex* volverimus *a. m. E*/HL: LH *R* 152 faciet *corr. ex* fiet *O*/circumferentie *corr. ex* cirferentie *F*/circumferentie illius *transp. R* 153 LH: LB *O* 154 fuerit: fuit *O* /in . . . fuerit (156) *mg. F*/O: eo *FP1* 155 TK *corr. ex* TH *a. m. E* 156 visa *om. FP1S*/LH *inter. E*/*post* et[1] *scr. et del.* si *F*/et fuerit: fueritque *ER* 157 ymago erit *transp. ER*/*post* fuerit *add.* visa *L3*/dextro: dextra *R*/erit[2]: et si *O*/T: et *FP1; corr. ex* et *C1*; L *O*/*ante* in[2] *add.* fuerit *O*/sinistro: sinistra *R* 158 *post* fuerit[1] *add.* visa *L3*/sinistro: sinistra *R*/T erit *transp. L3ER*/dextro: dextra *R*/H[2] *corr. ex* B *a. m. E*/supra: extra *L3* 159 *post* lineam *add.* erit T N *P1*/L: B *E* 160 LH: LB *O* 161 forma . . . LH *mg. a. m. E*/erit[1]: est *R*/post *om. R*

detur ante rem visam, sicut declaravimus in capitulo ymaginis quinti tractatus, et visus comprehendet H, quod est ymago T, in linea BO, et L quod est ymago K, in LO.

[7.30] Patet ergo quod in speculis concavis comprehenditur res visa quandoque equalis sibi.

[7.31] **[PROPOSITIO 26]** Item extrahamus BH recte, et in ipsa signemus R, et continuemus RE. Sic ergo angulus REB erit obtusus.

[7.32] Et extrahamus RE ad N. Sic ergo RB erit maior quam BN. Et proportio RB ad BN est sicut proportio RE ad EN; ergo linea RE est maior quam EN.

[7.33] Et extrahamus AL recte, et sit AM equalis BR. Et continuemus ME, et transeat usque ad U. Erit ergo ME maior quam EU. Et continuemus MR, NU. Erit ergo MR maior quam NU.

[7.34] Si ergo MR fuerit in aliquo visibili, et visus fuerit in D, erit NU dyameter ymaginis MR, et NU est minor quam MR. Et si visus fuerit in O, et NU fuerit in aliquo visibili, erit MR ymago NU, et est maior quam NU.

[7.35] Sed cum MR fuerit visibile, et fuerit ymago NU, tunc ymago erit conversa, et si NU fuerit visibile et MR fuerit ymago, ymago erit recta, nam ymago, si fuerit ultra visum, videbitur ante, et omne punctum ymaginis videbitur in linea in qua est de lineis radialibus.

[7.36] **[PROPOSITIO 27]** Item signemus in linea OH punctum Q. Et continuemus QE, et transeat ad C. Et sit OF equalis OQ, et continuemus EF, et transeat ad I. Erunt ergo due linee CE, EI maiores duabus lineis EF, QE, et erit linea CI maior quam linea FQ.

[7.37] Si ergo visus fuerit in O, et CI in aliquo visibili, erit FQ ymago CI, et FQ est minor quam CI. Et FQ videbitur super duas lineas

163 tractatus: tractus *F* / H: B *O* / quod *inter. a. m. E*; Q *P1* / BO: HO *R* 164 quod: et *FP1* / *post* in *add.* linea *C1* / LO *corr. ex* BO *C1* 165 comprehenditur: comprehendetur *E*; comprehendatur *R* 167 BH: LH *P1* / recte *corr. ex* recta *a. m. E* 168 signemus *corr. ex* assignemus *O* / R: K *L3* / continuemus: continemus *R* / RE: BE *L3* 169 *post* ergo *add.* TB erit maior BN ergo linea *R* / RB: TB *L3E* / erit: est *R* / BN . . . quam (171) *rep. E; rep. et del. F* / quam *om. R* 170 et . . . EN (171) *rep. et del. C1* / RB: TB *L3; corr. ex* TN *a. m. E* / ad BN *rep. P1* / BN: DN *S* / EN *corr. ex* N *S* / ergo: quare *R* 171 *post* quam *add.* linea *R* / EN: BN *S; corr. ex* BEN *L3* 172 *post* recte *add.* in M *R* / sit: sicut *FP1* / BR: BZ *L3* 173 transeat: transeamus *C1* 174 maior . . . MR (175) *om. S* / NU: MU *C1* 175 MR: ME *FP1* 176 NU: MR *C1* / minor: maior *FP1SL3C1E* / MR: NU *C1* / visus fuerit (177) *transp. FP1C1* 177 fuerit *om. S* / *post* visibili *add.* et *L3* / NU²: NB *FP1* / et . . . NU (180) *mg. O* 178 NU: QM *FP1*; MR *O* 179 et . . . visibile (180) *om. FP1SL3E; mg. C1* / fuerit . . . NU: NU fuerit ymago et D visus *R* / tunc *om. R* / ymago erit conversa (180): erit ymago conversa *R* 180 *post* si *add.* res visa *R* / NU fuerit *transp. R* / visibile et: et visus O *R* / MR fuerit ymago: ymago MR *R* / ymago² *scr. et del. F; om. P1R* 181 recta: erecta *L3* / si *om. S* 182 in¹ *om. L3* 184 C: P *R* / et³. . . linee (185) *mg. a. m. E* / OQ: EQ *S* 185 I: L *E* / CE *om. FP1*; QE *SL3E*; TI *O*; PE *R*; *corr. ex* QE *C1* / maiores *corr. ex* maior *C1* 186 QE: EQ *R* / CI: Q *L3E*; PI *R* / linea² *om. C1* 187 visus: visusus *F* / visus fuerit *transp. L3* / CI: Q *L3E*; PI *R* / visibili: visibile *FP1SO* / FQ: QF *C1* 188 CI¹: Q *L3E*; PI *R* / minor: maior *E* / CI²: PI *R* / *post* FQ² *scr. et del.* est minor quam TI *P1* / *post* videbitur *add.* semper *L3*

AO, OB. Erit ergo forma ante visum et minor quam res visa, et erit recta.

[7.38] Et si visus fuerit in D, et FQ fuerit in aliquo visibili, erit CI ymago FQ. Et est maior quam FQ, et erit forma ante visum conversa.

[7.39] Patet ergo quod in speculis concavis comprehenditur forma rei vise minor, et maior, et equalis.

[7.40] **[PROPOSITIO 28]** Item sit speculum concavum AB [FIGURE 6.7.28, p. 319], et centrum G, et habeat superficiem equalem transeuntem per centrum, et faciat circulum AB. Et extrahamus lineam GD, quomodocumque sit, et transeat in parte G ad E, et sit visus in E, et sit T in superficie visus.

[7.41] Et extrahamus TH perpendiculariter super lineam ED, et sit ZT equalis TH, et comprehendat E punctum H ex A. Sic ergo erunt duo puncta A, H a duobus lateribus puncti G, nam si in eodem essent, tunc linea que exierit a speculo ad A non divideret angulum quem continent due linee radiales.

[7.42] Et extrahamus lineas EA, AH, GA, GH, et transeat GH recte ad K. Duo ergo anguli qui sunt apud A erunt equales, et erit K ymago H.

[7.43] Et sit arcus BD equalis arcui DA, et continuemus lineas EB, BZ, BG, et extrahamus ZG ad L. Erunt ergo duo anguli apud B equales, et comprehendetur Z a visu ex B, et erit L ymago Z.

[7.44] Et continuemus KL. Erit ergo KL dyameter ymaginis ZH, et quia ZTH est perpendicularis super DE, et ZT est equalis TH, erunt due linee EA, AH equales duabus lineis EB, BZ, et duo anguli apud A sunt equales duobus angulis apud B, et linea GH est equalis linee ZG.

189 OB: BO *R*/ante: retro *R* 190 recta: erecta *L3* 191 visus *inter. a. m. E*/fuerit[1] *rep. P1*/FQ: QF *C1*/CI: PI *R; corr. ex* Q *E* 192 *post* et[1] *add.* T *L3; add.* R *E* 193 forma rei vise (194): rei vise forma *C1* 194 *ante* minor *scr. et del.* mor *C1* 195 sit: si *L3; corr. ex* si *O* 196 *post* habeat *scr. et del.* f *F*/equalem: planam *R* 197 AB: OB *L3*/lineam *om. C1* 198 quomodocumque: quocumque modo *OC1ER*/*post* quomodocumque *add.* quisque *L3*/transeat: transeant *E*/in: ex *R* 199 T *om. S*/*post* visus *inter.* vel rei vise *O* 200 perpendiculariter *corr. ex* perpendicularium *P1* 201 ZT: ZD *L3E*/*post* A *add.* et GH producta in P comprehendat arcum AP maiorem quarta circuli *R* 202 G *corr. ex* H *L3*/si *mg. F* 203 exierit: exiret *R*/ad A *rep. P1*/divideret: dividet *L3* 204 quem *corr. ex* quoniam *E*/*post* continent *scr. et del.* duo anguli *L3*/*post* radiales *add.* per equalia *R* 205 EA: EH *L3; corr. ex* TA *a. m. C1*/AH: HAH *P1; corr. ex* HAH *F; inter. E*/transeat *corr. ex* transeant *S*/GH[2]: EH *P1* 206 qui sunt *om. R*/erunt: essent *FP1* 208 BD: HD *S*/*post* equalis *scr. et del.* anguli *S* 209 *post* BZ *add.* GZ *SO*/BG: BZ *F; om. P1; corr. ex* BZ *E*/*post* BG *add.* GZ *C1*/L: I *E*/*post* L *add.* et secet ZB dyametrum DG in F *R*/duo *om. P1* 210 equales *om. P1*/Z[1] *om. FP1S; inter. E*/B: A *FP1*/*post* erit *add.* punctum *R*/L: B *FP1SL3C1E* 211 dyameter: dyametri *S*/ymaginis: ymagines *FP1* 212 ZTH: ZH *L3*; TH *R*/DE: D *S* 214 duobus *rep. et del. E*/*post* B *add.* erit HE equalis ZE *R*/est *om. C1* 215 ZG: ZH *FP1SOL3ER*

[7.45] Ergo due linee AG, GH sunt equales duabus lineis BG, GZ, et basis AH est equalis basi BZ. Ergo angulus AHG est equalis angulo BZL, et angulus HAK est equalis ZBL. Ergo HK est equalis ZL, et linea HG est equalis ZG; ergo GK est equalis GL. Ergo KL est equidistans ZH.

[7.46] Item angulus HGA est obtusus, et duo anguli apud A sunt equales; ergo linea GH est maior linea GK, et similiter ZG est maior quam GL. Linea ergo KL est minor quam ZH. Sed KL est dyameter ymaginis ZH. Linea ergo ZH videbitur minor quam sit secundum veritatem. Et linea ZH est superficies faciei aspicientis.

[7.47] Si ergo revolverimus circulum ad B, EG immobili, in circuitu ED, fiet circulus, et fiet ex duobus punctis A, B circulus in superficie speculi. Et erit situs visus E respectu cuiuslibet comparis linee ZH ex illo circulo quam signat ZH, et ex omni arcu compari arcui AB ex portione quam dividit circulus quem signant duo puncta A, B sicut est situs quem visus E habet ex linea ZH et ex arcu AB. Et similiter declarabitur si posuerimus lineam maiorem quam ZH aut minorem.

[7.48] Patet ergo ex hiis omnibus quod dyameter superficiei faciei aspicientis comprehenditur in speculo concavo minor quam sit. Sequitur ergo quod si visus fuerit in E, tunc aspiciens comprehendet suam formam in tali speculo minorem quam sit, et quia K est ymago H, et L est ymago Z, erit ymago conversa.

[7.49] Et sic visus E comprehendet suam formam, scilicet quod est in dextro comprehendet etiam in sinistro et sursum deorsum, et econverso. Et similiter si visus fuerit in quolibet puncto inter quod et superficiem speculi fuerit centrum speculi, comprehendet suam formam conversam, et hoc est quod voluimus.

216 AG: EAG *FP1* / GZ: GH *FP1* 217 AH *corr. ex* AG *O* / *post* basi *scr. et del.* L *F* / AHG: ABG *S*; AHK *R* 218 ergo HK *rep. C1* 219 *post* linea *scr. et del.* HK *P1* / ZG: ZH *FP1SOL3ER* / GK: HK *FP1O*; KH *L3E*; KG *C1* / GL: HL *O* / GL ergo *om. P1* / KL: ZL *P1*; *corr. ex* LK *a. m. E* 221 item *om. P1* / HGA: LGA *S* / apud *om. O* 222 linea[2] *inter. P1* / GK *corr. ex* KG *S* / ZG: ZK *P1* 223 GL: GK *FP1*; *corr. ex* AGL *S* / sed . . . ZH[1] (224) *mg. a. m. E* 224 linea *om. R* / videbitur: videtur *R* 226 EG: ED *R* / in *om. P1* / in . . . et (227) *om. R* 227 *post* ED *scr. et del.* fa *O* / fiet[1] *corr. ex* fiat *a. m. E* / fiet[2] *corr. ex* fient *O*; *corr. ex* fiat *a. m. E* 228 *post* ZH *scr. et del.* exit *F*; *add.* et *L3* 229 quam: quem *SL3ER* / signat: signant *R* / ZH: puncta Z H *R* / compari: comparari *S* / *post* arcui *scr. et del.* ex *C1* 230 *post* portione *add.* speculi *R* 231 quem: quoniam *FP1S* / E: est *FP1* (*scr. et del. F*) 232 *post* lineam *add.* ZH *R* / quam ZH *om. R* 234 sequitur: sciendum *FP1R* 235 si visus: similis *L3E* / visus fuerit *transp. R* / fuerit *om. L3E* 236 suam formam *transp. C1ER* / in tali speculo *om. R* / *post* et *scr. et del.* linea *P1* / K *om. L3* 238 *post* comprehendet *scr. et del.* in sinistro *S* / *post* formam *inter.* conversam *O* / scilicet: secundum *P1L3ER* 239 in dextro: dextrum *R* / comprehendet etiam *om. R* / etiam *om. SO* / sursum *corr. ex* sur *O* / *post* sursum *add.* et *FP1* 240 econverso: econtrario *R* / et[1] *om. ER* / inter quod et: uterque *L3* 241 speculi[1] *corr. ex* speculum *S* / comprehendet: comprehendit *C1* / suam formam *transp. ER* 242 est *inter. O* / voluimus: volumus *C1E*

[7.50] Patet ergo ex hiis quattuor figuris quod in speculo concavo quandoque comprehenditur maior, quandoque minor, quandoque equalis, et nunc recta, et nunc conversa.

[7.51] Et in capitulo de ymagine diximus quod in speculo concavo quandoque ymago erit una, quandoque due, et quandoque tres, et quandoque quattuor, et hoc idem accidit in hiis predictis.

[7.52] Illud ergo quod habet ymaginem se maiorem forte habebit alias minores et equales, et quod habet minorem ymaginem forte habebit alias maiores et equales, et quod habet equalem forte maiorem et minorem, et quod rectum videtur forte videbitur sub alia ymagine conversum, et econverso. Restat ergo declarare formas eorum que comprehenduntur in huiusmodi speculis.

[7.53] **[PROPOSITIO 29]** Sit ergo speculum spericum AB [FIGURE 6.7.29, p. 320], et extrahamus in ipso speculo superficiem equalem transeuntem per centrum, et faciat circulum AB circa centrum E. Et extrahamus in hoc circulo duos dyametros se secantes AEO, BED, et speculum non excedat arcum BADO. Et ponamus in BE punctum Z, quomodocumque sit, et ponamus in linea AE punctum K, et sit AK maior quam KE. Et continuemus ZK, et transeat ad F. Et continuemus EF, et sit angulus EFG equalis angulo EFZ.

[7.54] Quia ergo FK est maior quam KA, et KA est maior quam KE, erit FK maior quam KE. Angulus ergo FEK est maior angulo EFK; est ergo maior angulo EFG. Linea ergo FG concurret cum linea KE. Concurrant ergo in G. Due ergo linee ZF, FG convertuntur per angulos equales; K ergo est ymago G, si visus fuerit in Z.

243 *post* concavo *add.* ymago *R* 245 nunc^2: ? *S* 246 *post* concavo *add.* ymago *R* 247 *post* una *add.* et *FP1* / et^1 *om. SE* 248 et *om. S* / idem: quidem *FP1* / *post* hiis *add.* speculis *L3* 249 quod *corr. ex* quo *L3* / *post* maiorem *add.* et quod habet equalem *SL3C1E* (habet: habebit *E*) / habebit *corr. ex* habet *F* 250 minores *corr. ex* maiores *C1* / et^1 *inter. E* / habet . . . ymaginem: ymaginem habet minorem *R* / minorem . . . habet (251) *mg. a. m. E* / *post* ymaginem *add.* ne *L3* 251 et^1. . . maiorem *om. R* / forte . . . videtur (252) *om. L3* 252 minorem: minores *R* / rectum: recte *P1E* (*alter. in a. m. E*) / videtur: videbitur *R* / videbitur *corr. ex* videtur *a. m. E* 253 conversum *om. E* / econverso: econtra *SL3*; *alter. ex* econtra *in* conversum *a. m. E*; econtrario *R* / declarare: declare *F* 254 huiusmodi: hiis *R* 255 sit: si *SL3*; *corr. ex* si *E* / *post* spericum *add.* concavum *R* 256 speculo *om. O* 257 equalem: planam *R* / faciat: fiat *P1* / circa: citra *FP1*; *alter. in* citra *E* 258 *post* hoc *scr. et del.* speculo *E* / duos: duas *R* 259 excedat: ecedat *F*; excedet *S* / arcum *om. L3* / et^2 *inter. O* 260 quomodocumque: quocumque *FP1*; quocumque modo *C1ER*; *corr. ex* quocumque *O* 261 AK *corr. ex* KA *C1* 262 sit: si *O* / EFG: EFH *FP1SOL3*; EHF *E*; GFE *R* / EFZ: ZFE *R* 263 quam^1 *om. R* 264 erit . . . KE^2 *mg. a. m. E* / est maior *transp. R* 265 EFK *corr. ex* FK *O* / est *inter. O* / est ergo *transp. C1R*; *transp. deinde corr. E* / EFG: EFH *FP1SOL3E* / FG: FH *FP1O*; FR *L3E* 266 G *corr. ex* H *L3* / ergo^2 *om. L3* / ergo linee *transp. C1* / FG: FH *FP1O*; FR *L3E* / convertuntur per: reflectuntur propter *R* 267 *post* angulos *scr. et del.* ano *P1* / *post* equales *add.* ZFE GFE *R* / G: HG *O* / fuerit *om. P1* / *post* Z *add.* et R *E*; *scr. et del.* et R *C1*

[7.55] Et extrahamus lineam ZLH, quomodocumque sit, et continuemus EH, HG, ZG, et extrahamus FE usque ad M. Proportio ergo ZM ad MG est sicut proportio ZF ad FG. Et ZH est maior quam ZF, et GH est minor quam GF. Ergo proportio ZH ad GH est maior quam proportio ZF ad FG; est ergo maior quam proportio ZM ad MG. Linea ergo que dividit angulum ZHG in duo equalia secat lineam MG; secat ergo lineam EG. Angulus igitur GHE est maior angulo EHZ.

[7.56] Et ponamus angulum EHR equalem angulo EHZ. Linea ergo HR secat lineam GF, et secat lineam EG; secet ergo lineam EG in R. Ergo due linee ZH, HR convertuntur per angulos equales, et erit L ymago R. Dico ergo quod forma cuiuslibet puncti linee GR convertitur ad visum Z ex puncto arcus FH, et non ex alio.

[7.57] Huius rei demonstratio est quoniam in capitulo de ymagine quinto tractatu, in duabus figuris viginti septem et viginti octo, dictum est quod duo arcus AB, DO non possunt esse tales quod ex illis convertetur aliquid de linea EO ad Z, et arcus BO non est de speculo. Non ergo remanet nisi arcus AD.

[7.58] Sed in tricesima quinta figura dictum est quod forma cuiuslibet puncti dyametri EO convertitur ad aliquod punctum arcus AD, et in tricesima sexta capitulo de ymagine patuit quod numquam convertitur forma puncti R ad Z ex arcu AD nisi uno solo puncto. Forma ergo cuiuslibet puncti linee GR convertitur ad Z ex uno solo puncto arcus AD.

[7.59] Et ponamus in linea GR punctum C. Forma ergo C convertitur ad Z ex uno puncto arcus AD. Dico ergo quod illud punctum

268 et[1] *om. E* / ZLH: ZLLY *P1; alter. in* ZLLY *mg. F* / quomodocumque; quocumque modo *ER* 269 ZG: ZH *FP1O; om. S* / FE: FS *L3* / ergo *rep. S* 270 et *corr. ex* Z *P1* / ZH *inter. a. m. C1;* ZG *FP1SL3E* / *post* ZH *inter.* G in arabico *O* / ZF[2]... quam[1] (271) *om. S* 271 GH[1] *corr. ex* HGH *F* / ZH: ZG *FP1SL3E* / GH[2]: DG *O* 272 linea ergo (273) *transp. R* 273 *post* equalia *add.* et *L3* / MG *corr. ex* M *P1* 274 secat *om. L3* / ergo *inter. a. m. E* / *post* EG *add.* secet ergo lineam EG in R ergo *R* / igitur *om. R* / est maior *transp. R* / EHZ: ZHE *R* / *post* EHZ *add.* et HZ secet AE in L *R* 275 et . . . R (277) *om. R* / EHR: EHI *S;* EAHR *O* / EHZ: EH *FP1;* EHG *S* 276 GF . . . lineam[2] *om. FP1* / GF: HQ *O* / EG: EH *FP1O* 277 ZH: ZB *FP1* / convertuntur per: reflectuntur propter *R* / et *inter. E* 278 convertitur: reflectitur *R* 279 *post* puncto *add.* aliquo *R* / FH: FGH *FP1; alter. ex* FDH *in* FDG *O* 280 de *inter. C1* 281 tractatu: tractu *O* / viginti[1]. . . octo *om. R* / viginti septem: ZA *FP1SL3E;* viginti sex *O* / et *om. SOL3C1E* / viginti octo: ZG *FP1SL3E;* viginti septem *O* / *post* viginti octo *add.* ZA ZG *FP1* / dictum est (282) *om. O* 282 non: item *FP1;* etiam *SL3E* 283 convertetur *inter. O;* reflectatur *R* / BO *inter. E;* EO *R* 284 AD *rep. S* 285 tricesima: trecesima *F;* triscesima *L3* 286 convertitur: reflectitur *R* / ad aliquod punctum: ab aliquo puncto *R* 287 et in tricesima *rep. L3E* / tricesima *corr. ex* tricesesima *F* / *post* tricesima *add.* et tricesima *SOC1* / sexta: quinta *O* / *post* quod *add.* punctus *FP1* / convertitur: convertetur *C1;* reflectitur *R* 288 R: HR *O;* linee GR *R* / *post* nisi *add.* ex *OER* (*inter. O*) / uno *om. ER* / forma[2]. . . puncto (289) *mg. a. m. E* 289 *post* puncti *scr. et del.* R ad Z ex arcu AD *C1* / linee GR *transp. FP1* / GR: HR *FP1O;* GX *L3* / convertitur: reflectitur *R* 291 et *inter. O* / GR: HR *FP1O;* GK *S* / forma . . . AD (292) *om. R* 292 *post* uno *add.* solo *P1* / illud *inter. O*

non erit nisi in arcu FH. Sin autem, convertatur forma C ad Z ex U, quod est in arcu AF, et continuemus lineas ZU, CU, GU, EU.

[7.60] Linea ergo GU erit maior linea GF, et ZU est minor quam ZF; ergo proportio GU ad ZU maior proportione GF ad FZ. Ergo est maior proportione GM ad MZ. Linea ergo que dividit angulum GUZ in duo equalia secat lineam ZM; secat ergo ZE. Angulus ergo GUE est minor angulo EUZ; ergo angulus CUE multo minor est angulo EUZ, et similiter de quolibet puncto arcus AU. Forma ergo C non convertitur ad Z nisi ex arcu HF.

[7.61] Et dico quod non potest converti ex arcu HD. Quod si fuerit possibile, convertatur ex Q, quod est in arcu HD, et continuemus lineas ZQ, CQ, RQ, ZR, EQ, et extrahamus EH ad N. Linea ergo ZQ est maior quam ZH, et linea QR est minor quam HR; ergo proportio ZQ ad QR est maior proportione ZH ad HR, que est sicut proportio ZN ad NR. Linea ergo que dividit angulum ZQR in duo equalia secat lineam NR; secat ergo lineam ER. Angulus ergo RQE est maior angulo EQZ; angulus ergo EQC est multo maior angulo EQZ. Hoc idem sequitur in omni puncto arcus HD; forma ergo C non convertitur ad Z ex arcu HD, neque ex arcu AF.

[7.62] Sed iam patuit quod omnino debet converti ex arcu AD. Forma ergo C non convertitur ad Z nisi ex aliquo puncto arcus FH. Convertatur ergo ex T, et continuemus lineas CT, ET, ZT. Quia ergo T est inter duo puncta F, H, erit linea ZT inter duas lineas ZF, ZH. Linea ergo ZT secat lineam KL. Secet ergo ipsam in I. I ergo est ymago C, et C nullam habet ymaginem nisi I.

[7.63] Et sic declarabitur quod ymago cuiuslibet puncti linee GR erit punctum linee KL. KL ergo est ymago GR, et KL est linea recta,

293 non erit *inter. a. m. E*/FH: IH *FP1*; GH *S*; BH *O*; KG *L3*; KH *E*/sin: sint *S*/autem: aut *FP1*/convertatur: reflectatur *R*/forma *inter. a. m. E* 294 *post* arcu *scr. et del.* ex quod in arcu *E*/lineas: linea *S*/ZU: ZTI *FP1*/CU: OU *S*; EU *R*/GU *om. SL3*/EU: CU *R*; *om. L3* 295 ergo *corr. ex* ego *L3*/linea[2] *om. R*/GF: AGF *FP1* 296 ZU *corr. ex* CZU *F*/*post* ZU *add.* est *SOL3C1ER*/FZ: ZF *L3*/est *om. L3ER* 297 ergo *om. S*/GUZ: GZ ZU *FP1*; GZ UZ *L3E* 298 in duo: per *R*/equalia: equa *C1*/secat[2] *mg. C1*/ergo[2]: GLO *S*; GO *L3* 299 EUZ *corr. ex* EUG *O*/CUE: QUE *FP1*; UE *L3E* 300 *post* puncto *scr. et del.* ? *P1*/arcus: arcu *L3*/convertitur: reflectitur *R* 1 HF: AF *O* 2 et: item *FP1*/non *om. O*/potest: poterit *P1*/converti: reflecti *R*/HD: GD *O*/si *om. S* 3 convertatur: reflectatur *R*/HD *alter. in* GD *O* 4 RQ: IQ *L3*/ZR *om. E*/ZR EQ *transp. R*/EH: ZH *FP1*; EG *O* 5 maior: minor *E* 6 ZQ *inter. C1*/proportione: proportionum *S*/HR: GR *O*/est[2] *inter. a. m. E* 7 ZN: ZM *FP1SL3E* (*alter. in a. m. E*)/ergo . . . lineam[1] (8) *mg. a. m. E* 8 NR . . . lineam[2] *om. L3*/ergo[1] *om. E*/RQE: IQE *FP1*; CQE *L3E* 9 angulus . . . EQZ[2] *om. P1*/ergo . . . idem *mg. a. m. E*/EQC: CQE *R*/est *inter. O*/est multo *transp. O* 10 convertitur: reflectitur *R* 11 ex[1]. . . neque *om. L3E*/neque: nec *C1*/AF *corr. ex* HF *C1* 12 converti: reflecti *R* 13 convertitur: convertur *S*; reflectitur *R*/*post* Z *scr. et del.* ex arcu HD *SC1* 14 convertatur: convertitur *L3E*; reflectatur *R*/CT: ED *FP1*; *alter. in* ET *E*/ET: et *L3ER* 15 inter[2]. . . ZF *om. P1* 16 KL *om. L3*/ergo *om. L3*/*post* ergo *add.* lineam *ER*/I[1]: R *L3*/I[2] *inter. O*/C: T *R* 17 C: T *R*/habet ymaginem *transp. C1* 18 sic *corr. ex* si *O*/GR: GI *S*/GR . . . KL[1] (19) *om. P1* 19 erit: est *ER*

quia est pars dyametri circuli. Et GR est linea recta, quia est etiam pars dyametri circuli. Z ergo comprehendit formam GR recte in speculo AB sperico, et hoc est quod voluimus.

[7.64] **[PROPOSITIO 30]** Et iteremus formam [FIGURE 6.7.30, p. 320], et revolvamus super lineam GR a duobus lateribus duos arcus, quomodocumque sit, scilicet GNR, GQR, et sit arcus GNR non secans lineam GH. Et ponamus in linea GR punctum M, quomodocumque sit. Forma ergo M convertitur ad Z ex puncto arcus FH. Convertatur ergo ex T, et continuemus lineas ZT, MT.

[7.65] Duo ergo anguli ZTE, ETM sunt equales; linea ergo MT secabit arcum GNR. Secet ergo ipsum in N, et extrahamus lineam TM in parte M. Secabit ergo arcum GQR; secet ergo in puncto Q. Et continuemus NE, et extrahatur recte. Secabit ergo ZT sub lineam KL. Secet ergo illam in I. Et continuemus QE et extrahamus ipsam recte. Secabit ergo ZT super KL. Secet ergo ipsam in C.

[7.66] Quia ergo duo anguli T sunt equales, erit I ymago N, et duo puncta K, L sunt ymagines duorum punctorum G, R. Ymago ergo arcus GNR est linea transiens per puncta K, I, L, ut linea KIL. Sed linea KIL est convexa ex parte visus, et arcus GNR est convexus ex parte speculi. Z ergo comprehendet formam linee GNR convexe lineam convexam.

[7.67] Et quia duo anguli T sunt equales, erit C etiam ymago Q, et erit linea LCK ex parte visus concava ymago arcus GQR concavi ex parte superficiei speculi. Z ergo comprehendet formam arcus GQR concavi lineam concavam.

[7.68] In speculis ergo concavis ex quibusdam sitibus comprehenditur linea convexa convexa, et concava concava.

20 quia[1]: que *E* / dyametri: semidyametri *R* / *post* circuli *add.* AE *R* / et . . . circuli (21) *mg. a. m. E; rep. P1; rep. et del. F* / est etiam *transp. C1* / etiam *om. FP1R* 21 dyametri: semidyametri *R* / *post* circuli *add.* OE *R* / Z *om. R* / ergo: G *FP1* / comprehendit: comprehendet *C1* / formam *om. FP1* / *post* GR *add.* est *L3* 22 AB sperico *transp. R* / est *mg. F; om. C1* / voluimus: volumus *E* 23 et: item *FP1* / formam: figuram *R* / *post* formam *scr. et del.* GR *F* 24 revolvamus: constituamus *R* 25 quomodocumque: quocumque *F* / sit[1] *inter. O;* sint *R* / non *inter. E* 26 GH: GPH *FP1;* GDH *O* 27 convertitur: reflectitur *R* / puncto: punctus *FP1* / convertatur: reflectatur *R* 28 T *corr. ex* TE *S* / *post* ZT *add.* et *OR* / MT: ME *FP1* 29 ergo anguli *transp. deinde corr. S* / *post* ZTE *scr. et del.* EC TM *F* 30 GNR: HNR *FP1O* / ipsum *om. R* 31 arcum *om. R* / GQR *corr. ex* GNR *a. m. E* / secet *om. S* / Q: F *O* / *ante* et *add.* de *O* 32 NE: FE *FP1* / sub: super *FP1SOL3E;* supra *C1* / KL *corr. ex* GL *C1* / *post* KL *add.* secabit ergo ZT sub KL secet ergo ipsam in T *FP1* 33 *post* ergo *scr. et del.* in puncto Q *S* / in *mg. F* 34 secabit . . . C *om. FP1* / super: sub *SOL3C1E;* supra *R* / C: P *R* 35 *post* anguli *add.* ad *R* / sunt: inter *L3* / N: H *S;* Z *O* 36 sunt *om. R* 37 ut linea KIL *om. L3; mg. a. m. E* 38 *post* visus *add.* Z *R* / GNR *corr. ex* GNT *a. m. E* / est[2] *rep. P1* / convexus: convexa *O* / ex: a *E* 39 Z ergo: ergo visus Z *R* / GNR: GM *L3* 41 *post* anguli *add.* apud *R* / T: R *S* / erit *inter. O* / C: P *R; om. S* 42 LCK: LIK *L3E;* LPK *R* / *post* LCK *inter.* concava *O* / concava *om. O* / *post* concava *add.* et est *R* / concavi: concava *O* 43 Z ergo: ergo visus Z *R* / GQR: GR *FP1* 44 concavam: concavi *S* 45 in . . . ergo: ergo in speculis *F* / in . . . concavis *om. P1* / quibusdam: quibus *FP1* 46 convexa[2] *om. FP1SL3* / concava[2] *om. P1;* concavam *F*

[7.69] **[PROPOSITIO 31]** Item sit speculum concavum in quo sit circulus ABD maximus [FIGURE 6.7.31, p. 321], et centrum G, et extrahamus lineam BG, quomodocumque sit, et dividamus ex ipsa lineam GT maiorem medietate. Et extrahamus ex T lineam ETZ perpendiculariter, et sit utraque ET, TZ equalis TG. Et continuemus ET, EG, GZ.

[7.70] Et describamus circa triangulum EGZ circulum. Secabit ergo circulum AB in duo puncta, nam punctus T est centrum huius circuli, et TG est maior TB. Secet ergo iste circulus circulum AB in duobus punctis A, D, et continuemus lineas GA, GD, EA, EB, ED, ZA, ZB, ZD.

[7.71] Quia ergo due linee ET, TZ sunt equales, erunt due linee EB, BZ converse per angulos equales. Et quia duo arcus EG, GZ sunt equales, due linee EA, AZ convertuntur per angulos equales, et due linee ED, DZ convertentur per angulos equales.

[7.72] Et quia GT est maior quam TB, erit GE maior quam EB. Angulus ergo EBG est maior angulo EGB, et angulus EGB est semirectus. Ergo duo anguli EGB, EBG simul sunt maiores recto. Ergo angulus BEG est recto minor, et angulus EGZ est rectus. Ergo due linee EB, GZ concurrent extra circulum in parte BZ. Concurrant ergo in M.

[7.73] Et quia ED est intra angulum MEG, concurret cum linea GM. Concurrant ergo in L. Et quia GB transit per centrum EGZ circuli, erit portio AEG minor semicirculo. Ergo angulus AEG est obtusus, et angulus EGZ est rectus. Ergo ille due linee AE, ZG concurrent in parte EG. Concurrant ergo in F. Si ergo visus fuerit in E, et Z in aliquo visibili, tunc puncta M, L, F erunt ymagines Z. Sic ergo Z comprehenditur in tribus locis.

49 *post* BG *scr. et del.* quocumque sit *S*/quomodocumque: quandocumque *FP1L3E*/ex ipsa lineam (50): lineam ex ipsa *C1* 50 ETZ: ET *C1*/perpendiculariter: perpendicularem *L3E*; perpendicularem super BG *R* 51 utraque ET TZ *mg. O*/*post* utraque *add.* linea *O*/ET^{2} *om. R* 52 EG *corr. ex* TG *a. m. E* 54 AB: ABD *R*/duo puncta: duobus punctis *R*/punctus: punctum *R*/est: etiam *FP1*; et *SOL3E* 55 et: Z *L3E*/TG *om. FP1*; *corr. ex* TH *O*/iste circulus *transp. R*/circulum *inter. E*/AB: ABD *R* 56 duobus *om. R* 58 *post* ergo *add.* iste *L3*/*post* linee^{2} *add.* EG GZ equales et similiter *R* 59 EB BZ: EA AZ *S*/*post* BZ *add.* etiam *E*/converse: convertuntur *S*/converse per angulos *om. R*/angulos *om. S*/et . . . equales^{2} (60) *om. S*/EG GZ: HE HZ *FP1O* (HE: EH *O*) 60 due^{1}. . . equales^{2} *om. FP1*/EA *corr. ex* A *E*/convertuntur per: reflectentur inter se propter *R*/et . . . equales (61) *om. L3*; *mg. a. m. E* 61 ED DZ: EB BZ *R*/convertuntur per: reflectentur inter se propter *R* 62 maior^{1}. . . GE *rep. S*/quam^{1} *om. C1*/GE: BGE *FP1*; *corr. ex* HE *SO* 63 *ante* angulus^{1} *scr. et del.* ergo *E*/EBG: OBH *P1*; EBH *O*; *alter. ex* H *in* OBH *F*/EGB^{1}: EBH *FP1*; EHB *SO*/EGB et angulus *mg. C1*/EGB^{2}: EHB *FP1SO*; EBG *L3C1E* 64 anguli *om. O*/EGB: EHB *FP1SO*/EBG: EBH *FP1O*/*post* EBG *add.* sumpti *C1* 65 BEG: HEB *FP1S*; BEH *inter. O*/EGZ: EHZ *FP1SO* 67 M: L *R* 68 intra: inter *FP1*/angulum: triangulum *R*/MEG: MEH *FP1SO*; LEG *R* 69 GM: HM *FP1O*/concurrant: concurrat *R*/L: M *R*/GB: GF *O*/EGZ: ZEG *R*; *alter. in* TEG *a. m. E* 70 AEG^{1}: AG *ER* 72 ergo^{1} *om. L3* 73 puncta *corr. ex* punctum *P1*/*post* puncta *scr. et del.* ? *F*/M L *transp. C1*/Z^{2}: et *FP1*/comprehenditur: comprehendetur *R*

[7.74] Item extrahamus ex E lineam ad arcum DZ, quomodocumque sit, et sit EK. Et continuemus GK, et secet arcum DZ in K, et continuemus lineas KZ, GK. Quia ergo arcus EG, GZ sunt equales, erunt duo anguli EKG, GKZ equales. Ergo angulus EKG est maior angulo GKZ. Sit ergo angulus GKN equalis angulo EKG. Due ergo linee EK, KN convertentur per angulos equales. Et extrahamus EK ad Q. Erit ergo Q ymago N respectu E.

[7.75] Et ymaginemur superficiem exeuntem a linea MGF perpendiculariter super circulum ABD, et extrahamus ex Z lineam in hac superficie perpendicularem super GZ, et transeat in utramque partem. Sit ergo CZR. Et ponamus G centrum, et in longitudine GN faciamus arcum circuli CNR. Secabit ergo lineam CR in duobus punctis, et sint C, R. Et continuemus lineas GC, GR. Erunt ergo in superficie perpendiculari super superficiem ABG. Et extrahamus GC, GR recte, et super G, et in longitudine GQ faciamus arcum circuli. Secabit ergo duas lineas GC, GR. Secet in S, O.

[7.76] Quia ergo superficies circuli ABD est perpendicularis super superficiem duarum linearum GC, GR erunt duo anguli EGS, EGO recti. Erit ergo utraque superficies EGS, EGO perpendicularis super superficiem SGO, et utraque istarum superficierum facit in speculo circulum magnum comparem circulo ABD. Punctum ergo compar puncto K circuli quem facit superficies EGC, convertuntur ex ipso secundum angulos equales due linee inter duo puncta E, C.

[7.77] Et linee GC, GR sunt equales, et linee GS, GQ, GO sunt equales, et Q est ymago N, et S est ymago C, et O est ymago R. Yma-

75 ex E *om. L3* / ad *inter. E* / DZ: DB *O* / *post* DZ *scr. et del.* et M B *C1* 76 et^{2} *inter. C1* / GK: GB *C1* / et^{3} *om. S* / K: H *FP1*; U *O*; B *L3E* 77 lineas: lineam *R* / KZ GK: KZIAEU *O* / GK *om. R* 78 EKG^{1}: EUG *O* / GKZ: GZ *FP1*; GUZ *O*; GBZ *L3*; *alter. ex* GZ *in* GBZ *E* / *post* equales *add.* sic ergo duo anguli EUK ZUK erunt equales sed linea EU est maior quam UZ *O*; *add.* et continuemus GK in R et extrahamus ER ZR *R* / ergo *corr. ex* angulo *F*; *om. P1* / EKG^{2}: ERG *R* / maior: minor *S* 79 GKZ: EKZ *FP1SOL3C1E*; GRZ *R* / sit ergo *transp. deinde corr. C1* / GKN: GKH *S*; GRN *R* / EKG: ERG *R* 80 *post* linee *scr. et del.* ergo *S* / EK KN: ER RN *R* / convertentur per: reflectentur inter se propter *R* / EK^{2}: ER *R* 81 Q ymago *transp. C1* 82 et *om. L3* / ymaginemur: ymaginemus *L3*; ymagine in *S* / superficiem: superficientem *F* / *post* superficiem *scr. et del.* existentem *C1* / perpendiculariter: perpendicularis *L3* 83 *post* super *add.* centrum *L3* / in hac: MH AC *S* 84 utramque *corr. ex* unamque *a. m. E* 85 CZR: CZP *R* 86 CNR: CZR *FP1SOL3E*; *corr. ex* CZR *C1*; CNP *R* / CR *inter. a. m. E*; CZP *R* / sint: situm *S* 87 R: P *R* / GR: GP *R* 88 ABG: ABD *R* / GR: GP *R* 89 G *corr. ex* GI *E* / *post* arcum *add.* cum *C1* 90 GR: GP *R* / secet *om. O* 91 circuli ABD *transp. R* / ABD: ABDG *FSL3C1E*; ABAG *P1*; ABG *O* 92 GR: GP *R* 93 recti *om. P1* / *post* recti *add.* et EG perpendicularis super superficiem GCP *R* / super *om. FP1* 94 superficiem: superficie *FP1* / *post* superficiem *scr. et del.* duarum linearum GT GR erunt duo anguli *S* / SGO *corr. ex* EGO *P1* / *post* istarum *add.* duarum *R* / facit: faciet *C1* 95 ABD: ABO *FP1*; ABGH *O* / compar . . . circuli(96): circuli compar puncto R est *R* 96 circuli: est *E* / quem: quoniam *FP1L3*; quod *R* / *post* superficies *scr. et del.* G *L3* / EGC: EGTO *FP1*; EGO *L3*; EGS *R* / *post* EGC *add.* ergo *R* / convertuntur: concurrunt *FP1SR*; *alter. ex* concurruntur *in* concurrunt *C1* 97 linee *rep. P1* / *post* C *add.* et similiter inter duo puncta D P *R* 98 *post* GC *add.* GN *O* / GR: GP *R* / GS: GC *deinde inter.* S in arabico *O* 99 N *inter. O* / est^{2,3} *om. L3ER* / C . . . ymago^{3} *om. S* / R: P *R* / R ymago *mg. C1*

go ergo arcus CNR convexi ex parte speculi est arcus SQO concavus ex parte visus.

[7.78] Et L est ymago Z, et duo puncta S, O sunt ymagines C, R. Ymago ergo linee CZR recte est linea transiens per puncta S, L, O, et talis linea est concava ex parte visus.

[7.79] Et signemus lineam transeuntem per puncta S, L, O, et extrahamus lineam EG ad H. Si ergo speculum non pervenit ad duo puncta B, H, sed alter duorum terminorum suorum fuerit inter duo puncta B, D, et reliquus fuerit infra H, et visus fuerit in E, et due linee RZC, RNC fuerint in aliquo visibili, tunc forma linee RZC recte erit concava, scilicet SLO, et forma arcus RNC convexi erit etiam linea concava, scilicet SQO. Et RZC recta habebit unam ymaginem, et arcus RNC habebit unam ymaginem.

[7.80] Item extrahamus BG ad I, et continuemus lineas EI, IZ. Iste ergo due linee convertentur secundum angulos equales, et EI secabit FG; secet ergo in T. T ergo erit ymago Z. Puncta ergo M, L, T, F sunt ymagines Z. Et si speculum excesserit duo puncta A, I, et visus fuerit in E, et deorsum aspicientis in speculo fuerit ex parte arcus AI, comprehenderit totum arcum IDA.

[7.81] Tunc Z videbitur in quattuor locis, scilicet in L, M, T, F, et videbit duo puncta R, C in duobus punctis S, O, et sic linea RZC habebit quattuor ymagines concavas. Una transit per puncta S, M, O, scilicet linea SMO; secunda pertransibit per puncta S, L, O, scilicet linea SLO; tertia transibit per puncta S, T, O, scilicet linea STO; quarta transibit per puncta S, F, O, scilicet linea SFO.

100 CNR: GT ZR *FP1*; CZR *OL3E*; CNP *R* / SQO: CQO *O* / concavus *corr. ex* concavis *P1* 102 R: P *R* 103 *post* linee *inter.* recte *mg. a. m. E* / CZR: CZP *R*; *corr. ex* EZR *a. m. E* / recte *om. ER* / L: I *FP1*; B *L3* 104 linea *om. L3R* 105 L: I *FP1* 106 H: B *S* 107 B H *transp. O* / duorum *om. R* / terminorum suorum *transp. R* / suorum *inter. a. m. E* 108 D: H *FP1R* / reliquus: reliquis *FP1* / et[2] *inter. O* 109 RZC[1]: IZC *FP1O*; *corr. ex* IZC *E*; PZC *R* / RNC: INC *L3E*; PNC *R* / forma: formam *FP1* / RZC[2]: IZE *L3E*; PZC *R* 110 scilicet *inter. C1* / SLO: SO *FP1* / RNC: ROC *FP1*; *corr. ex* RNO *E*; PNC *R* / erit: erunt *L3* / etiam: et *F*; *om. P1* / linea: linee *L3* 111 RZC: PZC *R* / et[2]... ymaginem (112) *mg. F*; *om. S* 112 RNC: RC *L3E*; PNC *R* 113 I: L *S* / et *om. C1* / IZ: EZ *FP1SOL3E* / iste ... EI (114) *mg. a. m. E* 114 ergo: autem *O* / convertentur: reflectuntur *R* 115 FG: ZG *SO* / T[1,2,3]: U *R* / T[2] *om. O* / ergo[3] *mg. C1* 116 et[1] *om. S* / excesserit: excessit *SE* / A I: AZ AG *FP1SL3C1E*; AL *O* / I: D *R* 117 E *alter. in* I *O* / deorsum: dorsum *L3C1ER* / in speculo *om. R* / ex parte *inter. E* / AI: ABG *FP1SL3C1E*; *corr. ex* AB *O* / *post* AI *add.* et *C1R* 119 in[2] *om. R* / T: U *R* / et... RZC (120) *rep. L3* 120 *post* puncta *add.* scilicet puncta [*inter.* scilicet] *E* / R: I *L3*; P *R* / sic: si *O* / *post* linea *add.* recta *R* / RZC: ZR ZT *FP1SL3C1E*; PZC *R* 121 *post* concavas *add.* et *C1* / transit: transibit *ER* / *post* O *add.* SCL *SO*; *add.* SO *L3E*; *scr. et del.* SCL scilicet per puncta MSO *C1* 122 scilicet ... SMO *om. C1* / post scilicet[1] *add.* per puncta vel per *FP1*; *add.* per puncta *SL3E* / linea: lineam *FP1*; *om. SL3E* / *post* SMO *add.* secunda pertransibit per puncta SMO *S* / pertransibit: transibit *OER* / S: F *FE* / S L O *om. P1* 123 SLO: SFO *FP1* / tertia ... transibit (124) *mg. a. m. E* / tertia ... SFO (124) *om. FP1* / transibit: pertransibit *L3* / *post* puncta *add.* scilicet *L3E* / S: F *S* / scilicet ... O *om. E* / per puncta: linea *O* / T: U *R* / STO: FTO *O*; SUO *R* 124 linea *om. S*

[7.82] Patet ergo ex hac figura quod linea recta in speculis concavis comprehenditur concava, et convexa comprehenditur concava, et quod recta habet plures formas concavas.

[7.83] **[PROPOSITIO 32]** Item sit speculum concavum [FIGURE 6.7.32, p. 322] per cuius centrum transeat superficies, et faciat circulum ABG, et sit centrum D. Et extrahamus ex D lineam, quomodocumque sit, et sit DG, et transeat extra circulum. Et extrahamus ex D in superficie huius circuli lineam perpendicularem super lineam DG, et sit DA. Et abscindamus de angulo ADG recto particulam parvam, quomodocumque sit, et sit angulus GDE, ita quod inter angulum rectum et angulum ADE sit multiplum anguli EDG, et dividamus angulum ADE in duo equalia per lineam DB. Et abscindamus distinctionem equalem angulo EDG, et extrahamus ex D lineam continentem cum DB angulum rectum, et sit DT.

[7.84] Et extrahamus AD in parte D, et sit DK, et extrahamus ex Z lineam continentem cum ZD angulum equalem angulo KDT. Hec ergo linea concurret cum DA, nam duo anguli KDT, ADZ sunt minores duobus rectis. Concurrant ergo in H. Angulus ergo ZHD est equalis angulo ZDT.

[7.85] Et extrahamus ex Z lineam continentem cum ZH angulum equalem angulo BDK obtuso, et sit ZL. Duo ergo anguli LZD, BDZ sunt minores duobus rectis; linea ergo ZL concurret cum DB. Concurrat ergo in L.

[7.86] Et continuemus LH, et circa triangulum HLD faciamus circulum DHL. Transibit ergo per Z, quia duo anguli LZH, LDH sunt equales duobus rectis. Anguli ergo LHZ, LDZ sunt equales, quia basis eorum est idem arcus. Sed angulus ZHD est equalis angulo ZDT; remanet ergo angulus LHD equalis angulo LDT. Et angulus LDT est rectus; ergo angulus LHD est rectus.

126 comprehenditur[1,2]: comprehendatur *R* / et[1]. . . concava[2] *om. S; mg. a. m. E* 127 formas concavas *transp. C1* 128 sit: scit *S* 129 et *inter. a. m. E* 130 lineam . . . D (131) *mg. F* / quomodocumque: quocumque modo *C1R* 131 et[1] *inter. P1* 132 huius *om. S* / super lineam *om. L3* 133 DA: EA *L3* 134 quomodocumque: quocumque *L3* / sit[1]: sint *L3; corr. ex* sint *C1* / quod: ut *R* / inter: intra *E* / *post* angulum *scr. et del.* ai *P1* 135 sit: et *S* / multiplum: multi plurii *FP1;* multiplicatio *S* / anguli *om. S* 136 distinctionem: de angulo BDA *R* 137 EDG: EDB *L3E* / continentem *corr. ex* contineam *mg. F;* contingentem *SL3; corr. ex* concurrentem *C1* 138 DB: BD *FP1SOL3ER* / DT: DD *FP1;* DX *R* 139 D: TB *E* 140 continentem: contingentem *L3* / KDT: KDX *R* / *post* KDT *add.* et sit ZH *R* 141 concurret: concurrit *FP1L3* / KDT: KDX *R* 142 ZHD: ZDH *S;* DZH *C1* / ZDT: KDT *FP1SL3C1E* 143 ZDT: ZDX *R* 144 continentem: contingentem *SL3* / *post* ZH *inter.* G in arabico *O* 145 ergo *om. FP1L3* / anguli: guli *F* / BDZ: OBZ *FP1;* DBZ *SL3E; om. O* 146 duobus rectis *transp. C1* / ZL *corr. ex* LZL *F* / concurret: continet *S* / concurrat: concurrant *OL3C1ER* 148 circa: citra *E* / triangulum: circulum *SOL3E* / HLD: GD *O* 149 DHL: GHL *FP1;* DGL *O* / LZH: LZG *O* / LDH: LDG *O* 150 ergo *om. L3* / LHZ: LGZ *O* 151 ZHD: ZGD *O* / est equalis *transp. FP1* / ZDT: ZDX *R* 152 LHD: LGD *O* / LDT[1,2]: LDX *R* 153 LHD: LGD *O;* LDH *S*

[7.87] Et abscindamus ex linea DE lineam DM equalem DH, et continuemus LM. Angulus ergo LMD est rectus; circulus ergo LHD transit per M et secat arcum HE in puncto compari Z. Secet ergo in F, et continuemus DF. Angulus ergo LDF erit equalis angulo LDZ, quia arcus LM est equalis arcui LH, et arcus MF est equalis arcui ZH. Ergo arcus FMD est equalis arcui ZHD.

[7.88] Et continuemus lineas HB, HF, FM, FZ, FB. Angulus ergo BHD erit acutus, et angulus GDH erit rectus. Ergo linea HB concurret cum linea DG extra circulum. Concurrant ergo in Q. HF ergo concurret etiam cum DG extra circulum; concurrant ergo in N.

[7.89] Et extrahamus FB quousque secet arcum LZ. Secet ergo in R, et continuemus RM. Angulus ergo FRM, qui est in circumferentia, respicit arcum FM, et angulus FBM est maior angulo FRM, et angulus FBM est in circumferentia ABG. Ergo si BM linea extrahatur in parte M, abscindet de circulo ABG arcum maiorem simili arcus FM.

[7.90] Et arcus FM est duplus similis arcus FE. Et arcus FE est equalis arcui ZA, et arcus ZA est equalis arcui EG, et arcus FE est equalis arcui EG. Ergo arcus GF est duplum arcus GE; ergo arcus GF est similis arcui FM.

[7.91] Si ergo BM extrahatur recte in partem M, abscindet de circulo ABG arcum ultra punctum F maiorem arcu FG. Linea ergo BM secabit lineam DG inter duo puncta G, D. Secet ergo in O. Et extrahamus lineam FM, et secet DO in U; et extrahamus BM in parte B, et secet arcum LR in C. Et continuemus CD.

[7.92] Quia ergo angulus BFZ est in circumferentia ABG, erit angulus BFZ dimidium anguli BDZ. Sed angulus BDZ est multiplus anguli ZDA; ergo angulus RFZ est multiplus anguli ZDH. Ergo arcus

154 abscindamus *corr. ex* abscimdamus *S* / DH: DHG *FP1*; DG *O* 155 LM: LHD *FP1*; LH *SL3E*; LG *O* / circulus ergo *transp. L3* / LHD: LGD *O* 156 per *rep. C1* / et *mg. a. m. E* / secat *corr. ex* sedcat *F* / HE: BE *R* / puncto compari *transp. R* 157 LDF *corr. ex* LDFD *P1* / LDZ: ADZ *FP1* 158 LH: LB *S*; LG *O* / ZH: ZG *O* 159 FMD: LF *R* / ZHD: ZGD *O*; LZ *R* 160 HB: GB *O* / HF: GF *O* / *post* FM *add.* BM *R* 161 BHD: BGD *O*; *corr. ex* HBD *E* / erit^{1}: est *R* / GDH: DGH *L3E* / erit^{2} *om. R* / HB: GB *O* 162 DG: DH *L3* / concurrant . . . circulum (163) *rep. S* / HF: GF *O* 163 concurrant: concurrunt *FP1*; *corr. ex* concurrent *C1* / N: B *FP1* 164 et *inter. C1* / FB: FH *C1*; *corr. ex* MFB *F* 165 R *corr. ex* K *a. m. E* / RM: NAR *L3* / ergo *mg. C1* 166 FBM: FOM *FP1* 167 FBM: FOM *FP1* / in^{1} *inter. a. m. E* / in^{2}: ex *R* 168 abscindet: abscondet *FP1* / *post* abscindet *scr. et del.* in L *F* / de: TE *L3* / simili arcus *rep. et del. F* / arcus: arcui *R* / *post* FM *add.* circuli FHD *R* 169 duplus similis: similis duplo *R* / FE^{1}: FR *L3* / et arcus FE *om. S* 170 ZA^{1}: AZ *L3R*; *alter. ex* AH *in* AZ *E* / ZA^{2}: AZ *R* / arcui^{2} *om. P1* / EG: ZEG *FP1L3*; *corr. ex* ZEG *E* / et^{2}: ergo *R* 171 *post* arcui *add.* FG *FP1C1E* / ergo^{1}. . . GE *rep. S* / arcus^{1} *inter. a. m. E* / duplum: duplus *R* 173 *post* ergo *add.* arcus *FP1* / de *om. FP1* 174 *post* ABG *scr. et del.* maiorem simili arcus FM et arcus FM est duplus similis arcus FE et arcus FE est equalis arcui EG ergo arcus GF est duplum arcus G ergo arcus GF est similis arcui FM si ergo BM extrahatur recte in partem M [EG *corr. ex* EGD] *S* / F: G *R*; *om. S* 175 secabit *corr. ex* secat *E* / duo *inter. E* / G D *transp. C1* / in O *rep. et del. F* / extrahamus *corr. ex* extramus *F* 176 et^{1} *inter. C1* / B *om. P1* 177 LR: LU *L3* 178 est *om. FP1*; *inter. O* 179 BFZ: BFGZ *P1*; *corr. ex* BFGZ *F* / dimidium: dimidius *R*; *corr. ex* dimium *F* 180 ZDA . . . anguli^{2} *om. S* / angulus *om. L3* / RFZ: BFZ *R* / ZDH: ZDG *O*

RZ est multiplus arcus ZH, et arcus CZ est maior arcu RZ; ergo arcus CZ est multiplus arcus ZH.

[7.93] Et continuemus CH. Angulus ergo CHD cum angulo CMD sunt equales duobus rectis; ergo angulus CHD est equalis angulo BME. Ergo angulus ZHD addit super angulum CHD angulum CHZ, qui est equalis angulo CDZ, et angulus CDZ est multiplus anguli ZDA. Ergo angulus CHZ est multiplus anguli EDG; ergo angulus ZHD excedit angulum CHD multiplo anguli EDG. Angulus ergo ZHD est equalis angulo FMD, quia arcus FMD est equalis arcui ZHD.

[7.94] Et angulus CHD, ut declaravimus, est equalis angulo BME. Ergo angulus FMD excedit angulum BME multiplo anguli EDG. Ergo angulus FMD excedit angulum OMD multiplo anguli EDG. Et MOG angulus excedit angulum OMD angulo EGD; ergo angulus FMD excedit angulum MOG multiplo anguli EDG.

[7.95] Et angulus FMD excedit angulum MUD angulo EDG solo. Ergo angulus MUD est maior angulo MOG; ergo angulus MOU est maior angulo MUO. Ergo linea MU est maior linea MO. Et quia arcus ZHD est equalis arcui FMD, erunt duo anguli HFD, MFD equales. Due ergo linee HF, FU convertentur equaliter, et similiter HB, BO convertentur equaliter. Q ergo est ymago O, et N est ymago U.

[7.96] Et extrahamus ex M lineam equidistantem linee HQ, et sit MS, et extrahamus ex M etiam lineam equidistantem linee HN, et sit MP. Quia ergo angulus HND est maior angulo HQD, erit angulus MPO maior angulo MSO. P ergo erit inter duo puncta S, U, et quia

181 RZ[1]: RZA *O*/*post* est[1] *scr. et del.* pl *F*/ZH *corr. ex* RZH *F*; ZG *O*/*post* et *scr. et del.* continuemus FH *C1*/RZ[2]: RCZ *L3*; *corr. ex* LZ *E* 182 *post* est *scr. et del.* maior arcu *S*/ZH: ZG *O* 183 CH: CG *O*; LH *L3*; SH *E*/CHD: CDG *O*/angulo: triangulo *L3E* 184 sunt *om. L3E*/sunt equales: est equalis *R*/CHD: CDH *FP1SC1*; CDG *O*/est *om. O*/angulo BME (185) *transp. deinde corr. S* 185 ergo: sed *R*/ZHD: ZDG *O*; *corr. ex* CHD *a. m. E*/super *inter. a. m. E*/CHD: CDG *O* 186 CHZ: CGZ *O*/est[2] *om. FP1* 187 ZDA *corr. ex* CDA *a. m. C1*/ergo[1]. . . EDG *rep. et del. E*/CHZ: CGZ *O*/EDG *corr. ex* ZDG *E* 188 ZHD: ZGD *O*/ZHD . . . ergo (189) *om. L3*/excedit: extendit *FP1O*/CHD: CGD *O*/EDG: EGD *C1* 189 ZHD: ZGD *O*; *corr. ex* ZDH *C1* 190 ZHD: ZGD *O* 191 et . . . CHD *mg. F*/CHD: CGD *O*/ut: aut *L3*; *corr. ex* aut *a. m. E*/declaravimus *corr. ex* declaramuus *P1*; *corr. ex* declarabimus *a. m. E* 192 FMD: FMB *P1*; *alter. ex* FBMB *in* FMB *F*/*post* FMD *scr. et del.* est equalis arcui ZHD *C1*/BME: FME *O*/multiplo: multiplus *L3* 193 OMD: CMD *S*/EDG: ODG *L3*; *corr. ex* edguli *F* 194 MOG: GOM *O*/MOG angulus *transp. OR*/EGD: EDG *L3R*; GDE *C1* 195 MOG: GOM *O*/MOG . . . angulum (196) *mg. a. m. E* 196 angulo: angulus *L3*/EDG: EBG *S*; *corr. ex* AEDG *F*; *corr. ex* ADG *P1*/solo: solum *FP1* 197 MOG . . . angulo (198) *mg. a. m. E* 199 ZHD: GZHD *FP1S*; GZD *O*/est *om. O*/FMD: BMD *O*/HFD: GHFD *FP1*; GLD *O*/MFD: MBD *O* 200 due ergo *transp. L3*/HF FU: GB BO *O*/convertentur[1, 2]: reflectentur *R*/et . . . equaliter (201) *om. S*/HB: GB *O*/BO: BA *FP1*; BQ *O*; BC *L3E*/convertentur[2]: convertuntur *E* 201 O: C *S*/O . . . ymago *mg. S*/N: non *SOL3C1E*/est[2] *om. R* 202 *post* lineam *add.* et similiter HB BO convertentur equaliter *S*/HQ: GQ *O*; HU *L3*; *corr. ex* LQ *P1*/et[2]. . . HN (203) *om. SL3* 203 MS: MF *FP1*/etiam: et *FP1*; *om. C1*/HN: GN *O* 204 MP: MN *O*/HND: GND *O*/HQD: GQD *O* 205 MPO: MDU *O*; CQPO *L3E*/MSO: SMO *E*/S: F *L3*

angulus HDN est rectus, erit angulus HND acutus. Ergo angulus MPD est acutus; ergo angulus MPS est obtusus. Ergo linea MS est maior quam MP.

[7.97] Sed MU est maior quam MO, ut diximus; ergo proportio SM ad MO est maior quam proportio PM ad MU, et proportio SM ad MO est sicut proportio QB ad BO, quia MS est equidistans BQ. Et similiter proportio PM ad MU est sicut proportio NF ad FU; ergo proportio QB ad BO est maior quam proportio NF ad FU. Et proportio QB ad BO est sicut proportio QD ad DO, et proportio NF ad FU est sicut proportio ND ad DU, ut declaravimus in vicesima quinta figura capitulo de ymagine. Ergo proportio QD ad DO est maior quam proportio ND ad DU.

[7.98] Hiis preostensis, iteremus circulum, et perficiamus demonstrationem, ne multiplicentur linee et dubitentur littere. Sit ergo circulus in secunda forma ABG [FIGURE 6.7.32a, p. 323], et centrum D, et extrahamus lineam DQ. Et sit DU equalis DU in prima forma, et DO equalis DO in prima forma. Et DQ sit compar sibi in prima forma, et similiter DN.

[7.99] Et extrahamus super DQ DH perpendicularem super superficiem circuli, et sit DH equalis sibi in prima forma. Angulus ergo HDQ erit rectus, et circulus quem facit HDQ in speculo erit ex circulis ex quibus forma convertitur. Et erit arcus quem mensurant linee HD, DQ equalis arcui AG in primo circulo. Et ex duobus punctis istius comparibus duobus punctis B, F convertentur linee ad duo puncta U, O equaliter. Erit ergo Q ymago O, et N ymago U.

[7.100] Et extrahamus ex U perpendicularem lineam in superficie circuli ABG super lineam DU, et sit ZUE. Et sit D centrum, et in longitudine DO faciamus arcum circuli. Secabit ergo lineam ZUE in

206 HDN: HND *FP1*; GDN *O* / HND: GUD *O* / angulus[3] *om. L3C1E* 207 MPD: MOP *FP1*; MDU *O* 208 *post* maior *add.* linea *C1* / MP: MD *O* 210 SM[1]: FM *P1S* / est *rep. et del. F* / est . . . MO (211) *transp. ad* 211 *post* et [SM: FM] *S* / proportio[1] *om. FP1* / et . . . MU (212) *om. L3* 213 FU: QF *L3E* 214 QB: BQ *C1* / QD: D *FP1* / et . . . DU (215) *mg. a. m. E* 215 ND: QD *O* / *post* in *add.* Z *FP1* / vicesima quinta: secunda *C1* / vicesima . . . figura *om. R* 217 *post* DU *add.* HR *FP1*; *add.* quare patet propositum *R* 218 *ante* hiis *scr. et del.* his *C1* / iteremus *corr. ex* iterremus *S* / perficiamus: proficiamus *FP1* / perficiamus demonstrationem *transp. C1* 219 *post* multiplicentur *add.* et *L3ER* / linee: line *S* 220 forma: figura *L3R* 221 et[1] *inter. E* / DU[1]: DB *ER* / DU[2]: DB *R* / *post* prima *add.* figura *L3* / forma: figura *R* 222 et[1] . . . forma *mg. F* / DO[1]: DE *FP1SL3E* / forma: figura *R* / et[2] . . . forma (223) *om. FP1* 223 forma: figura *R* / et . . . forma (225) *mg. a. m. E* / DN: DU *R* 224 et . . . sit (225) *om. L3* / DQ: D *O*; *om. FP1* / DH *om. R* / perpendicularem: perpendiculariter *C1* 225 forma: figura *R* 226 erit rectus *transp. L3* 227 ex *inter. a. m. E* / *post* forma *add.* punctorum O U *R* / convertitur: reflectitur *R*; *corr. ex* con *O* / *post* convertitur *scr. et del.* et DI *P1* / arcus *om. P1* / quem: que *S*; qui *O*; quam *L3* / mensurant: usurant *S* 228 DQ: QD *C1* / *post* DQ *scr. et del.* et equalis arcui ET *C1*; *scr. et del.* est *E* / AG: HG *L3E* / circulo *corr. ex* ergo *a. m. E* / duobus: duabus *P1* 229 convertentur: reflectentur duo puncta *R* / *post* linee *add.* U P *R* / duo: mo *S* / U: N *R* 230 O[1]: Q *R* / Q: quasi *F*; *corr. ex* quasi *P1* / O et N ymago *mg. a. m. E* 232 et[3] *om. S* 233 *post* faciamus *scr. et del.* circulum *E*

duobus punctis. Secet ergo in Z, E, et sit arcus ZOE. Et continuemus DZ, DE et extrahantur extra circulum. Et circa D et in longitudine DQ faciamus arcum TQK. Secabit ergo duas lineas DZ, DE in T, K. Et continuemus TK. Secabit ergo lineam DQ in L.

[7.101] Quia ergo HD est perpendicularis super superficiem circuli, uterque angulus HDT, HDK erit rectus. Et utraque superficies HDT, HDK facit in superficie speculi circulum, et arcus illius qui est inter duas lineas HD, DT erit equalis arcui qui est inter HD, DQ, et similiter arcus qui est inter duas lineas HD, DK. Et utraque linea DZ, DE est equalis linee DO. Ergo hii duo arcus sunt huiusmodi quod ex illis convertentur linee secundum angulos equales ad duo puncta Z, E. Et due linee DT, DK sunt equales linee DQ; ergo punctum T est ymago Z, et K est ymago E.

[7.102] Et quia linee DT, DQ, DK sunt equales, et linee DZ, DO, DE sunt equales, erit proportio DT ad DZ sicut proportio QD ad DO, et sicut proportio KD ad DE. Sed proportio QD ad DO, ut in prima figura preostendimus, est maior proportione ND ad DU. Ergo proportio DT ad DZ est maior proportione ND ad DU, et similiter proportio KD ad DE.

[7.103] Et quia due linee ZD, DE sunt equales, et due linee DT, DK sunt equales, erit linea TK equidistans linee ZE. Ergo utraque proportio DT ad DZ et KD ad DE erit sicut proportio LD ad DU. Ergo proportio LD ad DU est maior proportione ND ad DU; ergo linea LD est maior linea ND. Ergo N est inter L, U. Sed N est ymago U, et duo puncta T, K sunt ymagines Z, E. Ergo ymago linee ZUE recte est linea transiens per puncta T, N, K. Et linea que transit per hec puncta est convexa, ex quibus patet quod linea recta in speculis concavis quandoque videtur convexa in quibusdam sitibus.

[7.104] Item ponamus in linea ZU punctum M, quomodocumque sit, et circa centrum M et in longitudine MU, faciamus arcum RUF.

234 ergo: ? *S* 235 extrahantur: extrahatur *FP1O*; extrahamus *ER* / circa: a *R*; *corr. ex* citra *a. m. E* 236 TQK: THK *FP1*; TQ *R*; *corr. ex* TQH *E* 237 continuemus: continuetur *S* / L: B *L3E* 239 utraque *corr. ex* uterque *F* 240 HDT: HD *S*; HUT *O*; *mg. a. m. C1* / facit: faciet *OL3ER* / illius *om. R* / qui est: quem *FP1* / qui . . . arcui (241) *om. S* / qui . . . arcus (242) *om. L3E* 241 DT: DK *C1* / *post* inter^{2} *add.* duas lineas *R* 242 DK: DB *O*; *alter. in* DT *C1* 243 DO: ID *FP1* / *post* quod *add.* est *FP1* 244 convertentur: reflectentur *R* / linee *om. R* / angulos *rep. S*; *corr. ex* angulus *F* / ad: et *S* 245 DT: DE *FP1* / T *om. FP1* 246 *post* E *scr. et del.* ergo *E* 247 DT . . . linee *mg. F* / DK: ZK *O* / linee: linea *L3* 248 erit: ergo *L3* / sicut . . . et (249) *om. S* 249 KD: AD *S* / DO *alter. in* DA *O* 250 ergo . . . DU (251) *om. L3* 251 ND: HD *FP1*; ZD *O* / proportio *om. C1R* 252 DE: D *S* 253 due^{1}. . . equales *mg. a. m. E* / equales *corr. ex* est *F* / et^{2} *om. L3E* 254 linea *corr. ex* linee *E* / linee *om. R* 255 sicut *om. FP1* / LD: BD *L3* 257 *post* maoir *add.* quam *L3* / ND: HB *FP1*; NED *S*; ED *O*; *corr. ex* NE *C1* / N^{1,2}: non *L3E* / L U: B O *L3* 258 E: U *O* / linee *corr. ex* linea *C1* / ZUE *corr. ex* ZNE *E* 259 T N K: T C Q K *FP1*; C M K *S*; M K *L3* 260 recta *om. R* 261 in . . . sitibus *mg. a. m. E* 262 quomodocumque: quocumque modo *FP1R* 263 circa: citra *P1* / et^{2} *om. S* / MU *corr. ex* LN *a. m. E* / *post* faciamus *add.* ergo *FP1*

Iste ergo arcus secabit arcum UOE in duobus punctis. Secet ergo in R, F, et continuemus lineas DR, DF, et transeant recte usquoque concurrant in arcu TQK in C, I. Superficies ergo duarum linearum HD, DC faciet in speculo circulum a cuius circumferentia convertentur equaliter linee ad R, et similiter superficies duarum linearum HD, DI faciet in speculo circulum a cuius circumferentia convertentur linee ad F. C ergo est ymago R, et I est ymago F, et N est ymago U.

[7.105] Ymago ergo arcus RUF est linea transiens per C, N, I, sed hec linea erit convexa, et arcus RUF est concavus ex parte superficiei speculi. Cum ergo visus fuerit in H et unaqueque linea ZUE, ZOE, RUF fuerit in aliquo visibili, tunc linea ZUE recta comprehendetur convexa, et linea ZOE convexa comprehenditur concava, et concava convexa. Si ergo unaqueque linea ZUE, ZOE, RUF habuerit unam ymaginem, tunc forma illarum linearum erit eodem modo quo declaravimus. Et si habuerit alias ymagines, forte erunt similes aliis ymaginibus, et forte diverse.

[7.106] Patet ergo ex istis figuris quod linee recte in speculis concavis quandoque comprehenduntur recte, quandoque convexe, quandoque concave. Et linee convexe quandoque comprehenduntur convexe, quandoque concave, et concave quandoque comprehenduntur convexe quandoque concave.

[7.107] Forme ergo superficierum visibilium comprehenduntur aliter quam sint in huiusmodi speculis, nam linee recte non sunt nisi in superficiebus rectis, et cum linea recta que existit in superficie plana comprehendatur convexa aut concava, tunc superficies in qua

264 arcum *mg. F*/*post* punctis *scr. et del.* se arcum *S*/ergo^{2} *om. ER* 265 DR: DK *OC1*/ et *inter. C1*/usquoque: usquequo *FOE;* usque *S;* quousque *R*/concurrant: currant *FP1L3C1; corr. ex* currant *O* 266 arcu *corr. ex* actu *P1*/TQK: DTQK *F;* DTQD *P1*/C: Q *E;* P *R*/ I *om. E*/HD: HC *L3*/DC: DP *R* 267 faciet *om. L3E*/circulum: circuli *L3E*/convertentur: reflectentur *R*/equaliter *om. R* 268 R: G *S*/HD: DH *O*/*post* HD *scr. et del.* D *C1*/DI *corr. ex* DY *P1* 269 convertentur: reflectentur *R*/convertentur linee *transp. S*/C: P *R; mg. C1; corr. ex* E *a. m. E* 270 I: Q *S;* L *L3*/est^{2}. . . N *om. P1*/*post* N *scr. et del.* et *C1* 271 ergo *om. L3*/RUF: MF *L3E*/C: Q *L3E*/N: T *FP1;* Z *O;* M *L3E*/I *om. L3E*/C N I: I P N *R*/sed *inter. O* 272 convexa: concava ex parte visum *R*/*post* est *scr. et del.* linea transiens *S*/concavus: concava *O; corr. ex* convus *F*/superficiei: superficies *FP1* 273 unaqueque: unaquaque *SO*/ZOE: ZOZ *FP1;* DOE *O* 274 comprehendetur: comprehenditur *L3E* 275 ZOE: ZOZ *FP1*/comprehenditur: comprehendetur *R*/et concava *om. L3*/*post* et^{2} *add.* RUF *R* 276 linea: linearum *R*/ZOE: ZOZ *FP1;* DOE *O* 277 illarum: istarum *O; om. L3E*/linearum: rectarum *L3E*/erit *om. FP1*/modo *rep. et del. C1* 278 *ante* et *scr. et del.* mmmmm *C1*/si *inter. E*/forte *corr. ex* formmnte *C1*/similes aliis *transp. C1* 279 *post* ymaginibus *scr. et del.* mmn *C1* 280 patet *corr. ex* pate *O* 281 comprehenduntur: comprehenditur *FP1* 282 quandoque^{2}. . . convexe (283) *om. S* 283 et concave *mg. C1*/et . . . visibilium (285) *om. L3*/concave quandoque *inter. O; transp. E* 284 quandoque concave *inter. a. m. E*/concave *corr. ex* convecave *F* 285 comprehenduntur *inter. O* 286 aliter: alter *P1*/sint *mg. F; inter. O;* sunt *ER* 287 nisi *om. FP1*/*post* superficiebus *add.* rebus *FP1* 288 comprehendatur: comprehenditur *R*/aut: vel *FP1*/qua: quibus *FP1SOL3C1E*/*post* qua *add.* ipsa linea *R*

est comprehendetur convexa aut concava. Cum ergo visus comprehendit lineas convexas et concavas et rectas aliter quam sint, comprehendet superficies in quibus sunt aliter quam sint.

[7.108] Patet ergo ex predictis quod in omnibus que in speculis concavis comprehenduntur accidit fallacia, sed in quibusdam accidit semper et in omni positione, in quibusdam vero accidit in aliqua positione. Fallacie autem composite accidunt in hiis speculis eo modo quo in compositis, et hoc voluimus declarare.

CAPITULUM OCTAVUM

De fallaciis que accidunt in speculis columpnaribus concavis

[8.1] In hiis enim accidunt similia eis que accidunt in spericis concavis, accidunt enim fallacie que proveniunt ex conversione, scilicet debilitas lucis et coloris et diversitas situs et remotionis que accidunt omnibus speculis. Accidit autem in eis ex diversitate quantitatis simile illi quod accidit in speculis spericis concavis. Et videtur etiam unum visibile unum, et duo, et tria, et quattuor, et rectum et conversum secundum diversos situs, et planum videtur concavum et convexum. Ostendamus ergo qualiter in hiis speculis diversatur quantitas et numerus rei vise, et qualiter apparet rectum et conversum eo modo quo in speculis spericis concavis declaravimus.

[8.2] **[PROPOSITIO 33]** Iteremus ergo primam figuram ex duabus figuris premissis in fallaciis speculorum columpnalium convexorum, et eisdem litteris. In illa autem figura [FIGURE 6.8.33, p. 324]

289 est: sunt *FP1SOL3C1E* (*inter. E*) / comprehendetur *corr. ex* comprehendet *a. m. E* / convexa aut concava: concava aut convexa *C1* / comprehendit: comprehendat *R; corr. ex* comprehendat *E* 290 aliter: alter *P1* / comprehendet . . . sint (291) *mg.* [comprehendet: comprehendit] *C1* 291 aliter: alter *P1* / sint *corr. ex* sunt *E* 293 concavis comprehenduntur *transp. FP1* / *post* concavis *add.* quod *L3C1* / in . . . positione (294) *om. L3* 294 positione: portione *FP1SE* / *post* positione *add.* in quibusdam vero accidit in aliqua positione sed in quibusdam accidit semper et in omni portione *FP1* (sed in quibusdam accidit *rep. et del. F*) / quibusdam²: quibus *P1*) / vero *om. ER* / in aliqua positione *mg. F* / positione²: portione *L3E* 295 composite: posite *L3* 296 in *om. L3* / compositis: composite *R* / voluimus: volumus *FP1C1* 297 capitulum . . . concavis (298) *om. S*; de erroribus qui accidunt in speculis columpnaribus concavis capitulum octavum *R* 298 in *om. OL3C1* / columpnaribus: columpnalibus *L3C1* 299 enim: autem *R* / *post* accidunt *scr. et del.* in spericis concavis *S* / similia: similes *R* / que: qui *R* 300 proveniunt: perveniunt *S* / conversione: reflexione *R* 1 situs *corr. ex* lucis *a. m. E* 2 *ante* omnibus *add.* in *S* / accidit: accidunt *SOL3; corr. ex* accidunt *C1* / *post* autem *add.* in hiis speculis *FP1* / in *om. ER* 3 spericis *om. L3* 4 et² *om. E* / et⁵ *om. P1* / conversum: convexum *FP1SL3C1ER* 5 videtur: videbitur *O* 6 ostendamus: ostendemus *R* / diversatur: diversificatur *P1* 8 spericis *inter. a. m. E* 9 primam figuram *transp. FP1C1* 10 columpnalium: columpnalis *FP1;* columpnalibus *L3;* columpnarium *C1R; corr. ex* columpnarium *E* 11 autem figura *transp. E*

patuit quod linee EG, GT, EB, QB, EA, AH convertuntur secundum angulos equales; et quod linee EO, HA, BQ, TG coniunguntur in O; et quod linea ABG est linea recta extensa in longitudine speculi; et quod linee GZ, BL, AD sunt perpendiculares super superficiem contingentem superficiem que transit per lineam ABG; et quod linea ABG est perpendicularis super superficiem in quo est triangulus EBO; et quod linea TQ est equalis QH, et AB equalis BG; et quod S, C, I sunt ymagines H, Q, T; et quod C est propinquius puncto E quam linea SI; et quod linea SI est in superficie trianguli UHT; et quod due linee UH, UT sunt equales; et quod due linee US, UI sunt equales; et quod due linee ES, EI sunt equales.

[8.3] Et continuemus CU, et secet SI in F. Dividet ergo ipsam in duo equalia, nam HT est divisa in duo equalia in Q, et erit CU in superficie trianguli CUE, qui est superficies circuli B equidistantis basi speculi. Q ergo erit in superficie trianguli CUE, et C est in triangulo CEI. Ergo C est in differentia communi hiis duabus superficiebus. Sed hec differentia est linea EB; ergo C est in rectitudine EB.

[8.4] Et due linee HU, TU sunt sub duobus punctis D, Z, nam due linee HU, TU sunt perpendiculares exeuntes ex H, T super duas lineas contingentes duas portiones in quarum circumferentia sunt puncta A, G. Superficies ergo trianguli UHT est sub axe DLZ.

[8.5] Sed nullum punctum huius axis, quamvis exeat in infinitum, erit in superficie trianguli UHT, nam si esset, tunc, si continuaretur cum aliquo puncto linee HT linea recta, illa superficies in qua esset illa linea recta et linea HT esset tunc superficies trianguli UHT, et illa

12 quod linea *om. S*/linee: linea *FP1OL3C1E*/EG *corr. ex* GEG *F;* EH *O;* AG *L3*/GT: BT *FP1;* HT *O*/QB: RQB *FP1;* KB *O*/convertuntur: reflectuntur *R* 13 *post* linee *scr. et del.* GZ BL EK *C1*/EO: EK *FP1SR;* TK *OL3;* AH *E; om. C1*/HA: HK *FP1O;* HL *S;* AB *L3;* TK *E*/BQ: BD *FP1L3E;* LD *S;* BH *O;* QB *R*/TG *om. O* 14 recta *inter. a. m. E* 15 linee: linea *L3*/BL: BK *E* 17 perpendicularis *corr. ex* perpendicalaris *F*/quo: qua *ER*/triangulus: triangulum *R*/EBO: EBD *L3* 18 *post* linea *scr. et del.* ABG est perpendicularis *S*/TQ: TR *FP1S;* TK *O*/QH: RH *FP1S;* KH *O*/*post* BG *inter.* H in arabico *O*/S C I: E S I *FP1;* Q I *S;* C R I *O;* Q L *L3E* 19 Q: R *FP1;* K *SO*/*post* quod *add.* Q *L3C1E*/C: F *O*/SI: CI *O;* SL *L3*/*post* SI *add.* et quod linea S *FP1*/et[2]. . . SI (20) *om. S* 20 SI: CI *O* 21 sunt[1] *corr. ex* sint *FP1*/US: UC *O* 22 ES: EZ *FP1;* EG *S;* ET *O* 23 CU: KU *O*/SI: SIE *FP1L3E;* SEI *S;* CE *O; corr. ex* SIE *C1*/F: Æ *R* 24 Q: R *FP1;* K *O; scr. et del.* N *S*/CU: KU *O* 25 CUE: KEO *O;* QUE *R*/CUE . . . circuli *om. P1*/qui: que *SOER*/*post* est *scr. et del.* in *C1*/superficies circuli *transp. E*/*post* superficies *scr. et del.* peculi *C1*/circuli *mg. a. m. E*/B: U *L3;* BF *R*/equidistantis *corr. ex* equidistans *a. m. E* 26 Q *om. FP1L3E;* C *SC1R*/Q ergo *transp. R*/erit *om. P1*/trianguli: triangulus *E*/CUE: KEO *O*/et *om. L3E*/C: GT *FP1;* Q *O;* AT *L3;* EIT *E; om. R*/est *om. C1*/in[2] *inter. O*/triangulo: superficie trianguli *R* 27 CEI: CEL *FP1;* DEI *L3*/C: G *O*/in *om. L3E*/*post* in *add.* linea que est *R*/communi *alter. in* communis *E* 28 ergo: GQ *S*/C: OC *S;* Q *O;* CQ *L3; corr. ex* CQ *C1; alter. ex* EQ *in* CQ *a. m. E* 29 et . . . Z *rep.* [et[2]: nam] *L3*/sunt . . . TU (30) *rep. E; rep. et del. C1* 30 HU: CN *S* 32 G: H *FP1O*/*post* trianguli *add.* est *E*/UHT est *transp. L3* 33 in *om. L3; inter. a. m. E* 34 erit *corr. ex* erat *E*/*post* erit *scr. et del.* circulum *E*/*post* tunc *scr. et del.* si esset *F* 35 linea *om. S*/*post* recta *add.* tunc *R*/illa . . . esset (36) *om. FP1*/*post* superficies *add.* esset *OL3C1E* 36 HT: HZ *L3*/*post* esset *inter.* in *C1*/tunc *om. R*

superficies esset illa in qua sunt due linee equidistantes HT, DZ. Et sic superficies in qua sunt due linee HT, DZ est superficies trianguli HUT, et sic axis erit in superficie trianguli HUT.

[8.6] Sed axis est equidistans linee HT positione, et axis secat duas lineas HU, TU. Et linea TH est in superficie trianguli UEH, que est superficies conversionis, et superficies communis huic superficiei et superficiei columpne est aliquis sector. Superficies ergo EUH secat axem columpne in uno puncto, scilicet in D, ut preostendimus. Et cum axis secet lineam HU, punctus sectionis cum linea HU erit in superficie trianguli UEH. Sed in hac superficie non est punctum per quod axis transeat preter quam D. Ergo linea HU secat axem in D. Et iam ostendimus quod HU secat eum in puncto sub D, quod est impossibile.

[8.7] Ergo axis DZ est extra superficiem UHT et propinquior puncto E quam superficies UHT. Superficies ergo in qua sunt linee HT, DZ est propinquior puncto E quam superficies UHT. Et C est in superficie in qua sunt HT, DZ, quia est in linea QL, et QL est in superficie in qua sunt HT, DZ. Ergo C est propinquior puncto E quam S, I. Sed C est in rectitudine EB. Si ergo EB exiverit in parte B, perveniet ad C; perveniat ergo ad C.

[8.8] Hiis preostensis, dico quod linea SI, que est equidistans axi speculi, cum fuerit in aliquo visibili, et visus fuerit in O ex parte concavitatis columpne, et superficies speculata fuerit superficies concava, tunc SI comprehendetur ex O in speculo ABG concavo, et diversabuntur ymagines eius secundum diversitatem sue distantie ab axe.

[8.9] Cuius demonstratio est quod angulus EBM est acutus; ergo angulus LBC est acutus. Et linea EBC est in superficie circuli B, et LB est dyameter huius circuli. Ergo EB secat circulum; ergo CB est intra concavitatem speculi.

37 et . . . DZ (38) *mg. C1* 38 est: esset *R* 39 erit: esset *R* 40 HT: HDT *S;* HD *O* 41 TH: HT *O;* UH *C1* 42 conversionis: reflexionis *R*/superficies[2]: linea *R* 43 est: et *FP1*/sector: sectio columpnaris *R* 45 cum[1]: si *R*/HU: HTL *FP1*/punctus: punctum *R* 46 est *inter. E* 47 quam *om. ER* 48 eum: EN *P1;* UAD *O* 50 est: esse *S*/UHT *rep. et del. L3*/*post* UHT *scr. et del.* superficies ergo in qua sunt linee HT DZ est extra superficiem NHT et *E* 51 E *om. L3*/superficies[1] *inter. a. m E*/UHT: HUT *FP1L3R* 52 HT *corr. ex* HD *S*/est[2] *rep. P1* 53 in[1] *inter. O*/est: sunt *FP1SOL3C1E*/QL[1]: KL *FP1SOL3C1E*/QL[2]: DL *FP1;* KL *SOL3C1E* 54 C: R *FP1SC1;* K *OL3;* RI *E*/propinquior: propinquius *R*/puncto *om. ER*/S: G *S;* Q *O;* C *C1*/I *om. FP1SOC1* 55 C: Q *O*/exiverit: exierit *P1C1*/perveniet: pervenit *FP1* 56 C[1, 2]: Q *O*/perveniat: perveniet *R*/*post* C[2] *add.* S *FP1* 57 SI: S *FP1;* G *S;* CI *O* 58 fuerit[1] *mg. F*/in aliquo *om. L3*/*post* ex *scr. et del.* tunc *L3* 60 SI: GI *S;* CI *O;* SR *L3*/O: eo *FP1*/in speculo *inter. a. m. E*/ABG concavo *transp. R*/*post* ABG *scr. et del.* in speculo *C1*/*post* concavo *add.* a linea ABG *R* 61 sue *om. R*/*post* sue *scr. et del.* sub *C1*/sue distantie *om. P1* 62 quod: quia *R* 63 angulus *om. R*/LBC: LBQ *O;* LHC *L3*/EBC: ELC *SL3;* EBQ *O*/B: BF *R*/B et: HET *FP1;* BEC *E*/et *inter. O*/LB *corr. ex* LEB *E* 64 huius circuli *transp. FP1*/ergo[1]. . . circulum *rep. et del. F*/EB: EBC *R*/secat *corr. ex* secet *OC1*/CB: QB *O*

[8.10] Et similiter OB est intra concavitatem speculi, quia angulus OBL est acutus, et duo anguli OBL, CBL sunt equales, nam sunt equales duobus angulis EBM, QBM, et LB est perpendicularis super superficiem contingentem columpnam que transit per B. Forma ergo C extenditur per CB et pervenit ad B, et convertitur per BO et comprehenditur a visu ex O.

[8.11] Item in quinto capitulo, cum fuimus locuti de speculis columpnaribus convexis, declaravimus quod superficies contingens columpnam in G erit sub E. Ergo EG secat superficiem contingentem; secat ergo lineam contingentem circumferentiam sectoris in G. Secat ergo sectorem et cadit intra ipsum; cadet ergo intra concavitatem speculi. Ergo due linee OG, GI sunt intra concavitatem speculi, et ZG est perpendicularis super superficiem contingentem columpnam in G, et duo anguli OGZ, IGZ sunt equales. Ergo forma I extenditur per IG et pervenit ad G, et convertitur per GO et comprehenditur ex O per lineam GO. Et similiter S extenditur per SA et convertitur per AO.

[8.12] Et iam declaravimus, cum tractavimus de fallaciis speculorum columpnalium convexorum, quoniam due linee HU, TU sunt perpendiculares super duas superficies contingentes sectores transeuntes per duo puncta A, G. Ymago ergo S est in linea HU. Et OA linea radialis que extenditur ex visu ad punctum conversionis; ergo ymago S est in OA. H ergo est ymago S, et sic patet quod T est ymago I.

[8.13] Et continuemus CL. Quia ergo C convertitur ad O ex circumferentia B, erit ymago Q in linea CL. Et OB est linea radialis que extenditur inter visum et punctum conversionis; ergo ymago C est in linea OB. Ergo ymago C est in puncto sectionis inter QL et OB.

66 et . . . speculi *om. S* / OB: AB *FP1* / est: erit *R; om. E* / intra: circa *L3* 67 OBL[1]: ABL *FP1OL3E* / CBL: QBL *O; om. P1* / nam sunt equales (68) *om. R* 68 duobus *inter. a. m. E* / QBM: KBM *FP1O;* ABM *S;* TBM *L3;* IBM *E* 69 B: ? *S; inter. E* 70 C: Q *O* / extenditur: ostenditur *S* / CB: QB *O* / convertitur: reflectitur *R* 71 ex: in *R* / O: BO *O* 72 item *om. S* / item . . . GO (80) *rep. P1* / item . . . O (80) *rep. F* / speculis *om. P1* 73 columpnaribus: columpnalibus *SE* 74 G: H *FP1SO* / EG: EH *FP1SO* / contingentem . . . lineam (75) *om. C1* 75 secat ergo lineam *om. P1* / *post* contingentem *add.* superficiem *S* / sectoris: sectionis *R* / G: H *FP1SO;* S *E* 76 sectorem: sectionem *R* / ipsum: ipsam *R* 77 OG: OH *FP1SO* / GI: H *FP1;* HI *SO* / et *inter. a. m. E* / ZG: ZH *FP1SO* 78 super superficiem *om. P1* / G: H *FP1SO* 79 OGZ: OBZ *FP1S;* OHZ *O; corr. ex* GGZ *E* / IGZ: IHZ *FP1O; corr. ex* SIG *E* / I *om. P1* / IG: IH *FSO;* H *P1* 80 G: H *FP1* / convertitur: reflectitur *R* / GO: HO *FP1O;* SG *S* / ex: in *R* / per[2] . . . GO (81) *om. P1* 81 GO: HO *FO* / F: C *O* / SA: EA *S;* CA *O* / *post* et[2] *add.* pervenit ad A et *R* / convertitur: reflectitur *R* / per[2] *mg. E* / AO: CO *O;* SA *L3;* SO *FP1S;* SA *mg. E* / *post* SO *add.* et comprehenditur in O *R* 82 et *inter. O* / tractavimus: detractavimus *S; corr. ex* detractavimus *E* 83 *ante* columpnalium *scr. et del.* con *P1* / columpnalium: columpnarium *SC1R* (*alter. in C1*) / quoniam: quando *L3;* quod *R* 84 duas *om. R* / sectores: sectiones *R* 85 G: H *FP1O* / S: C *O* / est *om. L3E* / OA: AO *R* / *post* OA *inter.* est *O* 86 conversionis: reflexionis *R* / ymago *mg. a. m. E* / S: G *S;* C *O* 87 OA: EA *L3;* AO *R* / H: B *SL3* / S: C *O; corr. ex* Y *E* / est[2] *inter. O* / I: R *O* 88 CL: QL *O* / C: QC *FP1SL3C1E;* Q *O* / convertitur: reflectitur *R* / circumferentia: circumferentie puncto *R* 89 Q : C *C1R* / Q in: quoniam *FP1L3* / CL: QL *O;* CB *L3; inter. E* 90 conversionis: reflexionis *R* / ergo . . . OB[1] (91) *scr. et del. E* / C: CQ *S;* Q *O* / est *om. E* / *post* in *add.* puncto communi CL et OB nempe in puncto Q *R* 91 linea . . . OB[2] *om. R* / C: Q *O* / *post* puncto *scr. et del.* conversionis *C1* / QL: CL *C1*

[8.14] Sed in capitulo de ymagine, cum tractavimus de ymaginibus speculorum spericorum concavorum, patuit quod ymago puncti cuius forma convertitur a concavitate circuli forte concurret cum linea radiali que est inter visum et punctum conversionis ultra circulum, et forte inter visum et circulum, et forte in centro visus, et forte ultra centrum visus, et forte CL equidistans erit OB.

[8.15] Et in illo capitulo patuit quod forte ymago erit unum punctum, aut duo, aut tria, aut quattuor Ymago ergo C forte erit in BQ, forte ultra Q, et forte in BO, et forte in O, et forte ultra O. Et forte ymago C erit unum punctum, aut duo, aut tria, aut quattuor.

[8.16] Si ergo ymago C fuerit Q, tunc HT erit dyameter ymaginis SI. Si ergo omnes ymagines SI fuerint in linea HT, tunc forma eius erit linea recta. Sin autem, erit prope rectam, nam medium eius est in rectitudine duarum extremitatum. Si autem ymago C fuerit ultra Q, tunc ymago SI erit fere concava ex parte visus. Et si ymago visus fuerit in linea BO, tunc ymago SI erit convexa ex parte visus.

[8.17] Et si ymago C fuerit plura puncta, tunc ymago C erit plures linee quarum omnium extremitates coniunguntur in duobus punctis H, T, et media eorum sunt distincta separata. Et HT est dyameter ymaginis SI, quocumque modo fuerit ymago, et dyameter est communis omnibus ymaginibus eius si plures habuerit ymagines, et linea HT est maior quam SI modica quantitate.

[8.18] Patet ergo quod, cum linee recte equidistantes axi columpnali speculi concavi fuerit in aliquo visibili, ymago eius forte erit recta aut concava, et forte una erit aut plures.

[8.19] **[PROPOSITIO 34]** Item iteremus secundam figuram de fallaciis speculorum columpnalium convexorum. In hac autem figura [FIGURE 6.7.34, p. 325] dictum est quod due linee EB, BH conver-

92 *post* sed *scr. et del.* cum *P1* 93 *post* spericorum *inter.* et *C1* 94 convertitur: reflectitur *R* / *post* convertitur *scr. et del.* et *C1* 95 linea radiali *transp. R* / conversionis: reflexionis *R* / circulum: speculum *R* 96 inter *inter. E* / circulum: speculum *R* / et^3... centrum (97) *rep. S* 97 *post* visus *add.* EC *L3* / CL: QL *O* 99 C: CQ *FP1;* Q *O; om. ER* / BQ: BK *FP1SO* 100 ultra^1 *om. P1* / Q: R *FP1S;* K *O;* B *C1;* OQ *L3ER* / O^1: MO *E* / O^2 *om. R* 101 C: CQ *FP1;* TQ *SL3ER;* Q *O* 102 C: Q *O* / fuerit: fuit *FP1S* / Q: R *FP1SO* / *post* tunc *add.* ergo *C1* / HT: HQT *R* 103 SI^1: S *FP1;* ST *S;* CI *O* / omnes ymagines *transp. deinde corr. S* / SI^2: S *FP1;* CI *O* / fuerint *corr. ex* fuerit *O;* fiunt *L3* / HT: HQT *R* 104 sin ... rectam *om. R* / erit^2 *om. FP1* / eius *inter. E* 105 *post* extremitatum *add.* HT *R* / C: CQ *FP1;* Q *O* 106 Q: C *FP1;* R *S;* K *O* / SI: S *FP1;* CI *O* / fere *om. FP1* / et ... visus (107) *om. R* / ymago^2... visus (107) *om. L3C1E* 107 BO: FO *O* / SI: CI *O* 108 C^1: CQ *S;* Q *O; inter. C1* / fuerit: fuit *L3;* fuerint *R* / C^2: CI *O* / erit: erunt *R* 109 coniunguntur: coniungentur *R* 110 eorum sunt: earum erunt *R* / *post* sunt *add.* distantia *O* / *post* distincta *add.* et *R* 111 SI: sed *FP1;* CI *O* 112 habuerit: habuerint *S* 113 SI: CI *FP1O;* C *SL3E* 114 cum *om. L3* / linee recte *transp. FP1; alter. in* linea recta *C1* / equidistantes *alter. in* equidistans *C1* / columpnali: columpnaris *R; corr. ex* columpnalis *F; alter. in* columpnalis *O* 115 fuerit: fuit *S;* fuerint *R* / eius: earum *R* 116 aut^1: et *FP1* / erit *om. R* 118 columpnalium: columpnarium *SR* 119 EB: EG *FP1* / BH: UT *FP1;* HB *ER* (*alter. in E*) / convertuntur: reflectuntur *R*

tuntur secundum angulos equales; et quod due linee EG, GT convertuntur secundum angulos equales; et quod HB, TG perveniunt ad L; et HB continet cum BO angulum acutum. Ergo HB secat superficiem contingentem superficiem columpne in B; BL ergo est sub concavitate columpne. Et similiter GL, et similiter due linee BR, GY.

[8.20] Et duo anguli LBD, DBR sunt equales, et duo anguli LGD, DGY sunt equales. Si ergo RY fuerit in aliquo visibili, et visus fuerit in L, et superficies concava columpne fuerit tersa, tunc forma R extenditur per RB, et pervenit ad B. Et convertetur per BL, et perveniet ad L, et comprehendetur ex L. Et linea HU est perpendicularis super lineam contingentem sectorem ex cuius circumferentia convertentur due linee RB, BL. H ergo est ymago R. Et similiter declarabitur quod forma Y extenditur per YG et convertitur per GL, et ymago eius est T.

[8.21] Et continuemus KU. Secabit ergo RY in M. M ergo est in superficie transeunte per axem et per L, nam L et K sunt in hac superficie; ergo KU est in hac superficie. Et quia duo puncta M, L sunt in superficie transeunte per axem columpne, ideo forma M convertetur ad L in hac superficie. Et linea AZ est differentia communis inter superficiem columpne et superficiem transeuntem per suum axem et per L; forma ergo M convertetur ad L per AZ.

[8.22] Et continuemus EM, que est in hac superficie. Et EL etiam est in hac superficie, et punctum E est elevatum a superficie contingente superficiem columpne in linea AZ. Ergo si AZ extrahatur recte in parte Z, concurret cum duabus lineis EM, EL. Concurrat ergo cum EM in I, et cum EL in N. N ergo est inter duo puncta E, L, quia L est

120 et . . . equales (121) *om. FP1* / EG *corr. ex* EH *O* / GT: UT *L3E*; TG *R*; *corr. ex* HT *O* / convertuntur: convertantur *S*; reflectuntur *R* 121 *post* quod *add.* due linee *O* / TG: TH *FP1O*; G *S* / perveniunt *corr. ex* perperveniunt *P1* / ad *rep. FP1E* / L et (122) *om. S* 122 BO: BD *O* 123 superficiem columpne: columpnam *R* / in *corr. ex* N *E* / in . . . columpne (124) *mg. a. m. E* / B: H *L3* / *post* est *add.* in *FP1C1* / sub *om. FP1* 124 GL: BL *FP1*; HL *O*; GR *L3E* / BR: BS *O* / GY: HY *O* 125 DBR: DBS *O*; BDR *C1*; *alter. ex* GDY *in* DGY *a. m. E* / et^2 . . . equales (126) *scr. et del. E* / duo anguli: similiter *R* / LGD: LD *FP1*; GLD *S* 126 DGY: DG *FP1*; DGB *O*; TGY *E*; GDY *R* / RY: XXY *FP1*; SY *O* / *post* et *scr. et del.* f *P1* 127 R: S *O* 128 RB: RO *FP1SL3*; SB *O*; ID *E* / B: D *P1* / convertetur: reflectitur *R* / perveniet: pervenit *C1* 129 L^1 *om. S* / ex: in *R* / *post* ex *add.* B *O* / *post* HU *scr. et del.* est concava columpne *C1* / *post* perpendicularis *add.* est *L3* 130 sectorem: sectionem *R* / cuius *om. FP1* / convertentur: reflectentur *R* 131 RB: SB *FP1O*; FB *L3E*; BR *R* / BL: DL *L3* / H: HG *FP1SOL3E* / ergo est *transp. FP1L3* / R: S *O* / et *om. ER* 132 extenditur: extendetur *SOC1*; *corr. ex* extendetur *E* / YG: YB *FP1*; YH *O* / convertitur: reflectitur *R* / GL: HL *FP1O* / est *om. C1* / T: CT *FP1*; CS *L3*; OS *E* 133 KU: QU *R* / RY: SY *O* / est *om. O* 134 et^1 . . . axem (136) *om. FP1* / et^2: Z *SOE* / K: Q *R* 135 KU: QU *R* / M L *inter. E* 136 ideo: vel O *E* / convertetur: reflectetur *R* 137 ad L *om. P1* / *post* et *inter.* quia *a. m. E* / linea: quia *R* / superficiem *inter. a. m. E* / superficiem columpne (138) *transp. ER* 138 superficiem: superficie *L3* / transeuntem: transeunte *L3* 139 convertetur: reflectetur *R* / ad L per: a linea *R* 140 EM: OM *SOL3E* / est *inter. E* / EL: L *L3* / EL . . . superficie^1 (141) *mg. a. m. E* / etiam *om. SER* 141 elevatum: elongatum *R* 143 concurret: concurrat *FP1SL3*; *corr. ex* concurrat *E* / EL: L *L3* 144 EL: L *L3* / N^2 *mg. F*; *om. L3E* / N ergo *transp. R* / L^2: EL *S*

intra concavitatem columpne, et N est in superficie columpne, et E est elevatum a columpna.

[8.23] Et in demonstratione huius figure patuit quod circulus BZG est medius inter lineam HT et superficiem exeuntem ex E equidistantem basi columpne. Et perpendicularis que exit ex E super AZ est in superficie exeunte ex E equidistante basi columpne. Ergo perpendicularis que exit ex E super lineam AZN cadit extra triangulum EIN et in parte N. Angulus ergo EIN est acutus; ergo angulus MIA est acutus.

[8.24] Extrahamus ergo ex M perpendicularem super AI, et sit MQ. Q ergo erit ultra I respectu N. Et extrahamus MQ ex parte Q, et dividamus QS ad equalitatem QM. S ergo erit extra superficiem speculi et ultra concavitatem eius, et L erit sub concavitate eius.

[8.25] Et continuemus LS. Secabit ergo NQ in F, et ex F extrahamus FX ad equidistantiam QM. Ergo est perpendicularis super AN et in superficie transeunte per axem et per L; ergo est dyameter circuli exeuntis ex F equidistantis basi columpne. Linea ergo XF est perpendicularis super superficiem contingentem columpnam transeuntem per AZ.

[8.26] Et continuemus MF. Erit ergo equalis FS, et duo anguli qui sunt in M, S erunt equales. Et quia XF est equidistans MS, erunt duo anguli F equales duobus angulis qui sunt apud S, M. Due ergo linee MF, FL convertuntur per angulos equales, et XF est perpendicularis super superficiem contingentem superficiem speculi in F. Forma ergo M extenditur per MF, et convertitur per FL, et ymago eius erit S.

[8.27] Et quia due linee RY, HT sunt equidistantes et perpendiculares super superficiem transeuntem per axem et per L (quia HT fuit posita talis), ideo due superficies exeuntes a duabus lineis HT, RY erunt equidistantes. Et quia RY est perpendicularis super superficiem transeuntem per axem et per L, ideo superficies duarum

145 N: non *SL3* 146 elevatum: elongatum *R* 147 BZG: BZH *FP1O*; BZD *S*; BG *R* 149 basi: basibus *R* / et *om. S* 150 basi *om. FP1SL3C1ER*; *inter. O* 151 ex *inter. E* / extra: intra *FP1SL3C1E* (*inter. a. m. E*) / EIN: EIA *S* 152 et *inter. a. m. E* 154 M: EM *S* / super AI *transp. L3* 155 MQ^{1,2}: MK *R* / Q^{1,2}: K *R* / *post* Q^2 *add.* in S *R* 156 QS: QR *O*; KS *R* / QM: KM *R* / S: R *O* / S ergo *transp. R* 158 LS: LR *O* / secabit *corr. ex* secat *E* / NQ: NK *R* / F^2: S *FP1* 159 FX: BZ *O* / QM: cum *L3E*; MK *R* / ergo est: cum ergo FX sit *R* 160 per^2: P *FP1* 161 equidistantis basi columpne *om. P1* / XF: FX *R*; ZF *O*; *corr. ex* ZF *C1* 162 super *mg. E* / contingentem: continentem *L3* 164 *post* equalis *add.* quia duo latera SQ QF super equalia duobus lateribus MQ QF et anguli contensi quia uterque rectus *L3* / FS: FR *O* 165 in: apud *R* / S: R *O* / XF: ZF *FP1O*; EXF *S* / MS: MG *FP1L3ER*; MR *O* 166 *post* anguli *add.* apud *R* / equales duobus *transp. deinde corr. O* / qui *corr. ex* quod *O* / S: R *O* 167 convertuntur per: reflectuntur secundum *R* / et *inter. O* / XF: ZF *FP1SOL3E* 168 superficiem speculi: speculum *R* / *post* superficiem^2 *add.* circuli *P1* 169 MF: M *P1* / convertitur: reflectitur *R* / FL: F *FP1* / S: L *FP1SOL3E* 170 RY: XY *FP1S*; SY *O* / HT: HZ *FP1* 171 transeuntem *om. FP1* / *post* axem *add.* C *FP1* 172 *post* HT *scr. et del.* RY erunt *S* / fuit: sunt *L3* / duabus: duobus *S* 173 RY^1: XY *FP1*; SY *O* / *post* equidistantes *add.* et perpendiculares *R* / RY^2: SY *O*

linearum RM, MS erit perpendicularis super superficiem transeuntem per axem et per L. Et erit MS differentia communis hiis duabus superficiebus, et quia AQ est in superficie transeunte per axem, et est perpendicularis super MS, que est differentia communis inter superficiem transeuntem per axem et inter superficiem duarum linearum RM, MS, erit AN perpendicularis super superficiem duarum linearum RM, MS.

[8.28] Et linea AN est equidistans axi columpne; ergo axis columpne est perpendicularis super superficiem in qua sunt due linee RM, MS. Superficies ergo ista est perpendicularis super axem columpne. S ergo in superficie exeunte ex linea RY perpendiculariter super axem columpne.

[8.29] Sed linea HT est in superficie perpendiculari super axem columpne equidistanti superficiei exeunti ex linea RY. S ergo est extra HT et propinquior L quam HT. Et duo puncta H, T sunt ymagines R, Y, et punctum S est ymago M; ymago ergo linee RMY est linea transiens per H, S, T.

[8.30] Sed talis linea est arcualis, quia S est extra HT, et transeat per puncta H, S, T linea HST arcualis. Et quia HT, secundum positionem, fuit elevata a convexo columpne, erit HT ultra superficiem speculi respectu L. Et iam declaravimus quod S est ultra concavitatem speculi respectu L; ergo tota linea HST est ultra concavitatem superficiei speculi. Et L est sub concavitate speculi; ergo L est extra superficiem in qua est linea HST. Arcualitas ergo linee HST apparebit visui L manifeste.

[8.31] Et quia F est in superficie columpne, et TH est ultra columpnam, et TH est in superficie trianguli LHT, erit linea LFS altior quam superficies trianguli LHT. Linea ergo LS erit altior duabus lineis LH,

175 RM: EM *FP1SL3E*; CM *O* / MS: MG *FP1SL3E*; MR *O* / super *om. F*   176 per: P *FP1L3* / et^2 *inter. P1*; *om. S* / MS: MR *FP1SL3E*; ML *O*   177 AQ: AK *R* / superficie: superficiente *S*; *corr. ex* superficiente *F* / *post* transeunte *scr. et del.* in *E* / *post* axem *scr. et del.* est *P1*   178 MS: MR *O*
180 RM: CM *FP1SOL3E* / MS: MG *FP1L3E*; MR *O* / *post* erit *add.* linea *O* / AN *inter. a. m. E*; AKN *R*
181 linearum *om. FP1SC1E*; *inter. O* / RM: CM *FP1SOL3E* / MS: MR *O*; MG *L3E*   183 due linee (184) *om. R*   184 RM: CM *FP1SOL3E* / MS: MG *S*; MR *O* / superficies *corr. ex* superficieses *E*
185 S: R *O* / *post* ergo *add.* est *R* / ex *corr. ex* a *E* / ex linea *rep. et del. F* / RY: SY *O* / perpendiculariter: perpendiculari *ER*   188 columpne *om. R* / equidistanti: equidistante *R* / exeunti *om. R* / RY: SY *O* / S: R *O*   189 propinquior: propinquiori *FP1*; propinquius *R* / L *om. FP1* / *post* quam *add.* sint *R* / HT: H et T *R*   190 R: S *SO* / Y: I *S* / S: R *O* / RMY: RIAY *F*; NAY *P1*; SMY *O* / *post* RMY *scr. et del.* SMY *C1*   191 S: R *O*; A *L3*   192 linea est *transp. R* / quia . . . arcualis (193) *om. S* / S: R *O* / est *om. P1*; *inter. E*   193 S: R *O*; C *L3* / linea HST *rep. et del. F* / HST: HRT *O*; HCT *L3* / *post* HST *add.* est *P1* / et *om. E* / *post* HT *add.* est *P1*   194 elevata: elongata *R* / *post* elevata *add.* et *C1*; *add.* ei *E* / HT: HR *C1*   195 S: R *O*   196 HST: HRD *O* / est: erit *ER*   197 superficiei speculi *transp. S* / L^1: EL *R*   198 HST^{1,2}: HRT *O*   199 L: vel *S*   200 columpne . . . superficie (201) *om. L3* / TH: BH *FP1SOL3C1E* / est^2 *om. R* / ultra: intra *OC1*; inter *E*   201 et TH *om. R* / TH: BH *FP1SOL3C1E* / LHT: BHT *FP1* / LFS: LFR *O*   202 LS: LR *O* / *post* altior *scr. et del.* quod *E*

HT respectu visus L. S ergo est altior quam dua puncta H, T; linea ergo HST apparebit visui L concava.

[8.32] **[PROPOSITIO 35]** Item secemus columpnam per superficiem declinem super axem eius, et non transeat per totum axem. Faciet ergo sectorem. Sit ergo ABG [FIGURE 6.8.35, p. 326]. Sed in prima figurarum de columpnis concavis declaratum est quod in superficie cuiuslibet sectoris columpne erit perpendicularis super superficiem contingentem columpnam ex cuius extremitatibus convertuntur forme. Sit ergo perpendicularis GZ, et sit BE perpendicularis super lineam contingentem circumferentiam sectoris in B, et sit B prope G. BK ergo secabit perpendicularem GZ, et continebit cum ipsa angulum acutum. Secet ergo in E. Angulus ergo BEG erit acutus.

[8.33] Et extrahamus ex G lineam ad equidistantiam linee BK, et sit GD. Angulus ergo DGE erit acutus; ergo GD erit intra concavitatem columpne. Et ponamus angulum EGL equalem angulo EGD. GL ergo concurret cum BE in L. Et signemus M in linea LE. Erit ergo angulus MAG acutus, quia AM est intra sectorem.

[8.34] Et ponamus angulum GAD equalem angulo GAM. Ergo AD concurret cum GD, nam duo anguli qui sunt apud A, G sunt acuti. Concurrant ergo in D. AD ergo secabit BK. Secet ergo in T.

[8.35] Cum ergo BK fuerit in aliquo visibili, et visus fuerit in D, tunc forma L videbitur in G, quia forma L convertetur ad D ex G, et quia DG est equidistans perpendiculari LB. Et forma M videtur in T, quia forma M convertitur ad G ex A, et T est ymago M.

[8.36] Et transeat per D superficies equidistans basi columpne. Secabit ergo superficiem ABG et faciet in superficie columpne circu-

203 HT: T *FP1SOL3E* (*inter. F*); *corr. ex* T *C1*/L *om. R*/S: R *O*/S ergo *transp. R*/altior: altius *R*/dua: duo *OC1ER* 204 ergo HST *transp. deinde corr. C1*/HST: HRT *O*/L: I *S* 206 *ante* declinem *add.* contingentem *E*/et . . . axem[2] *om. R* 207 faciet: faciat *C1*/sectorem: sectionem columpnarem *R* 208 figurarum: figura *R* 209 cuiuslibet: cuiusque *FP1*/sectoris: sectionis *R*; *corr. ex* linee *L3*/*post* columpne *add.* exit a puncto reflexionis *R*/erit: exit *SOC1E*; *om. R*/super *om. L3* 210 columpnam *om. R*/convertuntur: reflectuntur *R* 211 GZ: HZ *FO*; GA *R*/GZ . . . perpendicularis *om. P1*/BE: BEK *R* 212 super: supra *SOL3C1*/sectoris: sectionis *R* 213 secabit: secet *E*/GZ: GA sub axe *R* 214 in . . . ergo[2] *om. S*/angulus ergo *transp. C1*/BEG: BG *E* 216 equidistantiam *corr. ex* equidistantem *C1*/et[2] *om. L3E* 217 ergo[1]: G *S*/DGE: GDE *E*/GD[2]: GDE *R* 218 EGL: EBL *FP1*; EHL *O*/EGD: EG *FP1* 219 GL: DG *FP1*/concurret *corr. ex* concurrat *P1*/*post* signemus *add.* punctum *R*/M: LN *FP1L3E*; FN *S*/ergo *mg. F*/ergo angulus (220) *transp. F* 220 angulus *om. R*/MAG *corr. ex* MAGI *F*/quia: ergo *R*/sectorem: sectionem *R* 221 ergo *rep. et del. F* 222 A: H *O*; *corr. ex* D *a. m. E* 223 AD: LD *O*/*post* AD *add.* D *L3E*/BK: BH *FP1L3E* 224 BK: HK *FP1L3E*; LEK *O*; LK *R*/D: C *FP1* 225 L[1] *inter. E*/videbitur: videbit *FP1*/quia *corr. ex* quod *a. m. E*/convertetur: reflectetur *R*; *corr. ex* contervertetur *F*/D: C *FP1* 226 LB: DB *SO*; BLK *R*/videtur: videbitur *R*/T: CT *FP1* 227 convertitur: reflectitur *R*/G: D *C1R*/est *inter. E*/est ymago *transp. ER* 229 superficiem: sectionem *R*/*post* superficiem *scr. et del.* EG *E*

lum COR. Superficies ergo huius circuli secabit BK, secat enim GD, que est ei equidistans. Secet ergo BK in K, et sit centrum circuli CR punctum H. Et continuemus DH, et transeat ad R. Et continuemus KH, et transeat ad C.

[8.37] Forma ergo K convertitur ad D ex circumferentia ex arcu RC, ut patuit in ymaginibus circulorum. Convertatur ergo ex O, et continuemus KO, DO, HO. Anguli ergo qui sunt apud O sunt equales, et DO secabit HC in N. N ergo est ymago K.

[8.38] Et continuemus KD. KD ergo erit differentia communis inter circulum RC et sectorem ABG, nam duo puncta K, D sunt in utraque superficie, nichil enim de superficie sectoris ABG est in superficie circuli RC nisi linea KD. G ergo est extra circulum, et similiter T, et sunt in superficie sectoris.

[8.39] Et N est in superficie circuli, et forma LMK transit per puncta G, T, N, et linea que transit per hec puncta est arcualis. Sed superficies sectoris est declinis super superficiem columpne; axis ergo sectoris non transit per totam axem columpne, nec est equidistans basi columpne.

[8.40] Patet ergo ex hac figura et duabus premissis quod linee recte equidistantes axi columpne et equidistantes basi eius, et etiam ille que declinantur super superficiem eius, forte videbuntur arcuales, forte recte, forte converse. Item, quia T est ymago M, et N ymago K, erit forma MK conversa.

[8.41] Et si linea etiam fuerit in superficie circuli equidistanti basi columpne, cuius superficies transit per centrum visus, ut dictum est in ymaginibus circulorum in septimo capitulo huius tractatus, forma forte erit equalis recta, forte conversa.

[8.42] Patet ergo quod forma eorum que comprehenduntur in speculis columpnalibus concavis forte erit recta, forte conversa.

230 COR: POR *R; corr. ex* COZ *a. m. E* 231 que: qui *C1* / ei *om. S* / ei equidistans *transp. C1* / *post* equidistans *inter.* est *mg. a. m. E* / secet ergo *transp. ER* / ergo *mg. E* / CR: POR *R; corr. ex* Q *a. m. E* 233 *ante* KH *add.* ad *S* / C: P *R* 234 ergo *inter. O* / convertitur: reflectitur *R* / ex arcu: arcus *R* 235 RC: RP *R* / in: de *R* / circulorum: speculorum *R* / convertatur: reflectatur *R* 237 HC: HP *R* 239 RC: IR *S*; RP *R* / sectorem: sectionem *R* 240 *post* superficie[1] *add.* et *R* / enim *om. R* / superficie[2]. . . in *rep. P1* / sectoris: sectionis *R* / ABG: AB *S* 241 RC: IT *E*; RP *R* / G *inter. O; corr. ex* ergo *P1* 242 T: B *R* / sectoris: sectionis *R* 243 N: enim *P1* / *post* circuli *add.* RP *R* / puncta: punctum *FP1L3E* 244 G T N: G M *FP1SL3E* 245 sectoris[1, 2]: sectionis *R* / declinis: declins *S* / axis . . . columpne (247) *om. S* 246 totam: totum *FP1C1ER* (*alter. ex* totam *a. m. E*) / nec: neque *OC1R* / basi columpne (247) *transp. C1* 248 figura: forma *C1* / duabus: dua *P1* 249 equidistantes[2]: equidistans *L3* 250 *post* ille *add.* linee *R* / declinantur: obliquantur *R* / videbuntur: videntur *C1* 251 converse *alter. in* convexe *O* / item: et *R* / ymago . . . N *om. FP1* / N *inter. O* / est[2] *om. R* 252 erit: est *FP1* / MK: in K *S* 253 si linea: similia *SOC1* / linea: ista *FP1* / fuerit: fiunt *SO* / circuli *inter. a. m. E* / circuli equidistanti *transp. E* / equidistanti basi: equidistante basibus *R* 254 columpne: commune *L3* 255 in[1]: de *R* / circulorum *om. S*; speculorum concavorum *R* / tractatus: contractatus *L3* 256 forte[1] *inter. O; om. L3* / forte erit *transp. E* / forte[2] *inter. a. m. E* 257 comprehenduntur: comprehenditur *E* 258 columpnalibus: columpnaribus *P1L3C1ER*

[8.43] **[PROPOSITIO 36]** Item iteremus formam tertie figure de fallaciis speculorum spericorum concavorum, ipsis litteris existentibus. Et sit circulus BZA [FIGURE 6.8.36, p. 327] in superficie speculi columpnalis concavi, et sit visus in D. Erit ergo extra superficiem circuli, et erunt due linee EA, EB perpendiculares super superficiem contingentem superficiem columpne. Et erit superficies trianguli DGE perpendicularis super superficiem circuli, quia DG est perpendicularis super superficiem circuli.

[8.44] Superficies ergo trianguli DGE transit per totum axem et per D, et neutra superficies DBO, DAO, que se secant in linea DO, transit per totum axem. Et in neutra superficie est aliquid de axe columpne nisi E, quod est centrum circuli. Et utraque superficies DBO, DAO facit in superficie columpne sectorem, et forme convertuntur ex hiis sectoribus a duobus punctis A, B.

[8.45] Forma ergo R convertitur ad D ex B, et forma M convertitur ad D ex A, et NU erit dyameter ymaginis MR, et est minor quam MR. Et similiter duo puncta H, L convertuntur ad D ex duobus punctis A, B, et erit TK dyameter ymaginis LH, et est ei equalis. Et erit CI diameter ymaginis FQ, et est maior illa. Et omnes iste ymagines erunt converse.

[8.46] Et si visus fuerit in O, et linee CI, TK, NU fuerint visibiles, erunt econtra, tunc enim dyameter ymaginis CI erit minor ipsa, et dyameter ymaginis NU erit maior ipsa, et erit dyameter TK equalis ei, et omnes ymagines erunt recte. Et omnia ista ostensa sunt in predicto capitulo.

[8.47] Item cum utraque extremitas alicuius harum habuerit unam ymaginem, et aliquod punctum in medio habuerit plures ymagines, tunc illa linea habebit tot ymagines quot punctum medium habet. Et si utraque extremitas vel altera habuerit plures ymagines, et punc-

260 speculorum *mg. a. m. E*/spericorum *om. R*/spericorum concavorum *transp. L3*/ipsis: iisdem *R*/litteris: lineis *FP1*; litteribus *L3*; *inter. O* 261 BZA: BAZ *C1* 262 columpnalis: columpnaris *ER* (*alter. in E*) 263 super *inter. a. m. E*/superficiem: superficiens *S*; superficies *L3ER*/*post* superficiem *scr. et del.* circuli *C1* 264 contingentem: contingentes *SL3ER*/superficiem *mg. a. m. E*/erit *inter. a. m. E* 265 quia *scr. et del. C1*/DG: GD *OR*; GB *L3C1E*/*ante* est *inter.* quia *a. m. C1* 267 ergo *inter. a. m. E*/axem . . . axem (269) *scr. et del. E*/et *om. E*/et . . . axem (269) *om. R* 268 se *om. FP1*/secant: secat *FP1* 269 *post* superficie *add.* DBO DAO *R* 271 DBO: ABO *FP1*/facit: faciat *L3*/sectorem: sectionem *R*/et forme *om. L3*/convertuntur: reflectuntur *R* 272 sectoribus: sectionibus *R*/a: ad *C1* 273 R: I *L3E*/convertitur[1, 2]: reflectitur *R*/ex . . . D (274) *om. S* 274 ad D *om. L3ER*/et[2]. . . MR *inter. O*/minor: maior *L3* 275 convertuntur: reflectuntur *R*/D *inter. E* 276 est: erit *R*/CI: PI *R* 278 converse: convexe *O* 279 in O *om. L3*/CI: Q *L3*; PI *R*/TK: TH *FP1*/fuerint *mg. C1* 280 econtra: extra *FP1*; econverso *O*; econtrario *R*/*post* econtra *scr. et del.* extra *C1*/CI: PI *R*; *om. FP1* 281 erit[2]. . . ei: dyameter TK erit ei equalis *R* 286 medium habet *transp. C1* 287 plures *mg. C1*; *inter. a. m. E*

tum medium habuerit unam, tunc linea tot habebit ymagines quot habuerit punctum extremum. Et si utraque extremitas aut altera habuerit multas ymagines, et punctum medium habuerit multas ymagines, tunc linea habebit ymagines secundum maiorem numerum. Et hoc patebit ut de ymaginibus patuit speculorum spericorum concavorum.

[8.48] In speculis ergo columpnalibus concavis accidit fallacia in omnibus que in eis comprehenduntur sicut accidit in speculis spericis concavis, scilicet de formis specierum visibilium, et de quantitatibus et de numero suarum ymaginum, et de rectitudine et de conversione, cum fallaciis que appropriantur conversioni. Et fallacie erunt in hiis ut in speculis predictis, et hec sunt que voluimus declarare in capitulo hoc.

CAPITULUM NONUM

De fallaciis que accidunt in speculis piramidalibus concavis

[9.1] In hiis autem accidunt ille fallacie que accidunt in speculis columpnalibus concavis. Debilitas vero coloris et lucis, et diversitas positionis et remotionis accidunt in hiis sicut in omnibus speculis, nam causa huius est conversio. Accidit etiam in hiis speculis multitudo ymaginum, sicut in speculis columpnalibus et spericis concavis, sicut dictum est in capitulo de ymaginibus. Accidit etiam in eis ut columpnalibus concavis, scilicet quod rectum videtur convexum et videtur concavum.

[9.2] Huius autem demonstratio est quia linee recte que extenduntur in longitudine speculi que transit per caput piramidis, et que sunt prope illas, videntur convexe, et videntur concave, et forte recte.

288 linea: line *S* 289 habuerit: habet *L3ER*/aut: vel *R* 290 habuerit[1]: buerit *S*/multas ymagines *transp. C1* 291 *post* ymagines[1] *add.* et punctum medium medium habuerit multas ymagines *S*/*post* linea *add.* tot *L3ER* 292 hoc: hic *L3*/speculorum spericorum *transp. C1*/spericorum *om. P1* 294 columpnalibus: columpnaribus *L3C1ER* (*alter. in E*) 295 spericis concavis (296) *transp. S* 296 specierum: specie tamen *S* 298 conversioni: reflexioni *R* 299 hec sunt: hoc est *ER*/que: quod *L3ER*/voluimus: volumus *P1E*/capitulo hoc (300) *transp. P1L3ER* 1 capitulum . . . concavis (2) *om. S*; de erroribus qui accidunt in speculis piramidalibus concavis capitulum nonum *R* 2 accidunt in speculis: in speculis accidunt *FP1* 3 *post* in[2] *add.* hiis *P1* 4 columpnalibus: columpnaribus *C1ER*/concavis *inter. E*/coloris et lucis: lucis et coloris *FP1* 5 et *om. L3*/*post* hiis *add.* speculis *P1*/*post* omnibus *add.* predictis *C1* 6 conversio: reflexio *R*/in *om. FP1* 7 columpnalibus: columpnaribus *C1R*/et . . . columpnalibus (9) *om. S* 8 sicut *om. R*/est *om. R*/ut: in *L3*; quod in *R*; ut in *mg. a. m. E* 9 columpnalibus: columpnaribus *R*/quod: ut *R*/videtur: videatur *R* 10 videtur *om. R* 11 autem: aut *FP1*/quia: quod *L3ER*/linee recte *transp. ER*/recte *om. FP1* 12 que: qui *L3*/transit: transeunt *R*/per caput *om. P1*/caput: verticem *R* 13 videntur[1] *corr. ex* videretur *a. m. E*/et videntur concave *mg. a. m. E*; *om. R*

[9.3] **[PROPOSITIO 37]** Et demonstratio super hoc est ut demonstratio in speculis columpnalibus concavis, nam si iteraverimus secundam figuram de fallaciis speculorum piramidalium convexorum, inveniemus dyametrum ymaginis linee recte posite in illo speculo, qui est illic linea AY intra concavitatem speculi piramidalis, et inveniemus punctum quod est sub superficie contingente piramidem transeuntem per lineam ex qua convertitur forma linee recte ad visum, quod illic punctum F.

[9.4] Si fuerit punctum centrum visus, erunt omnia puncta que sunt in dyametro ymaginis conversa ad punctum F, et ymagines duarum extremitatum A, Y erunt extremitates linee recte AN, et loca ymaginis puncti quod est in medio AY diversabuntur. Et hoc declarabitur eadem via qua processimus in demonstratione prime figure speculorum columpnalium concavorum.

[9.5] Patet ergo ex hoc quod si AY fuerit in aliquo visibili, et visus fuerit F, tunc ymago forte videbitur convexa, et forte concava. Et patet etiam in secunda figura de fallaciis speculorum columpnalium concavorum quod linee posite in latitudine speculi apparebunt concave concavitate mirabili, et quod ymagines linearum rectarum que sunt in superficiebus transeuntibus per axem et per centrum visus erunt recte.

[9.6] **[PROPOSITIO 38]** Item iteremus tertiam figuram de fallaciis speculorum spericorum concavorum eisdem litteris. Si ergo aliquod punctum fuerit in axe piramidis, et due linee EA, EB fuerint perpendiculares super superficies contingentes piramidem (et hoc est possibile, quia sunt equales, possunt enim cum axe continere duos angulos acutos equales), cum ergo hee due linee fuerint perpendiculares, et visus fuerit D, tunc superficies in qua sunt linee GE, ED transibit per totum axem et per centrum visus.

15 speculis: spericis *E*/columpnalibus: columpnaribus *R*/iteraverimus: superconverimus *S*; iteravimus *E* 16 figuram: regulam *E* 17 illo *om. S* 18 qui: que *R*; *corr. ex* que *O*/ AY: AI *FP1SOE*; AL *L3*; AN *R*; *corr. ex* AI *a. m. C1*/et *om. L3E* 19 piramidem: piramidalem *S* 20 transeuntem: transeunte *S*/*post* lineam *add.* longitudinis *R*/ex: a *R*/convertitur: reflectitur *R* 21 *post* quod *inter.* est *O*/punctum *scr. et del. E* 22 si *om. FP1*/*post* si *add.* igitur *R*/*post* fuerit *inter.* positum *O*/punctum *om. O*/*post* punctum *add.* illud *R* 23 conversa: reflexa *R* 24 *post* A *add.* P *R*/Y: I *FP1OL3C1E*; L *S*/linee recte *transp. FP1* 25 *post* puncti *add.* P *R*/est *om. FP1O*/*post* medio *add.* puncti *FP1* 26 qua: quia *S* 27 columpnalium: columpnarium *R* 28 ergo *inter. C1*/ex hoc *inter. E*/AY: APY *R* 30 secunda figura *transp. ER*/columpnalium: columpnarium *R* 31 *post* concavorum *add.* et *P1* 32 rectarum *om. L3R* 33 sunt *inter. a. m. E*/transeuntibus: transeuntes *L3* 37 piramidis: piramidalis *S* 38 superficies contingentes: superficiem contingentem *L3*/*post* superficies *add.* concavas *P1* 39 possibile: impossibile *E*/ equales *om. P1* 41 visus fuerit *transp. ER*/GE: HE *FP1SOL3E* 42 transibit: transit *FP1*

[9.7] Et utraque superficies DAM, DBR erit declinis super axem piramidis, et erunt differentie earum due sectores piramidis. Et erit forma punctorum R, H, Q conversa ad D ex B, et forme punctorum L, M, F convertuntur ad D ex A. Cum ergo linee MR, LH, FQ fuerint in aliqua superficie visibili, et visus fuerit in D, tunc NU erit ymago MR, et TK erit ymago LH, et CI erit ymago FQ.

[9.8] Sic ergo ymago MR erit minor se ipsa, et ymago FQ maior se ipsa, et ymago LH equalis sibi ipsi, et omnes ymagines erunt converse.

[9.9] Et si visus fuerit in O et NU, TK, CI fuerint in superficiebus visibilium, tunc ymagines earum erunt MR, LH, FQ. Sic ergo erit ymago CI minor se ipsa, et ymago NU maior, et ymago TK equalis.

[9.10] Et iste ymagines erunt recte, nam iste ymagines erunt ultra centrum visus et comprehendentur ante visum super lineas radiales. Puncta ergo M, L, F comprehenduntur in linea AO, et puncta R, H, Q comprehenduntur in OB, et sic forma revertetur recta.

[9.11] Patet ergo ex hiis que diximus in hoc capitulo quod linee recte quandoque videntur in hiis speculis convexe, quandoque concave, quandoque recte, et quandoque maiores, et minores, et equales, et quandoque recte, et converse.

[9.12] Et in capitulo de ymagine declaravimus quod omne punctum visibile in huiusmodi speculis quandoque habet unam ymaginem, quandoque duas, et tres, et quattuor. Omnia ergo que comprehenduntur in huiusmodi speculis accidit in eis fallacia ut in columpnalibus concavis, et accidunt in eis etiam fallacie composite sicut in ceteris speculis. Et exempla et declaratio eorum sunt sicut in speculis planis. Et hoc intendimus declarare in hoc capitulo. Nunc autem finiamus sextum tractatum.

43 DAM DBR: DAO DBO *R* 44 *post* differentie *scr. et del.* ei *F* / earum *corr. ex* eorum *E* / due: duo *E* / sectores: sectiones *R* / erit forma (45): erunt forme *R* 45 R: B *E* / H: G *FP1OL3E;* E *SO* / R H *transp. R* / conversa: reflexe *R* / B: F *SOL3E; om. FP1* / et *inter. P1* 46 convertuntur: reflectentur *R* / D: T *L3* / cum *corr. ex* sum *F* / linee *om. S* / MR: MLR *FP1L3E;* LMR *S;* MLF *R* / LH: LG *O; om. FP1SL3ER* / FQ: GFQ *FP1SL3E;* RHQ *R* / fuerint: uerint *L3* 47 *post* D *scr. et del.* ex A *S* 48 et[1]... MR (49) *om. FP1* / TK: CR *L3* / LH ... ymago[2] *mg. a. m. E* / CI: PI *R* 49 MR *corr. ex* LH *S* 50 ipsi *om. P1* 52 CI: PI *R* 53 earum *om. P1* / erit *om. L3C1* 54 CI: Q *FP1SL3E;* FQ *R* / *post* CI *add.* erit *C1* / minor: maior *O; corr. ex* maior *a. m. E* / minor se ipsa: se ipsa minor *FP1R* / NU: MC *L3E* / et ymago[2] *inter. O* / TK: THK *S* 55 *post* recte *add.* N *S* 56 comprehendentur: comprehenduntur *R* 57 M L *transp. deinde corr. mg. E* / R: T *L3;* C *E* 58 OB: OBF *FP1SL3E;* OF *deinde inter.* B in arabico *O* / sic: si *FP1L3* / revertetur: reflectetur *R* 59 ex *om. FP1* 60 quandoque[2] *om. S* 61 et[1] *om. FP1* / *post* quandoque[2] *add.* minores *P1; scr. et del.* minores *F* 62 et[1] *om. FP1* / et[2] *om. L3ER* 63 *ante* et *add.* se *FP1* / declaravimus ... ymaginem (64) *mg. O* 64 huiusmodi speculis: huius speculum *L3* / *post* habet *scr. et del.* i *P1* / unam ymaginem *transp. C1* 65 *post* quandoque *add.* tres *P1* / et[1]: quandoque *P1C1* / omnia: in omnibus *R* 66 huiusmodi: huius *OL3ER* / in eis *om. R* / *post* fallacia *inter.* similis *O* / in[3] *om. FP1* / columpnalibus: columpnibus *L3;* columpnaribus *R* 67 et *om. ER* / accidunt: acciduntque *ER* / in eis etiam: etiam in eis *R* 68 sunt *mg. F; om. S* 69 hoc[2] *om. S*

FIGURES FOR TRANSLATION AND COMMENTARY

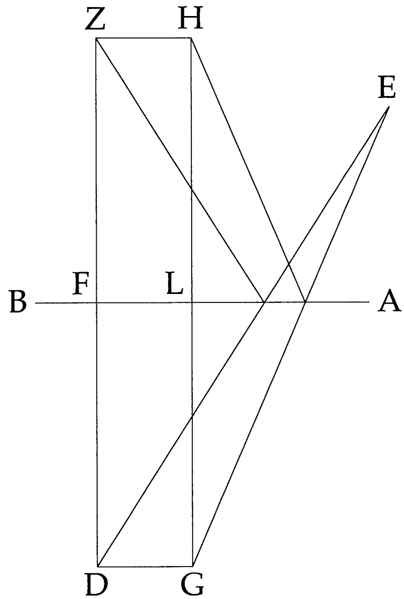


figure 6.3.1

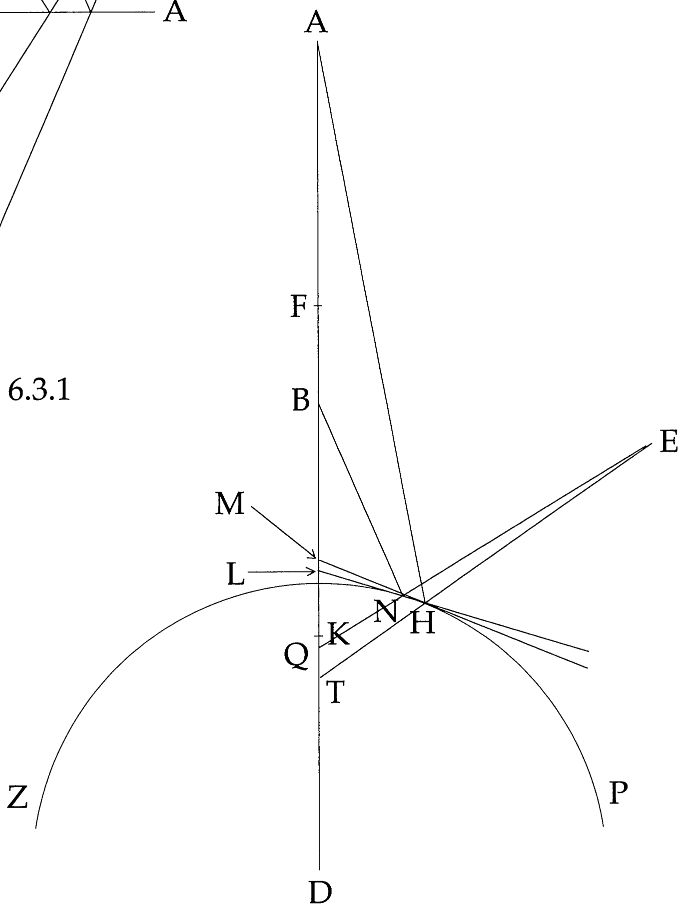
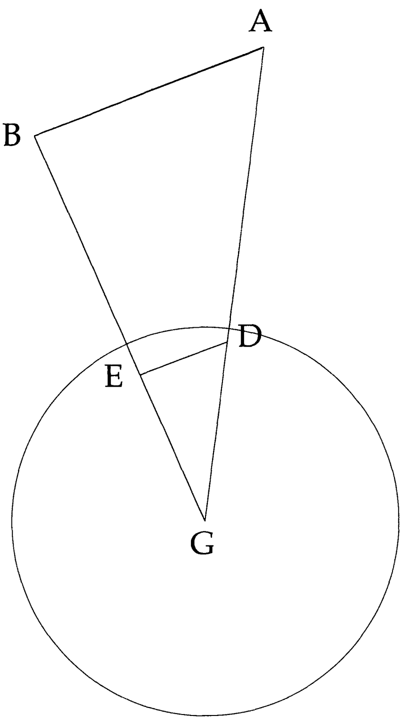


figure 6.4.2



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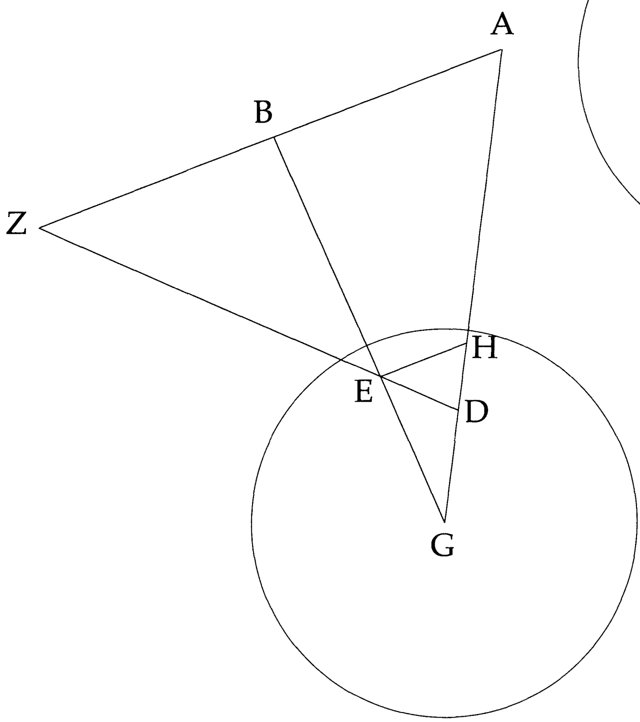

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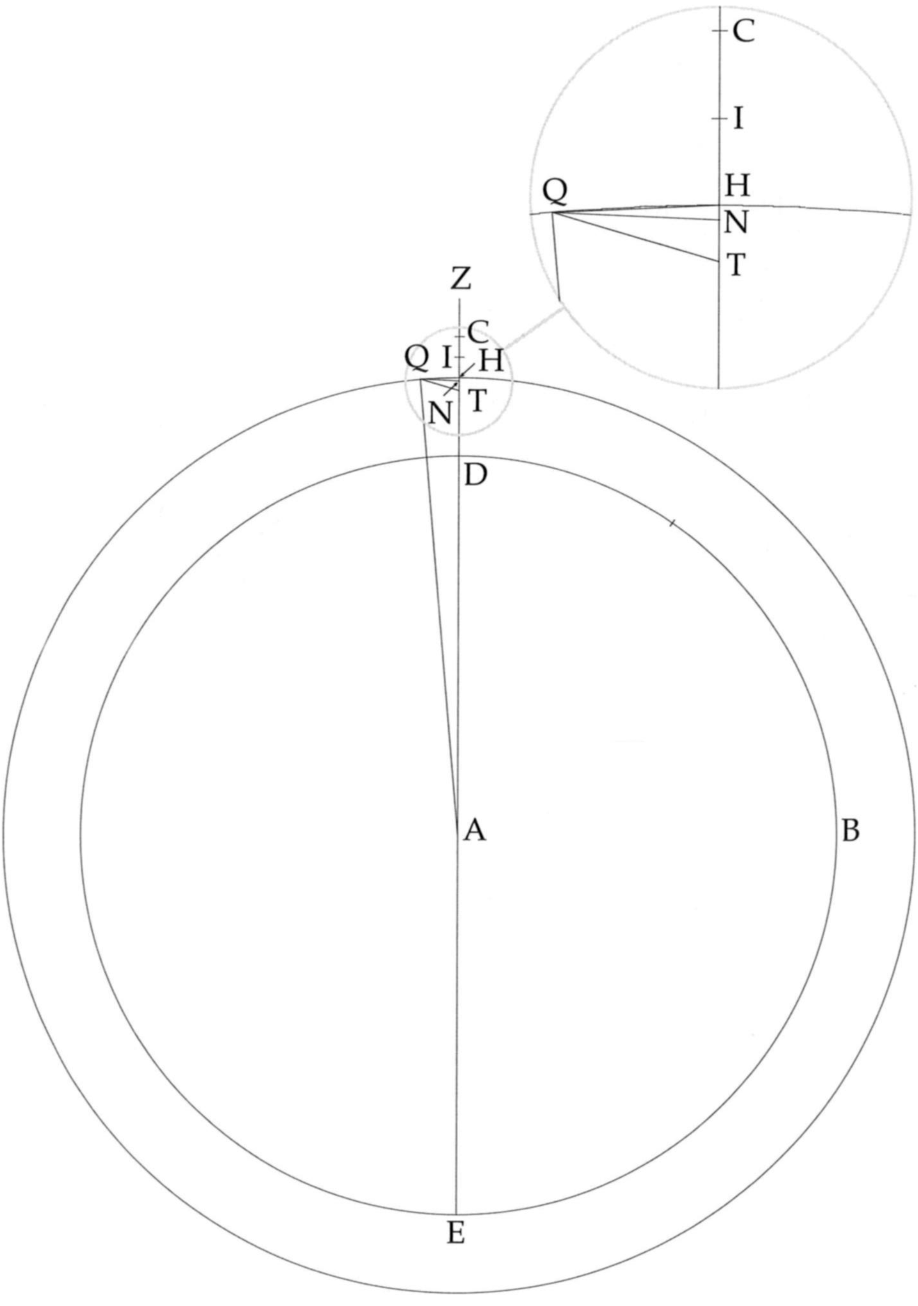

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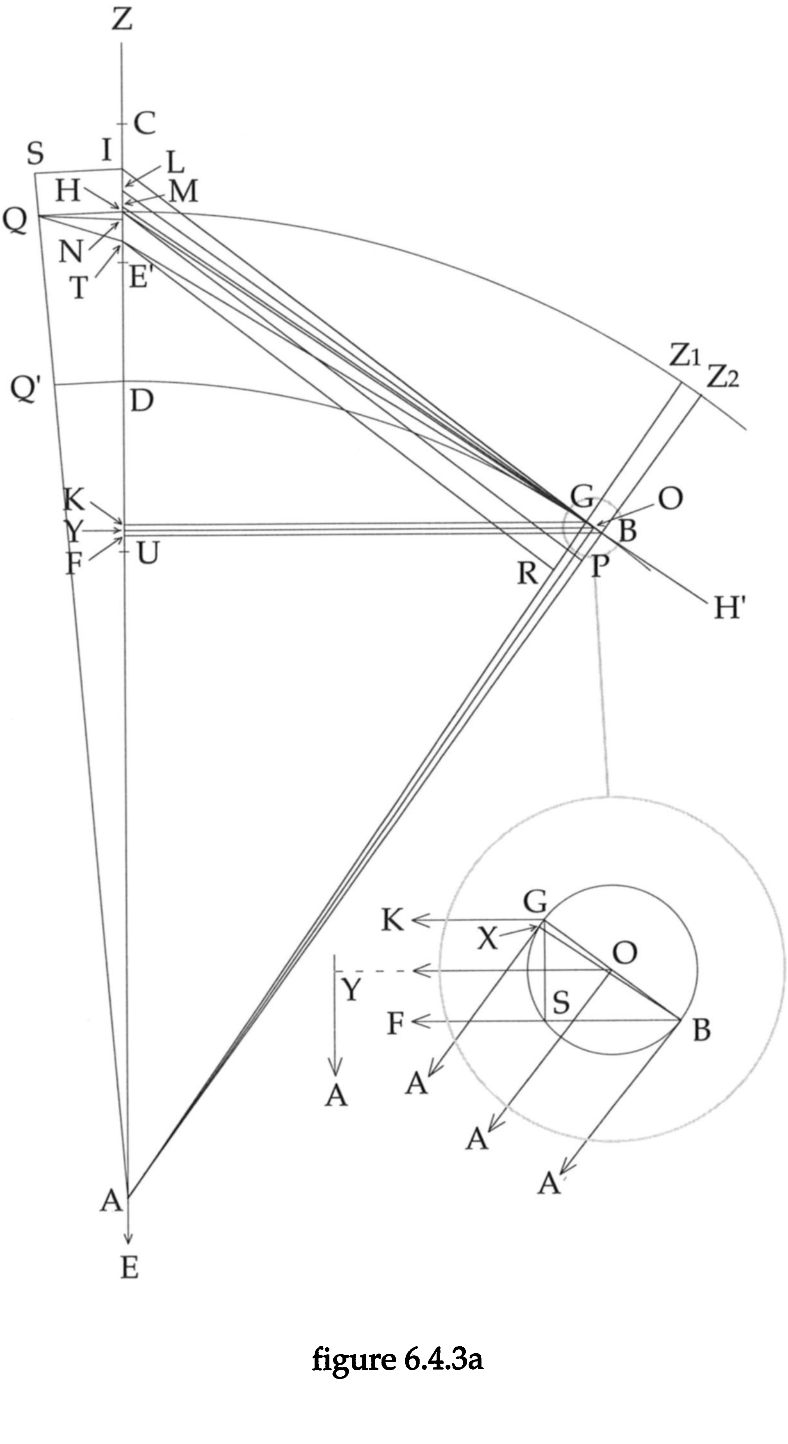

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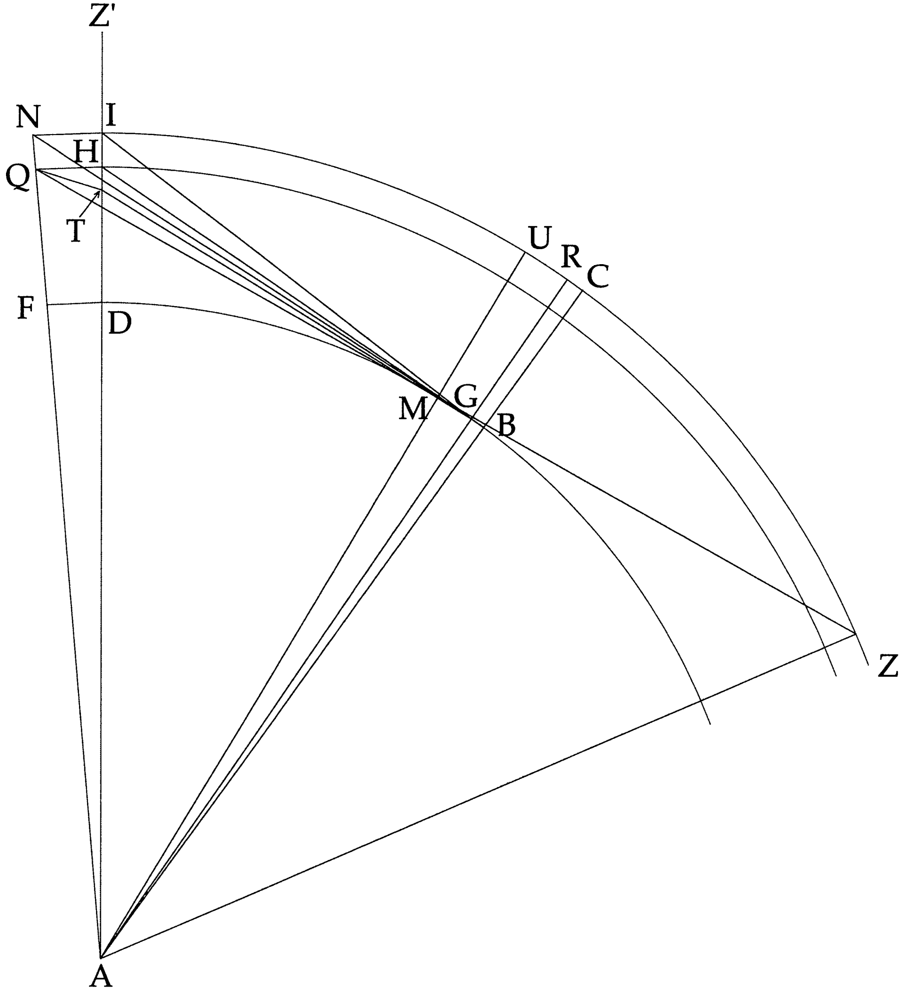

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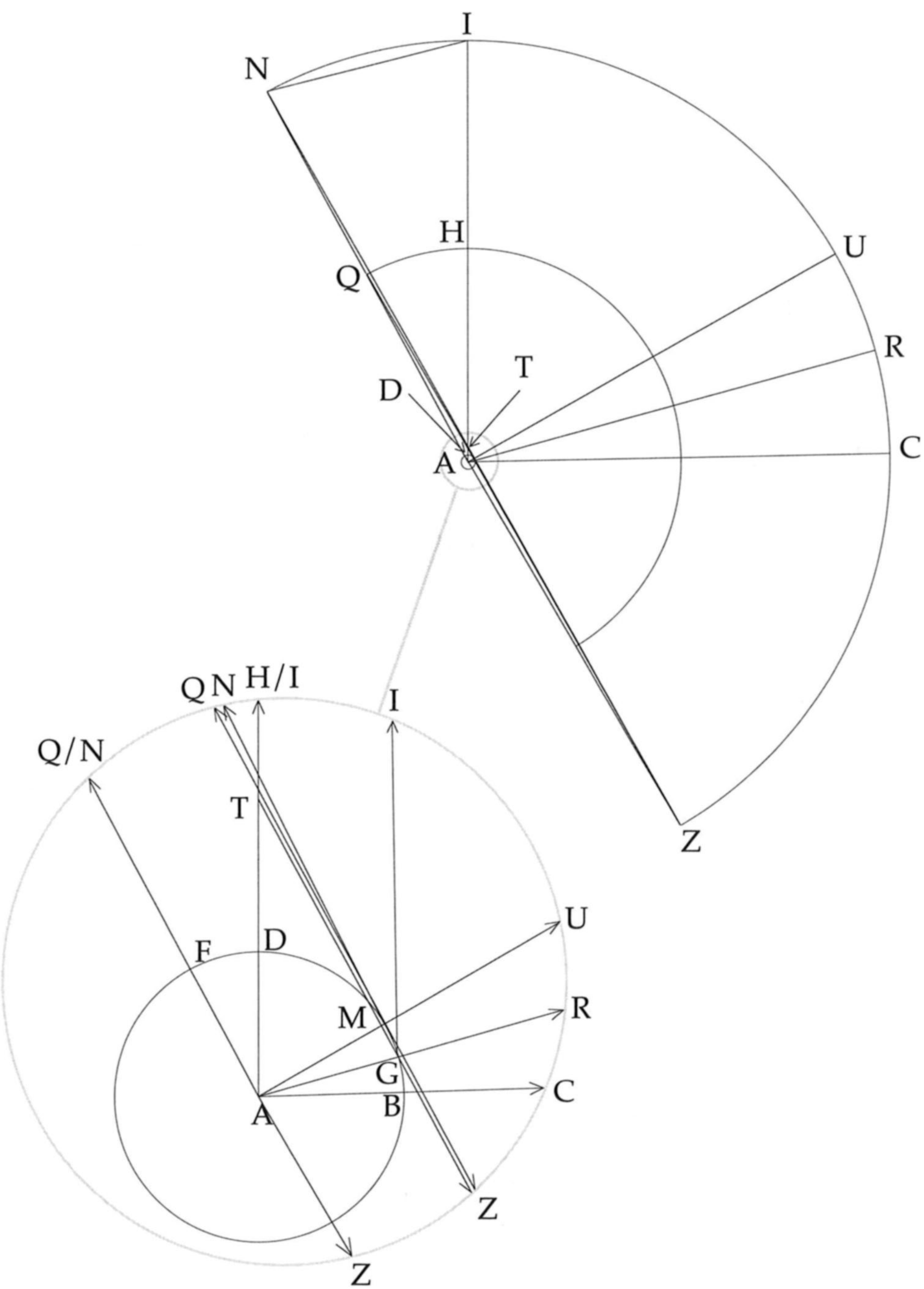

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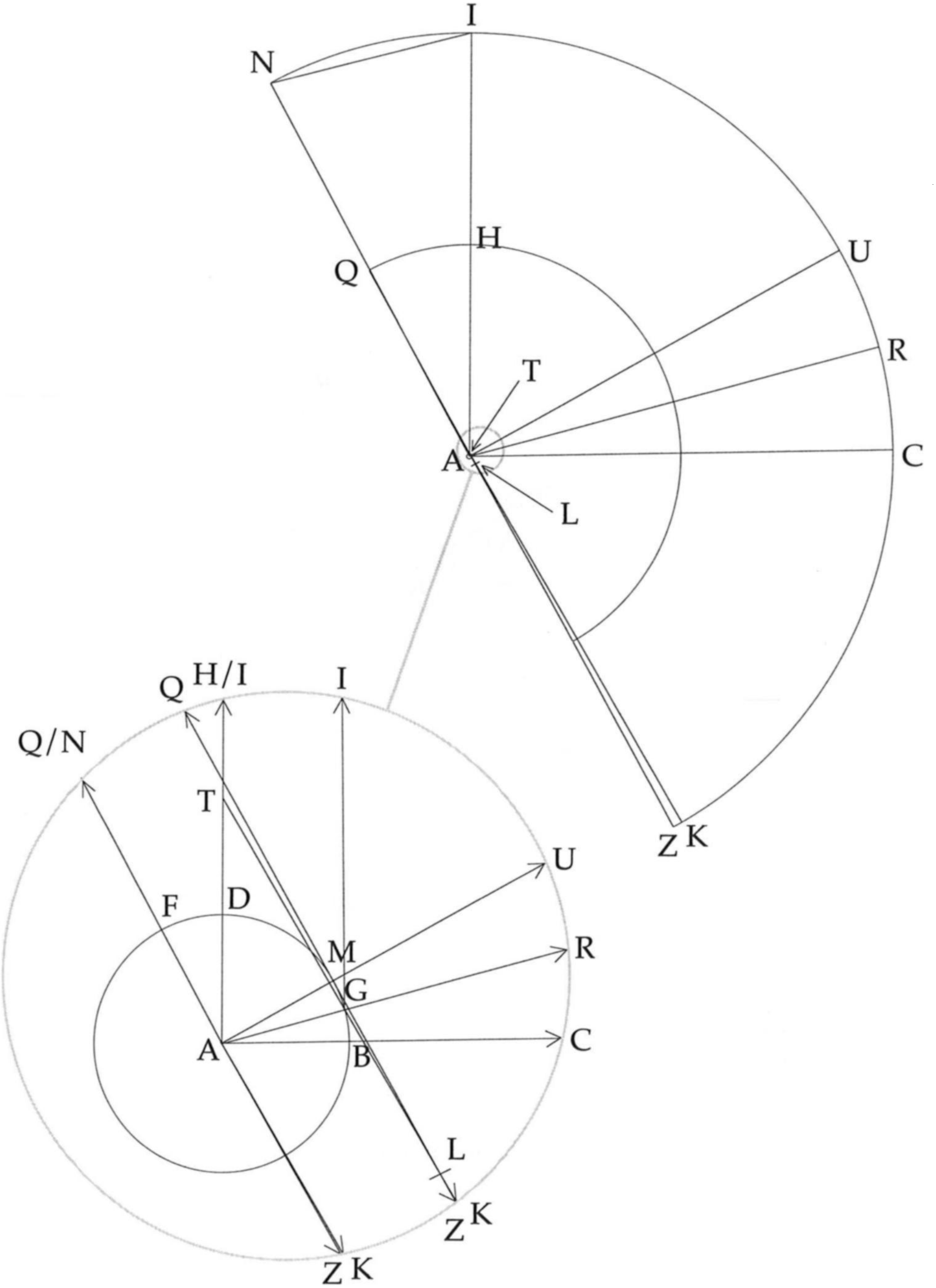

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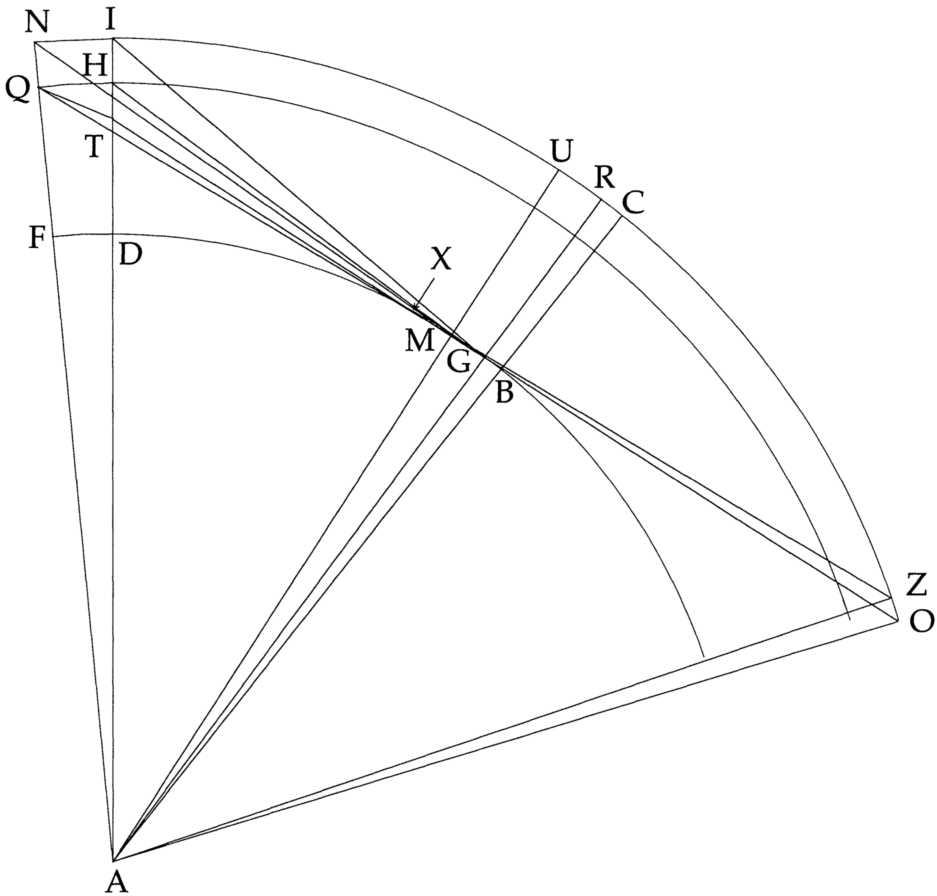

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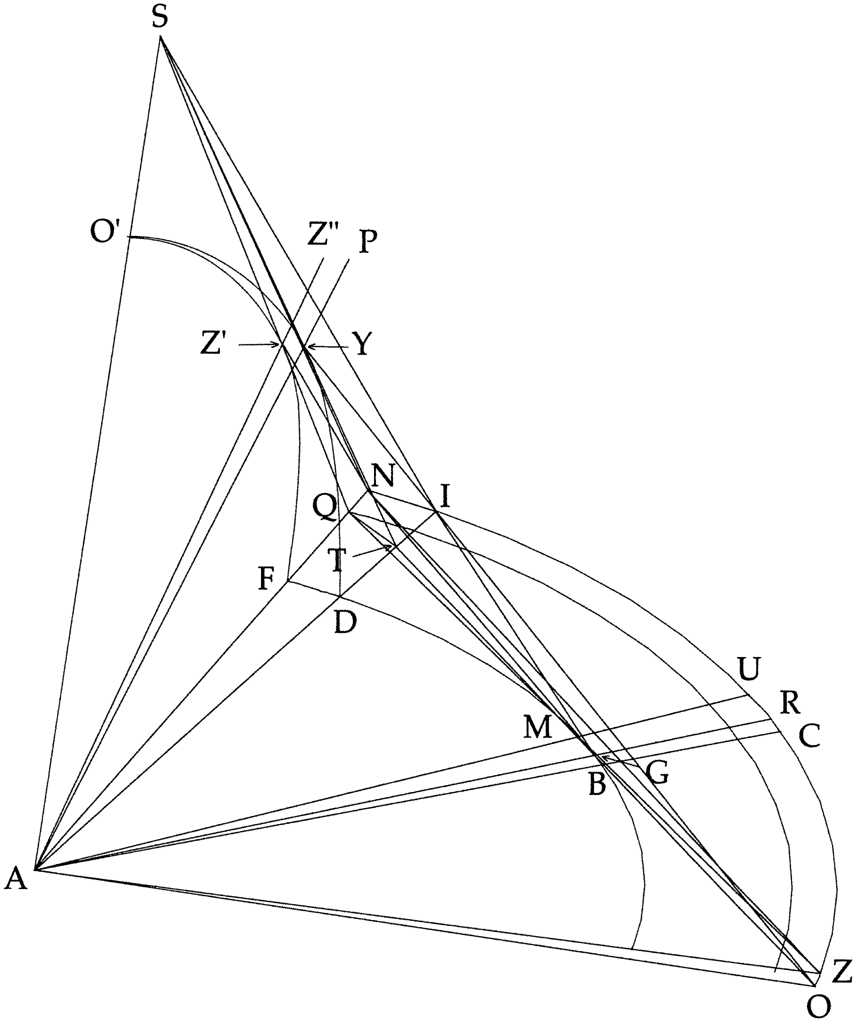

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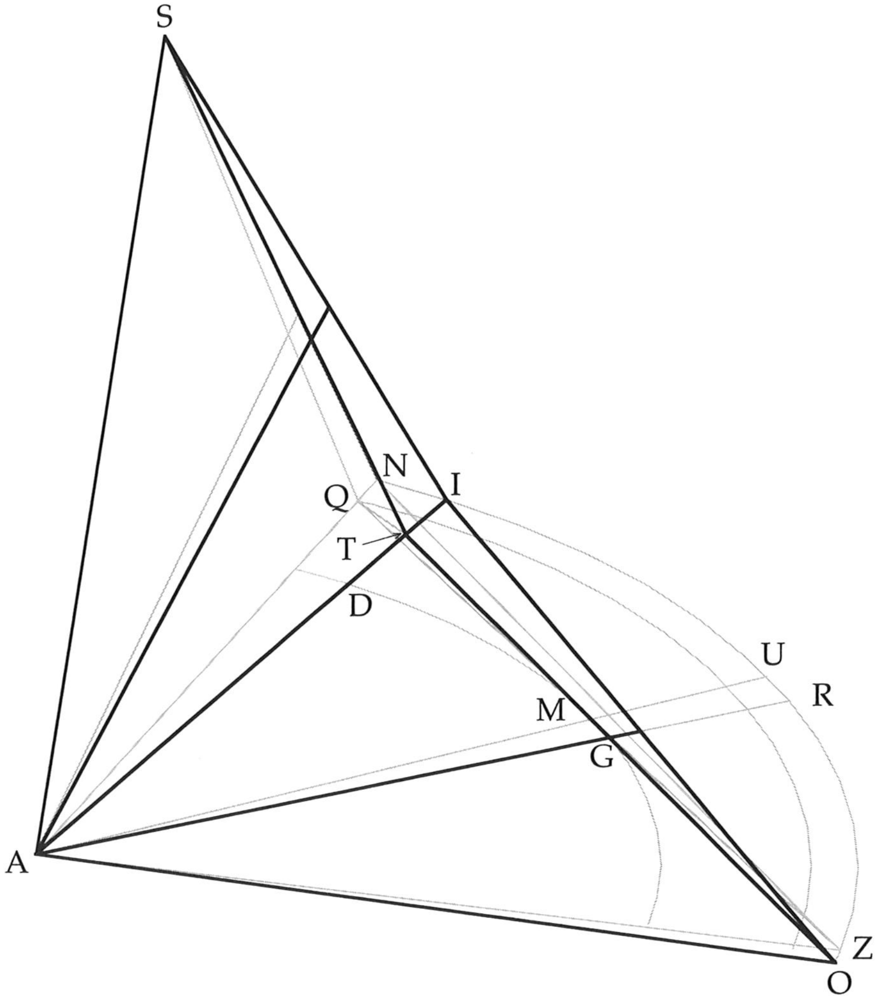

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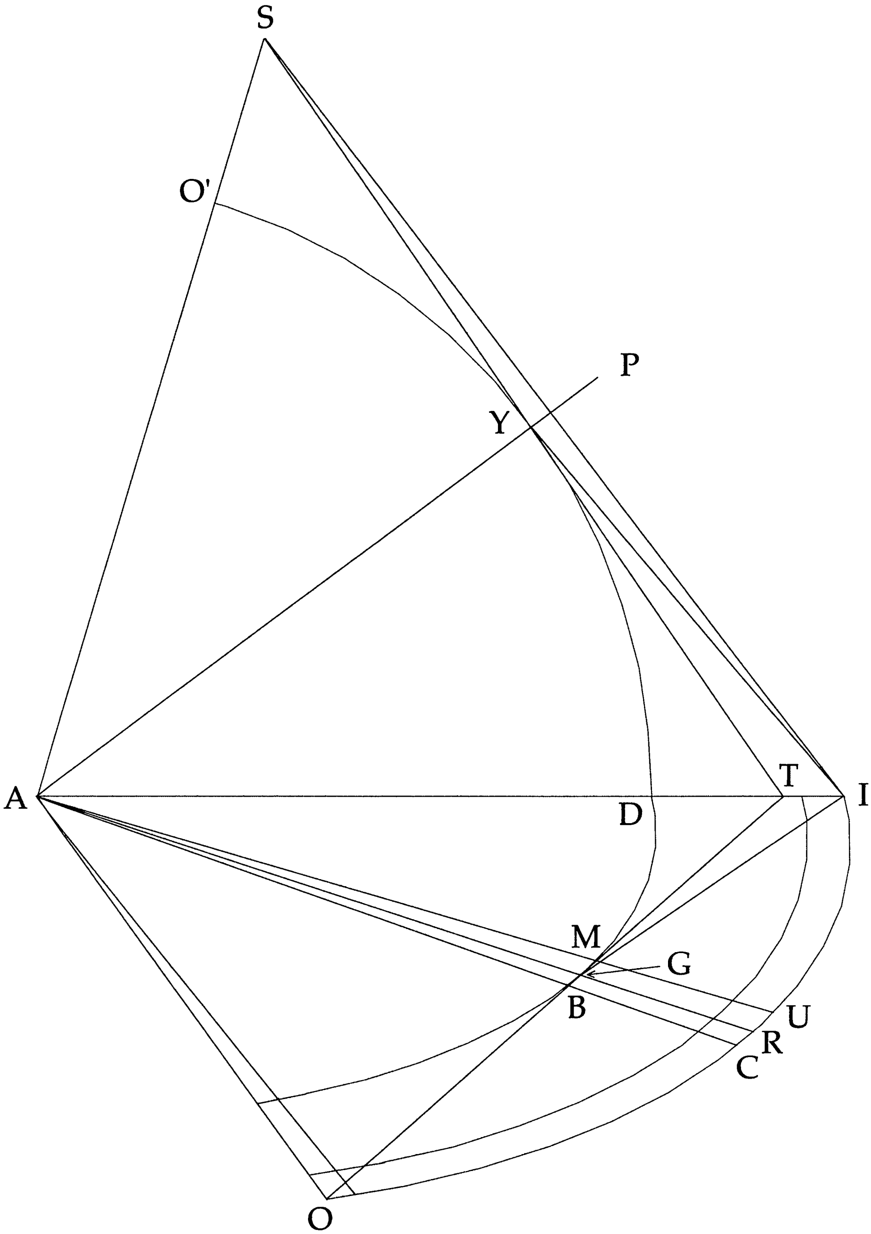

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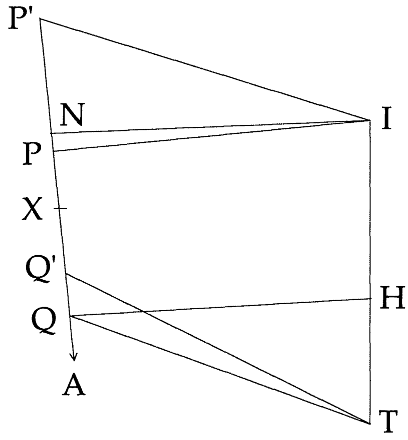

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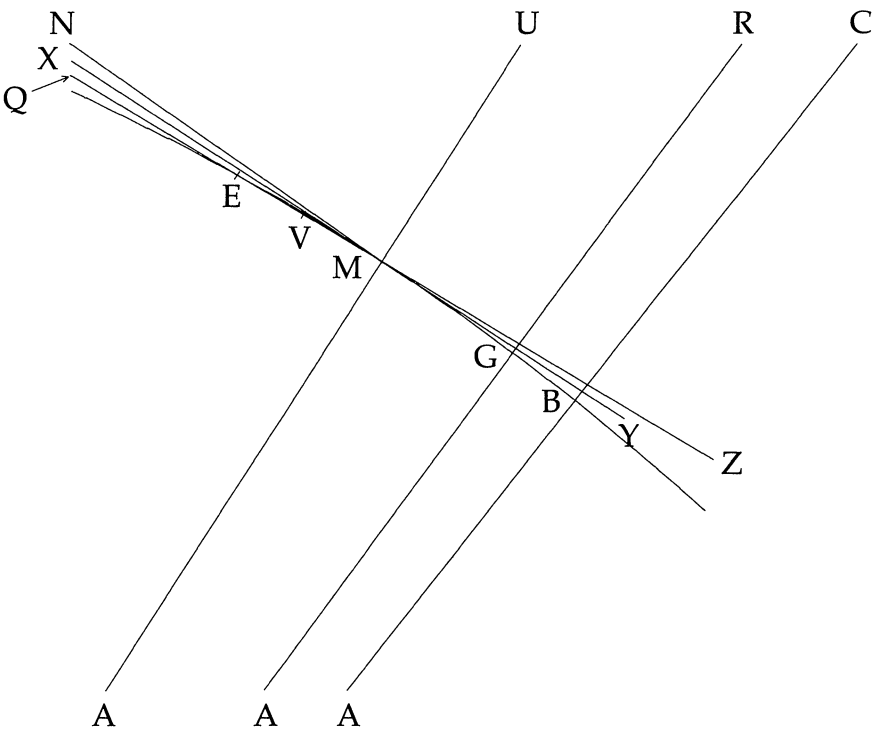

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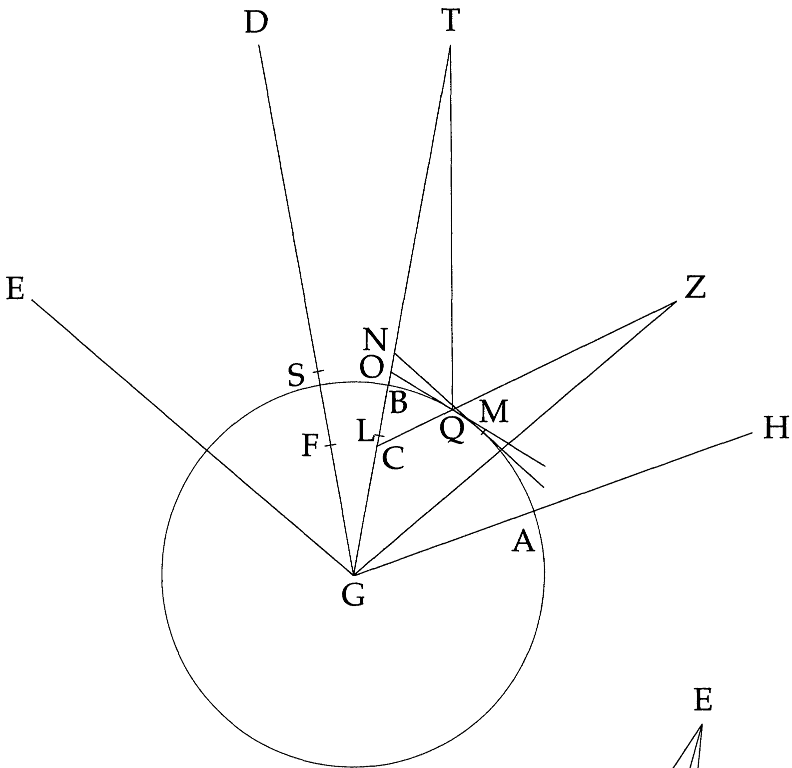

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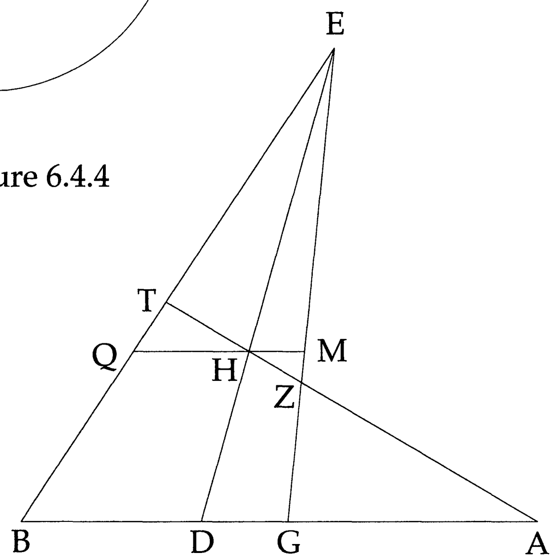

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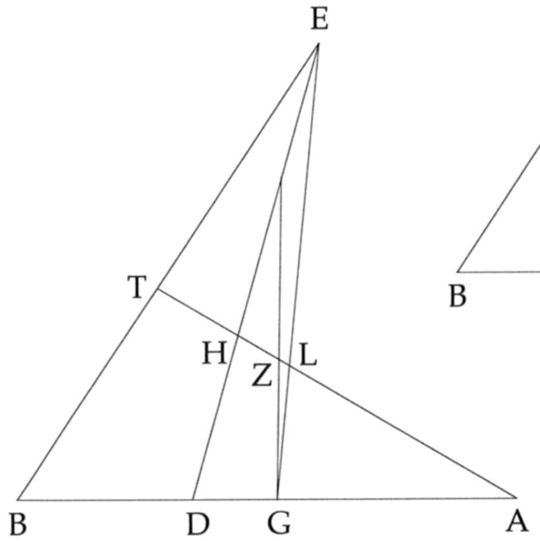

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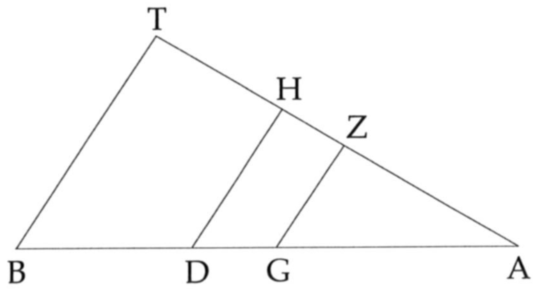

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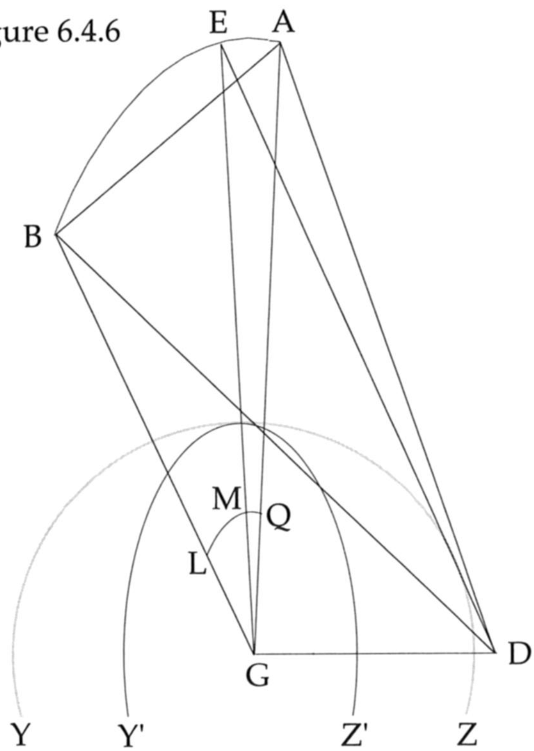

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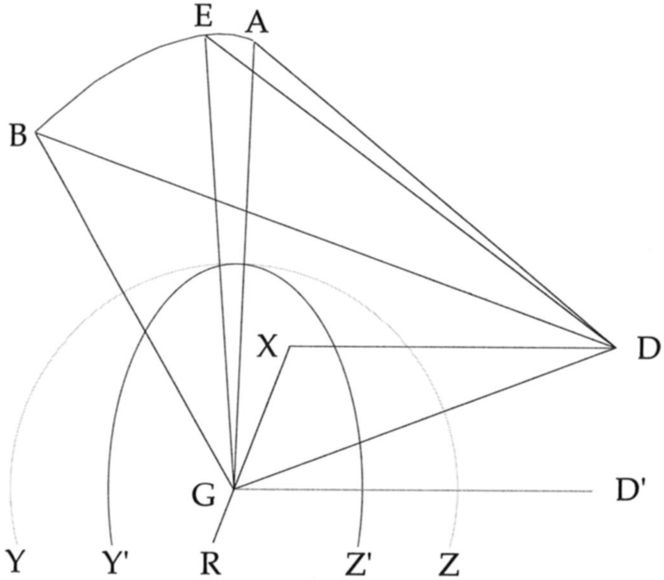

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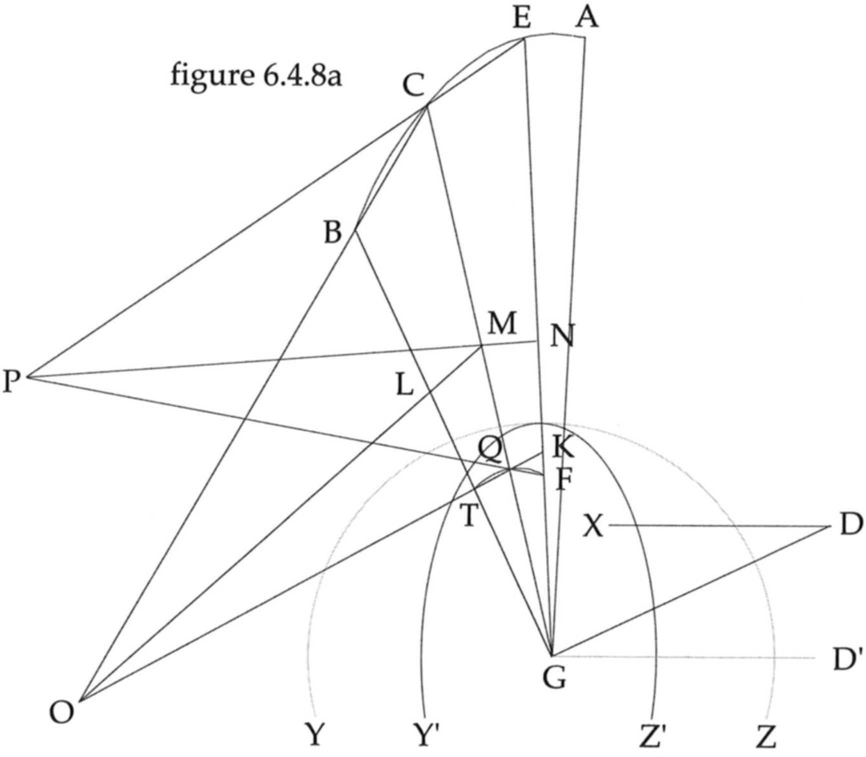

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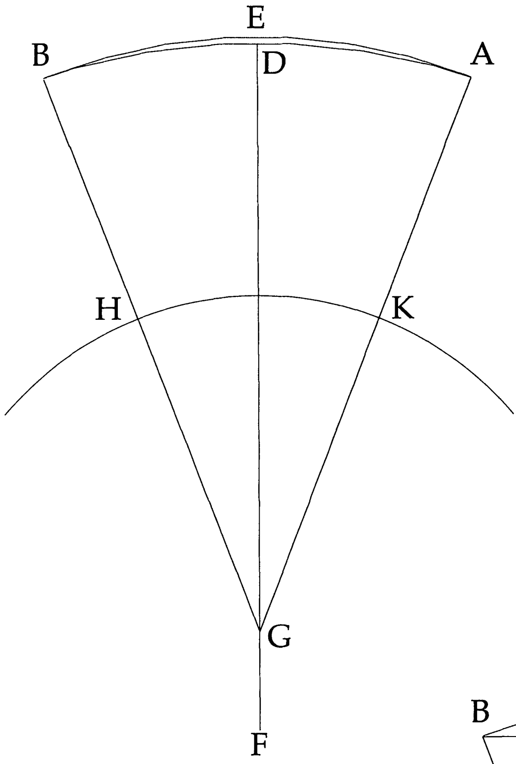

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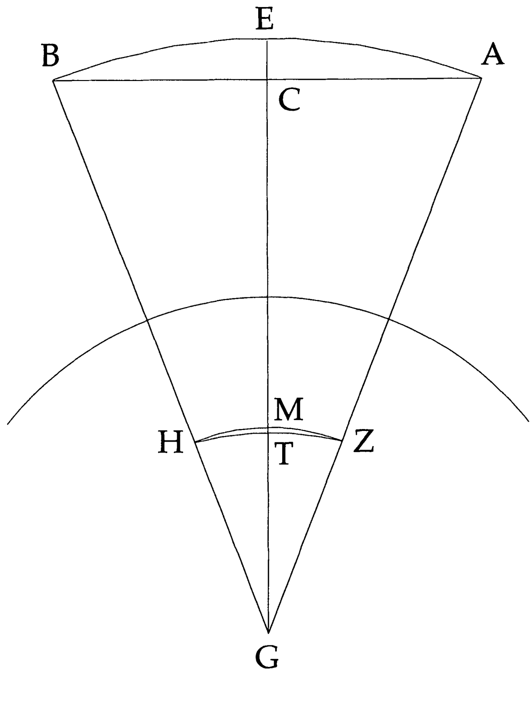

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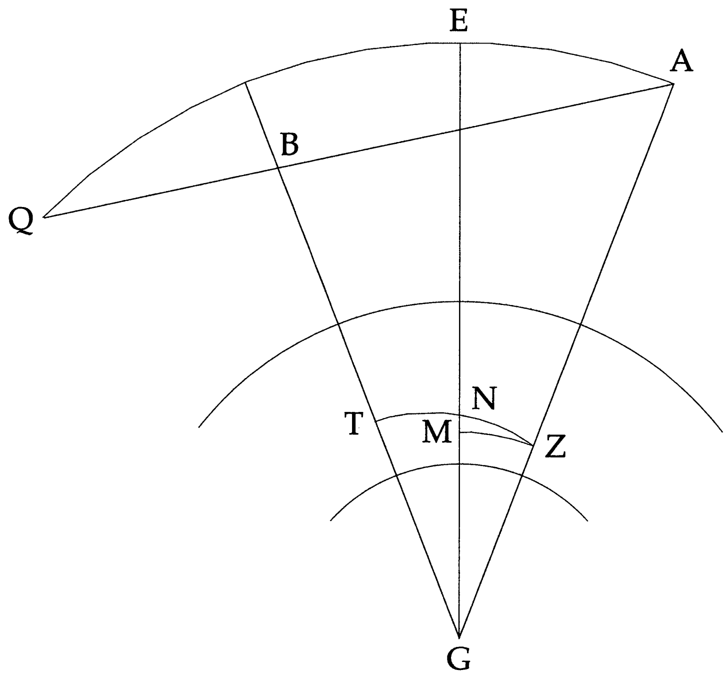

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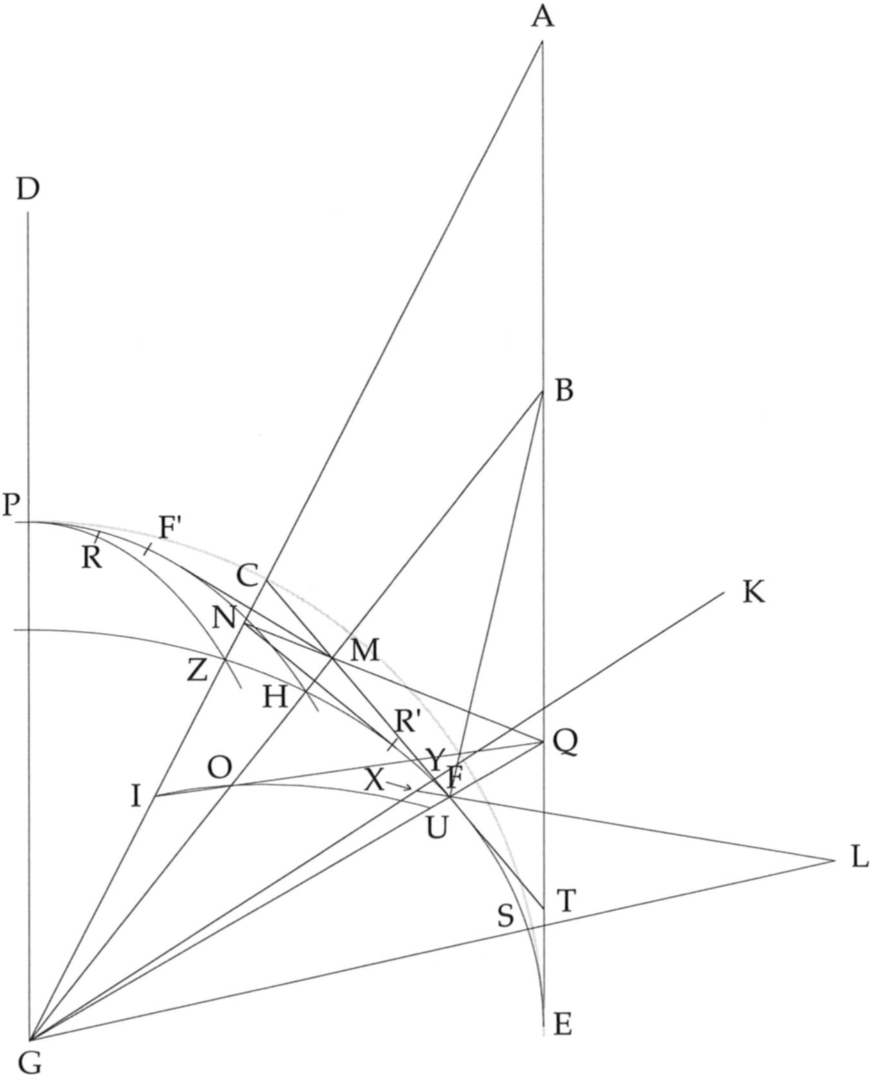

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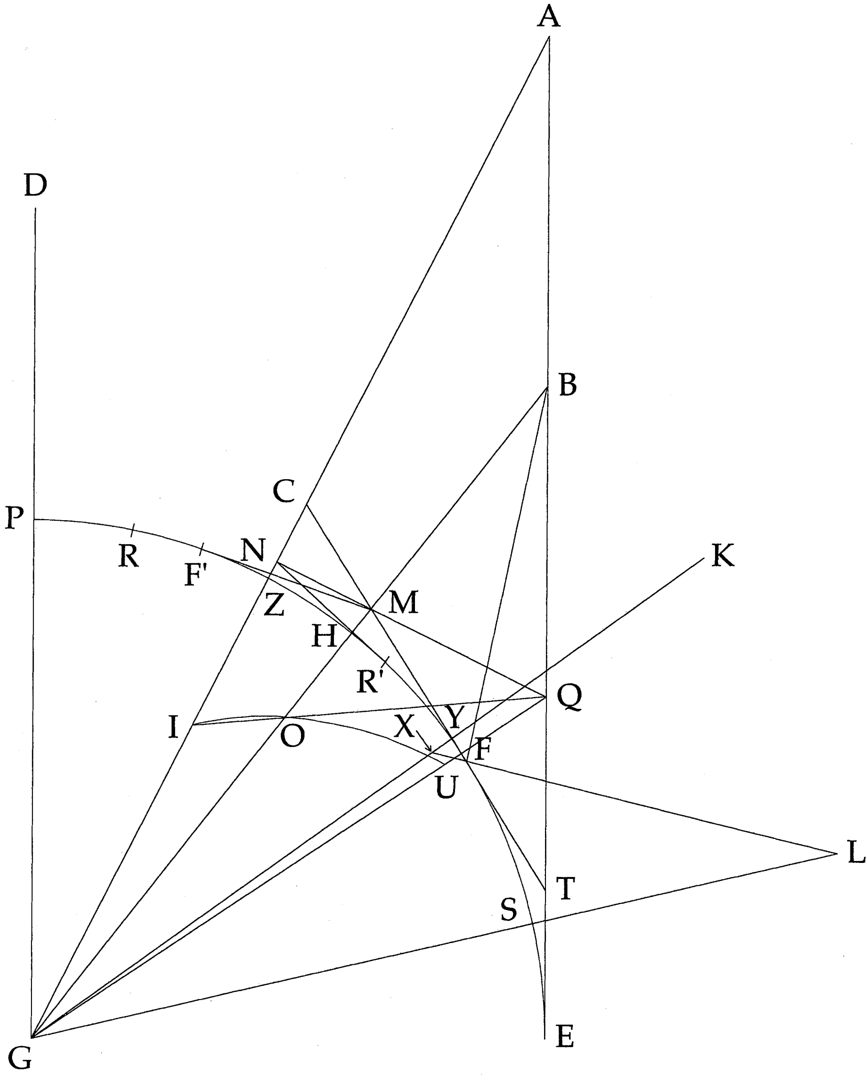

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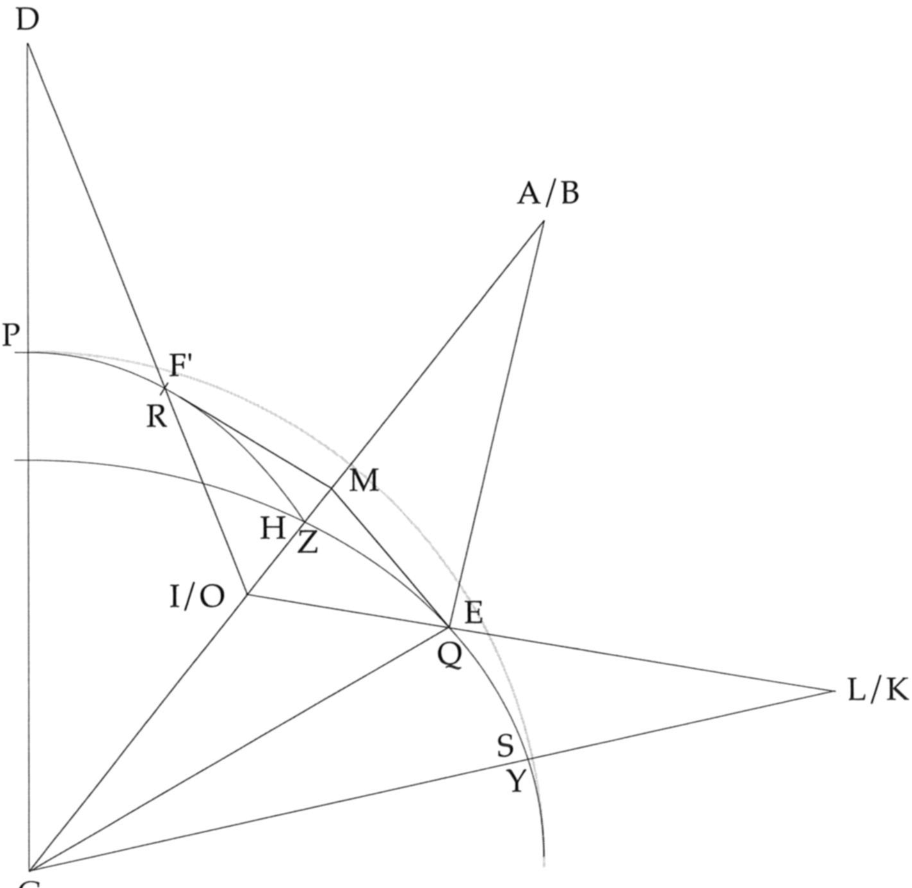

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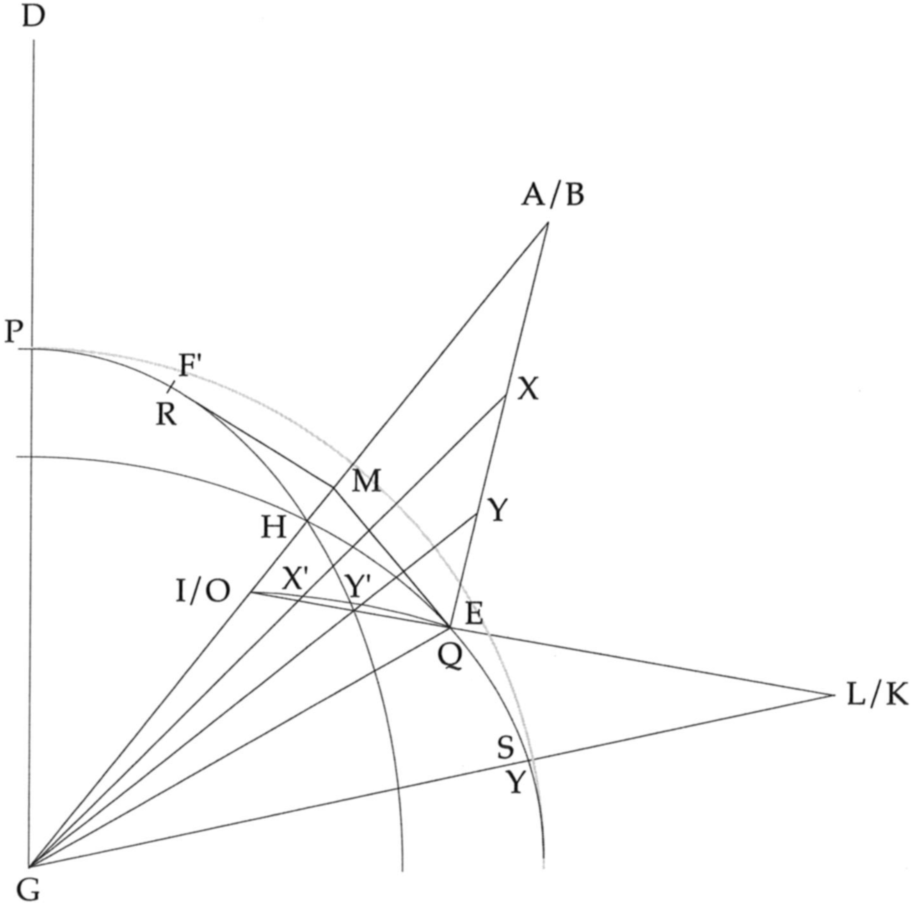

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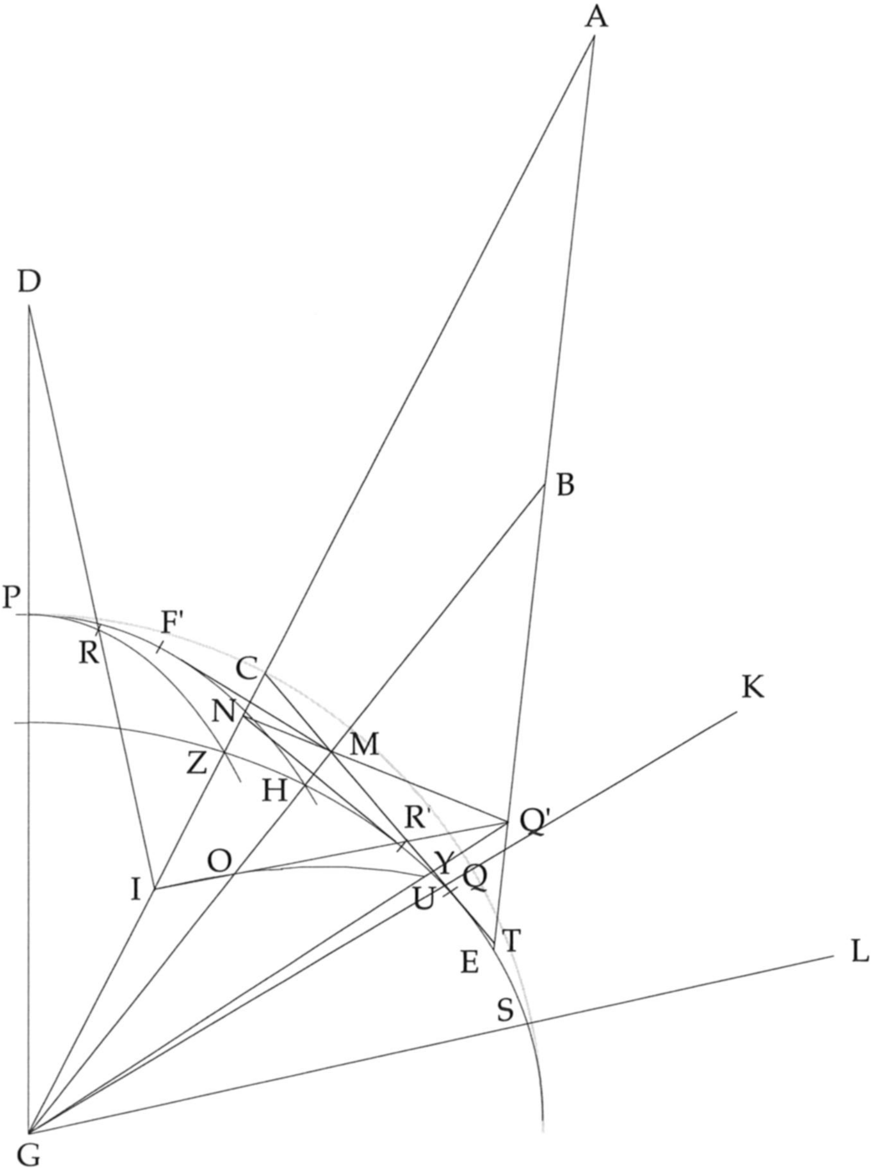

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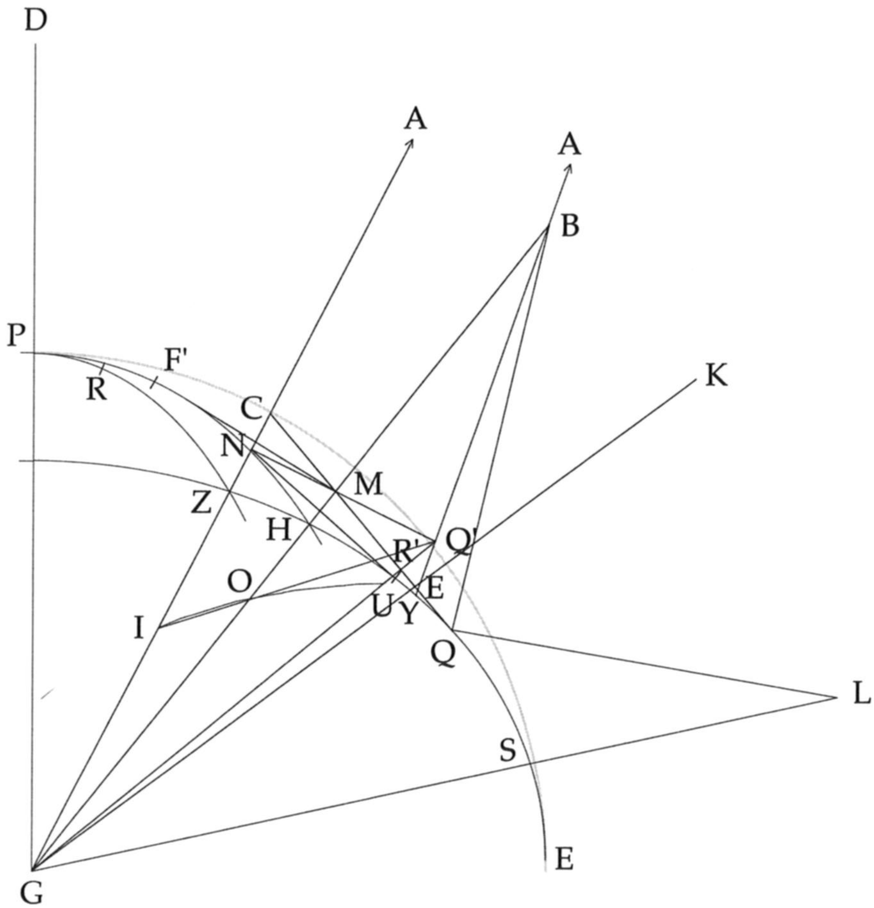

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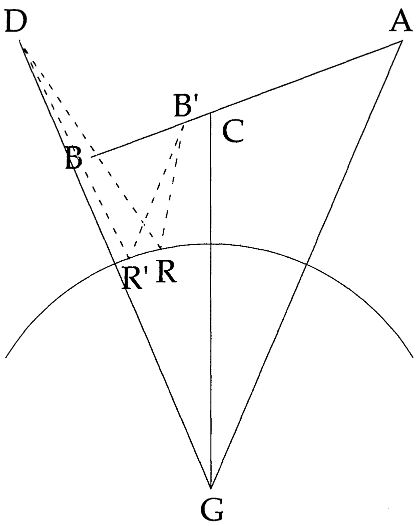

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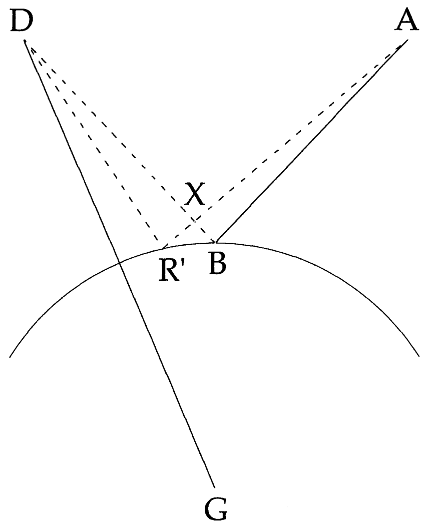

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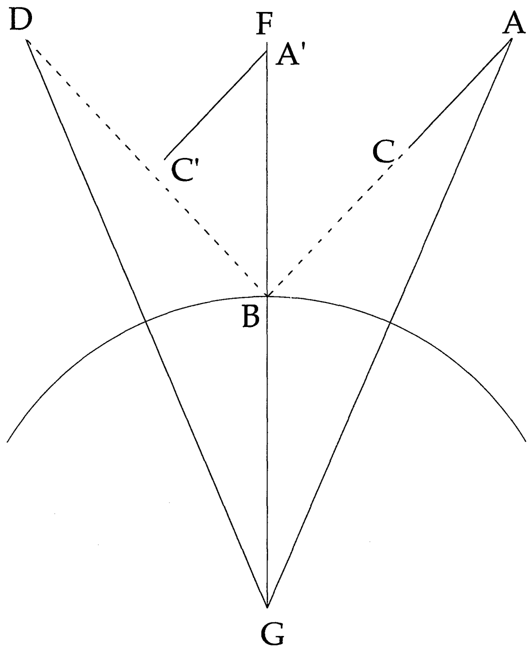

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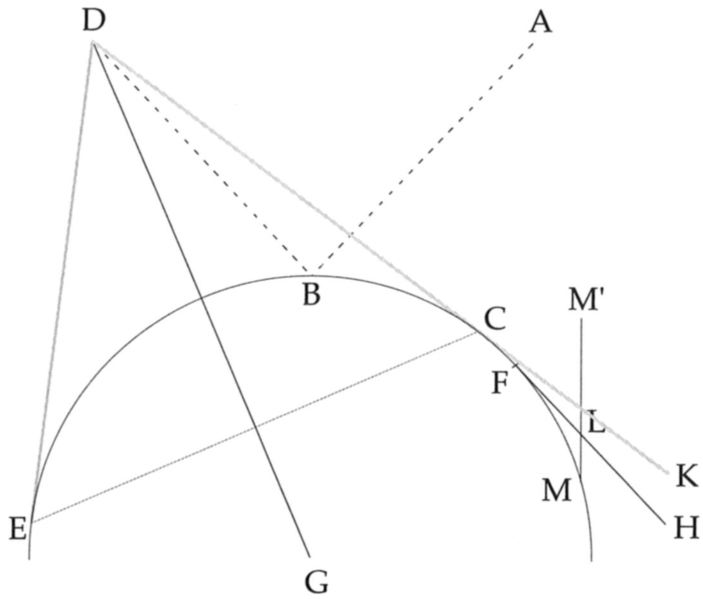

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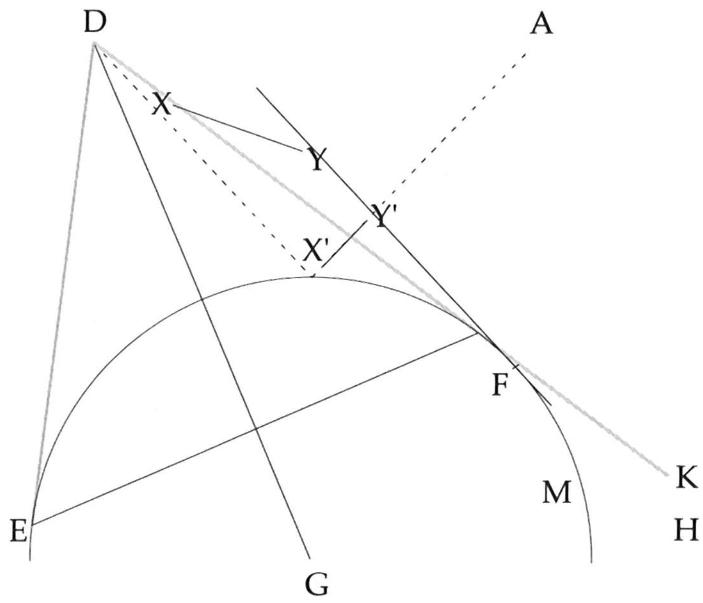

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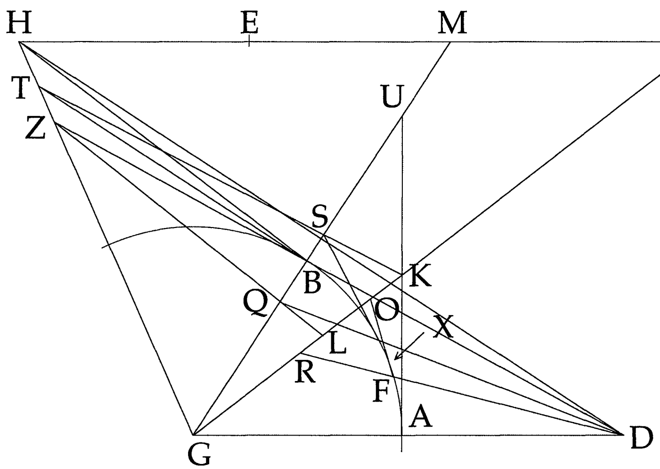

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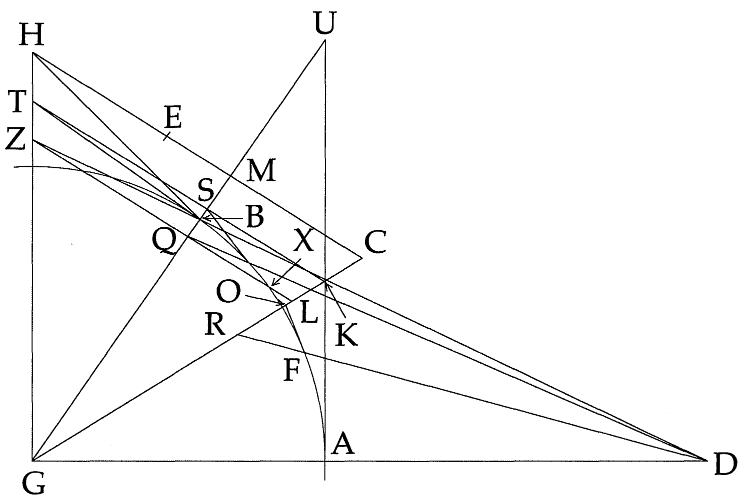

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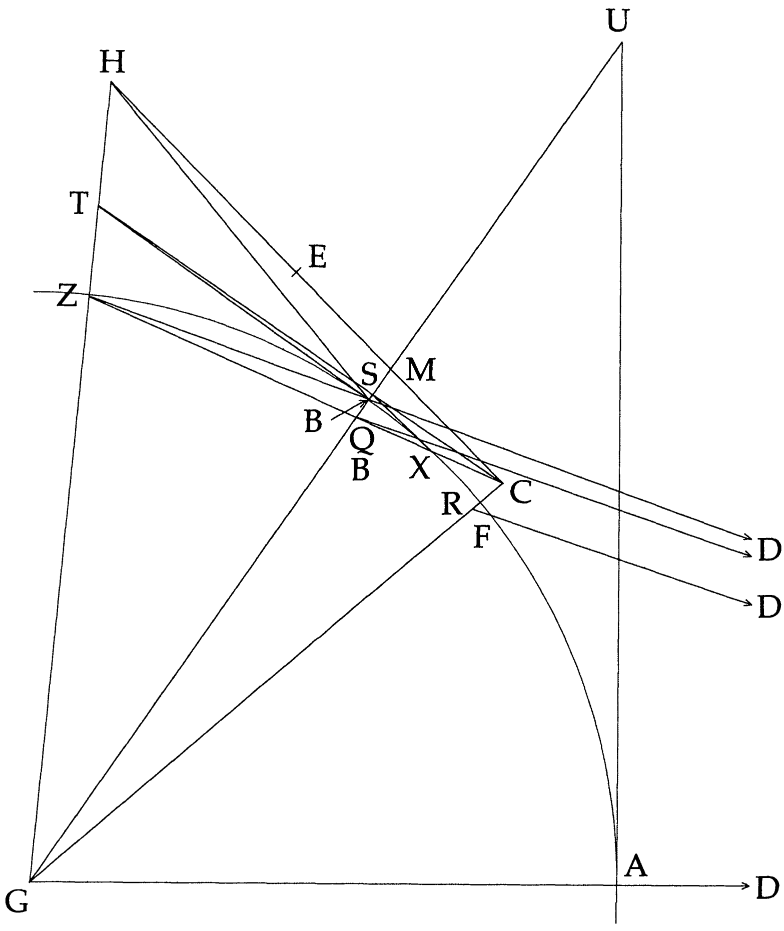

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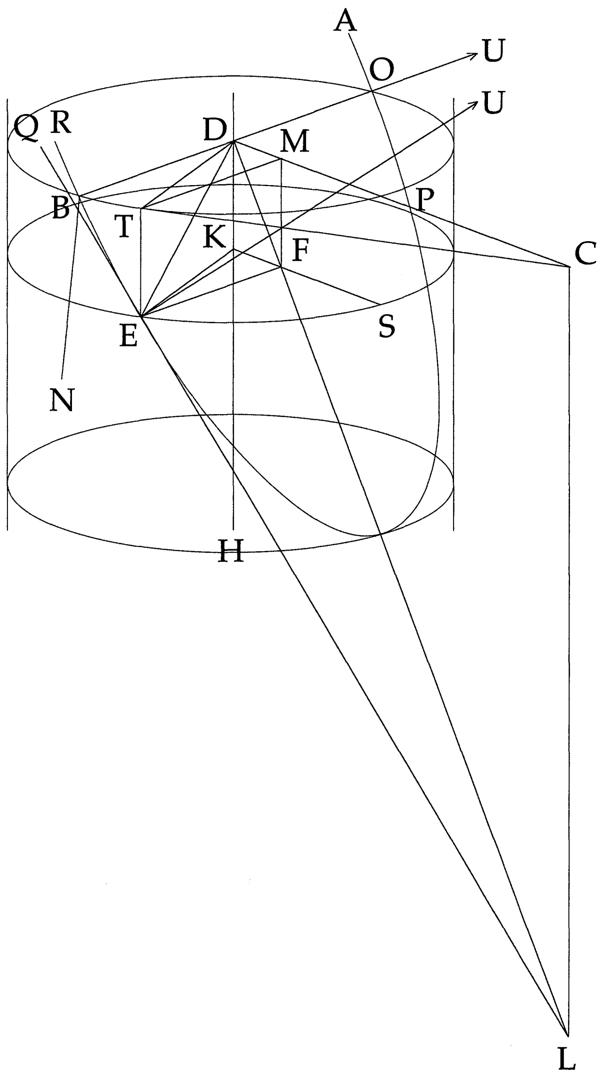

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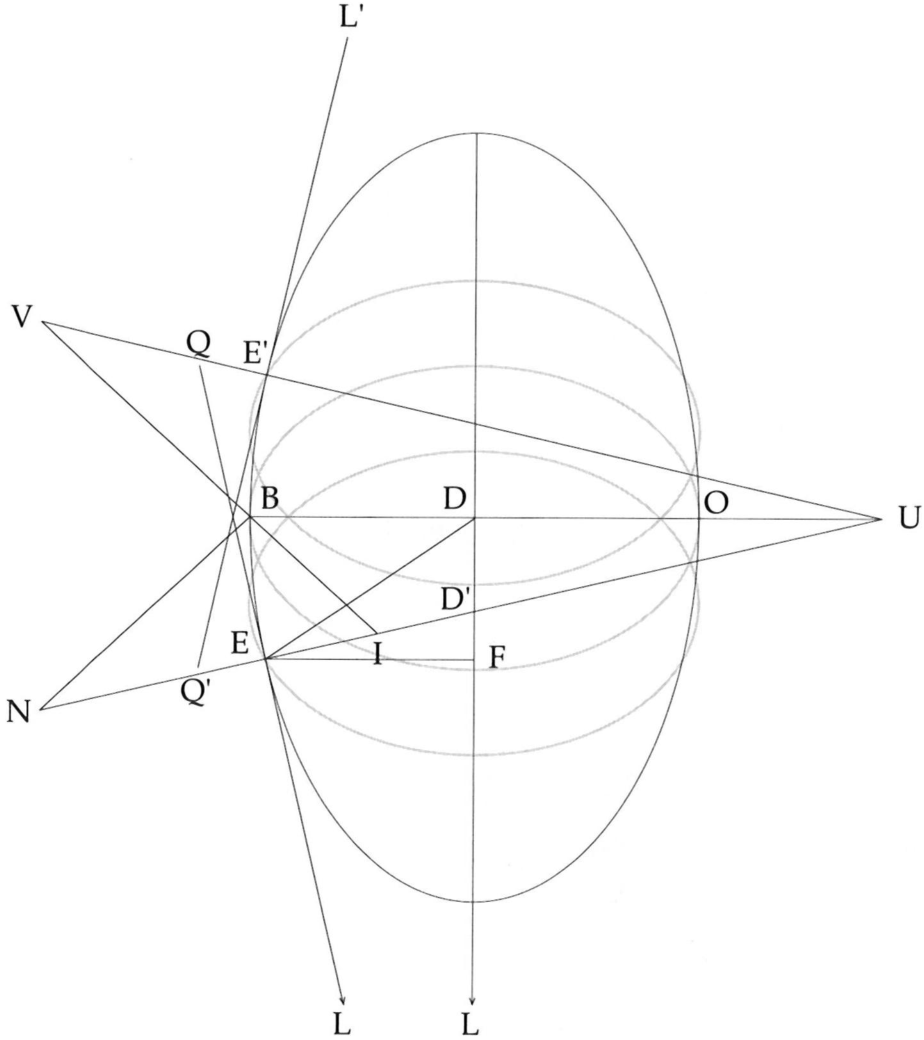

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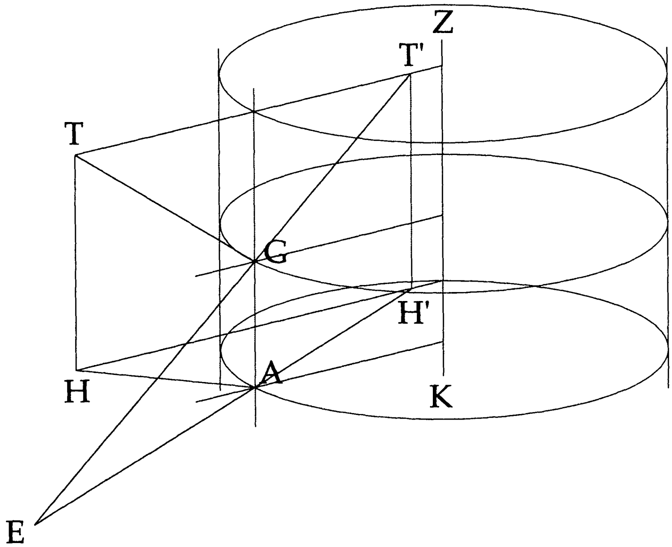

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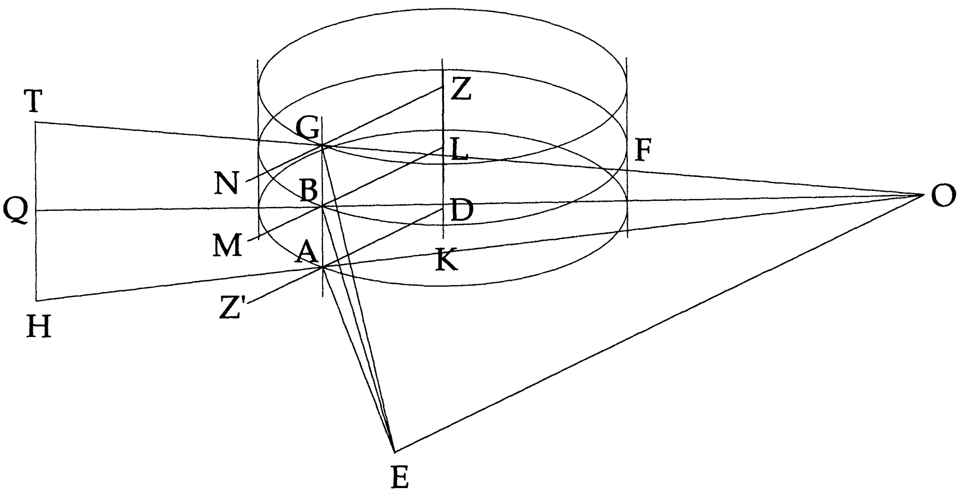

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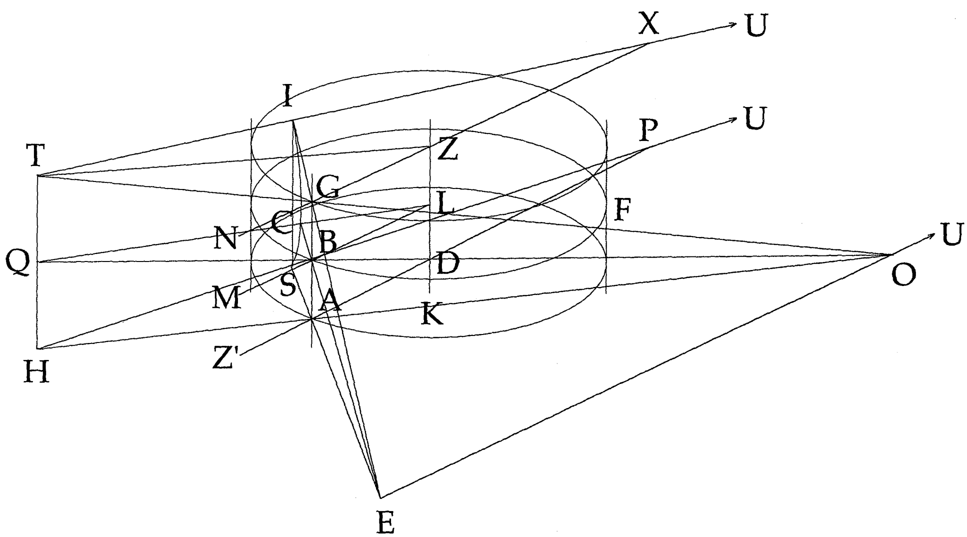

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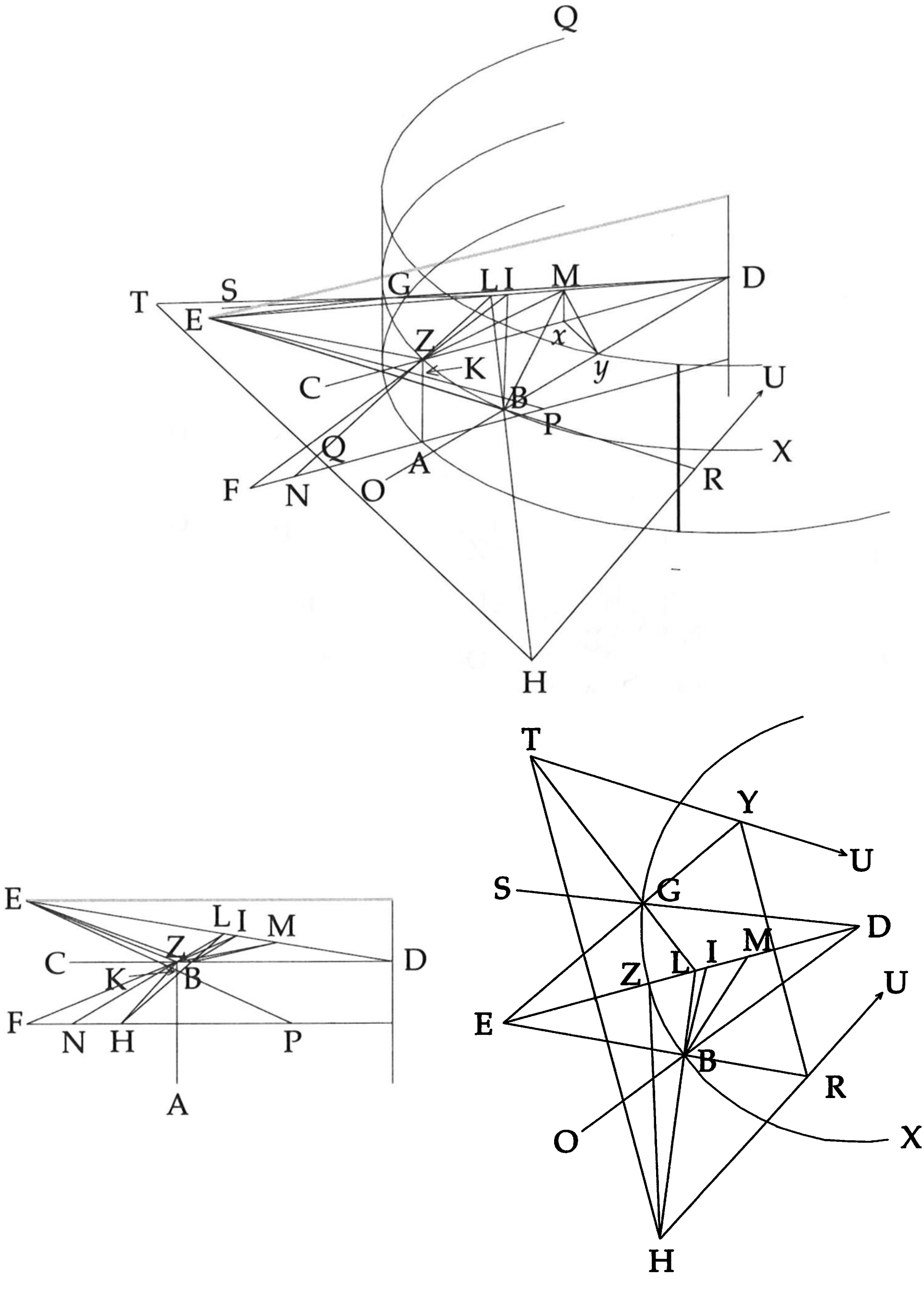
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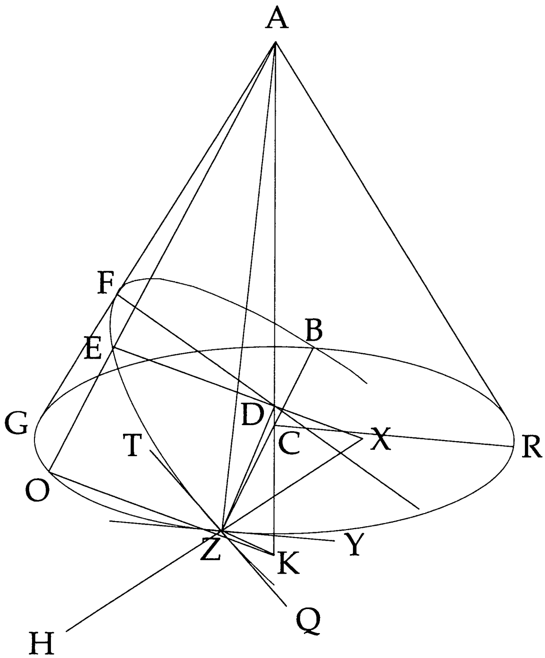
figure 6.5.19



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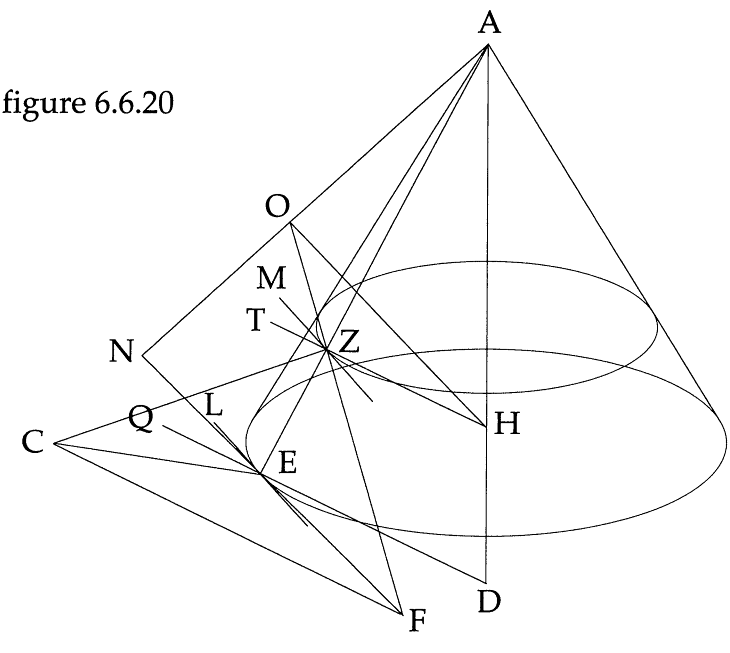

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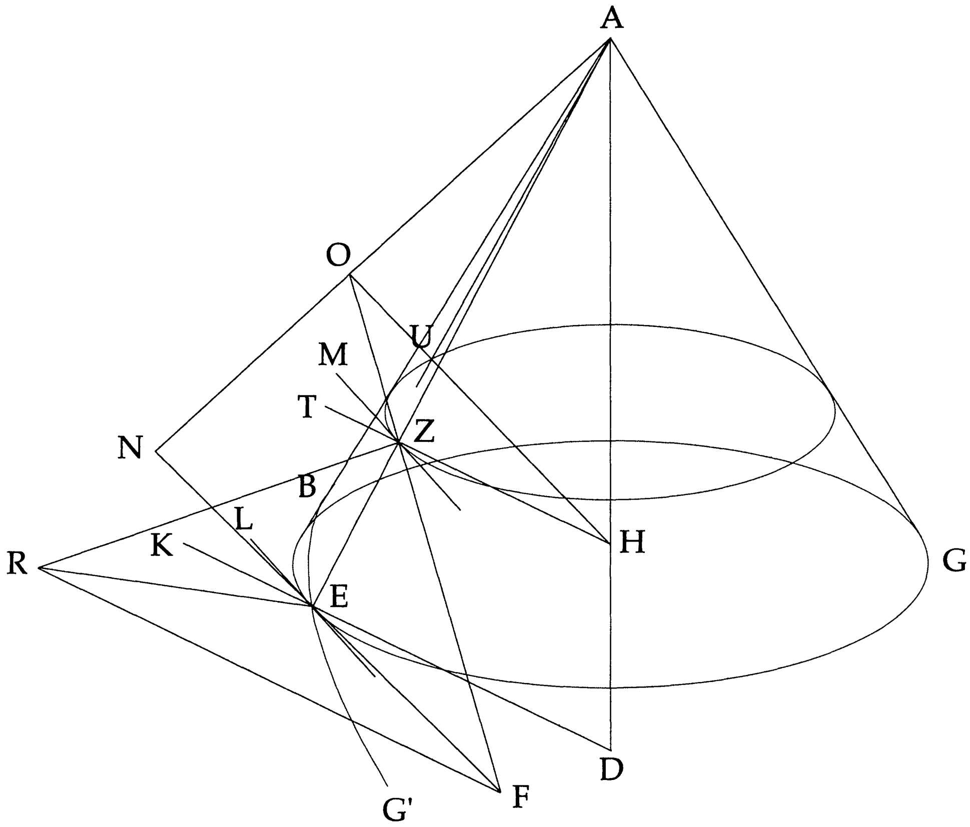

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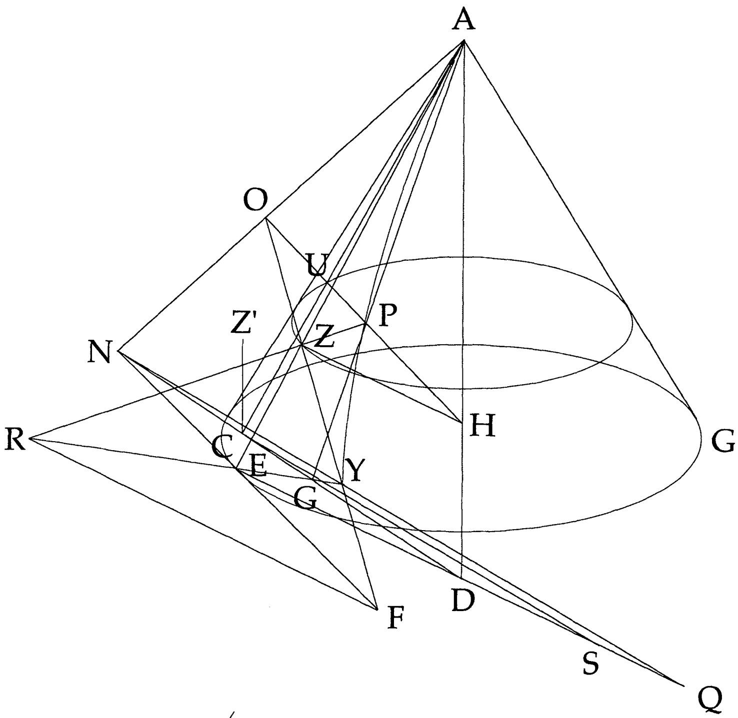

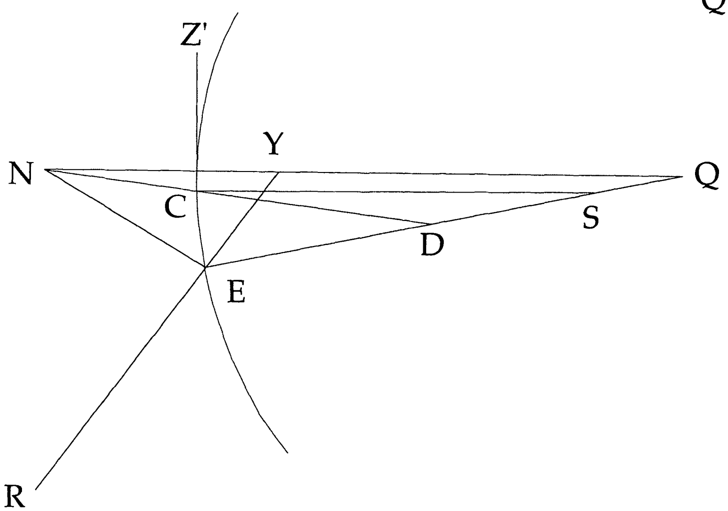

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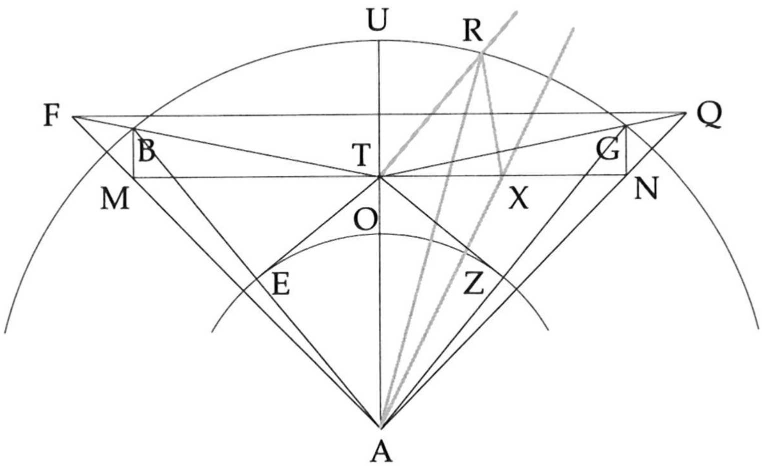

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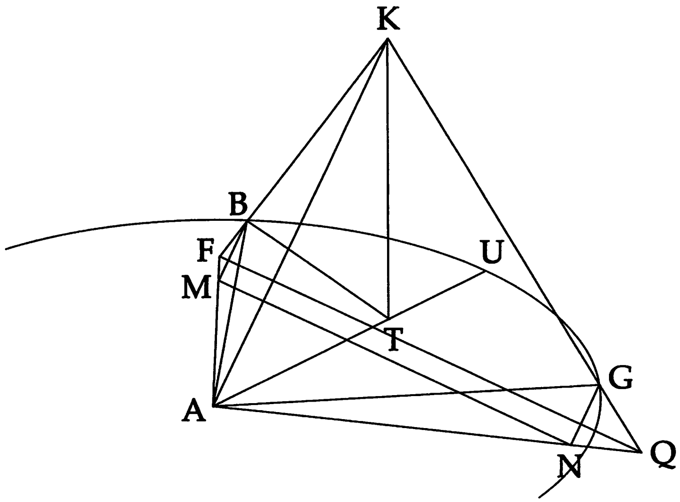

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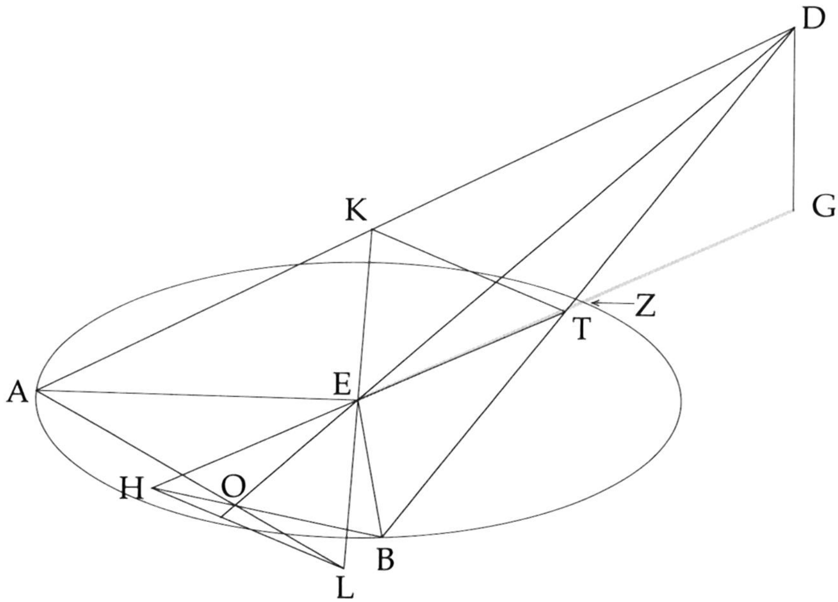

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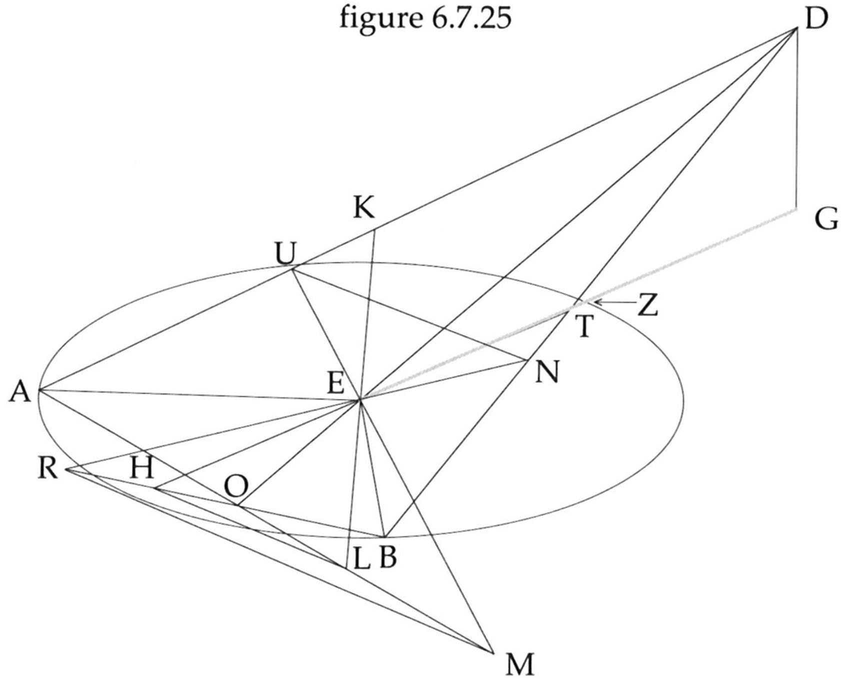

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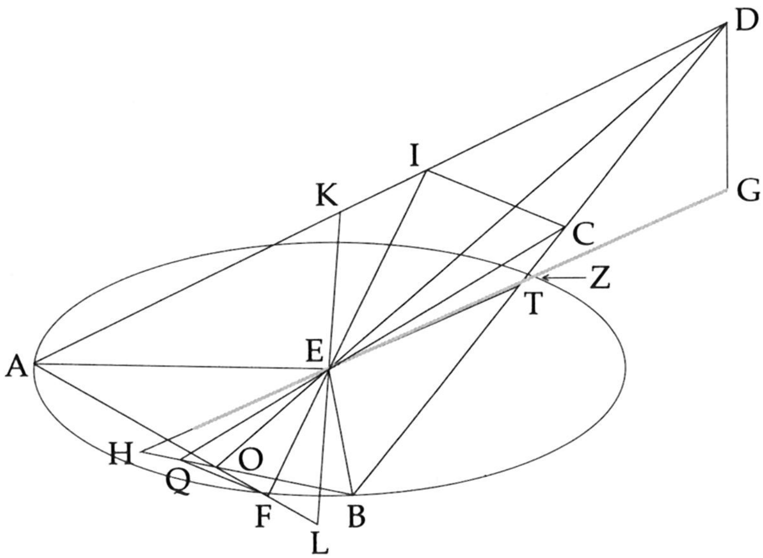

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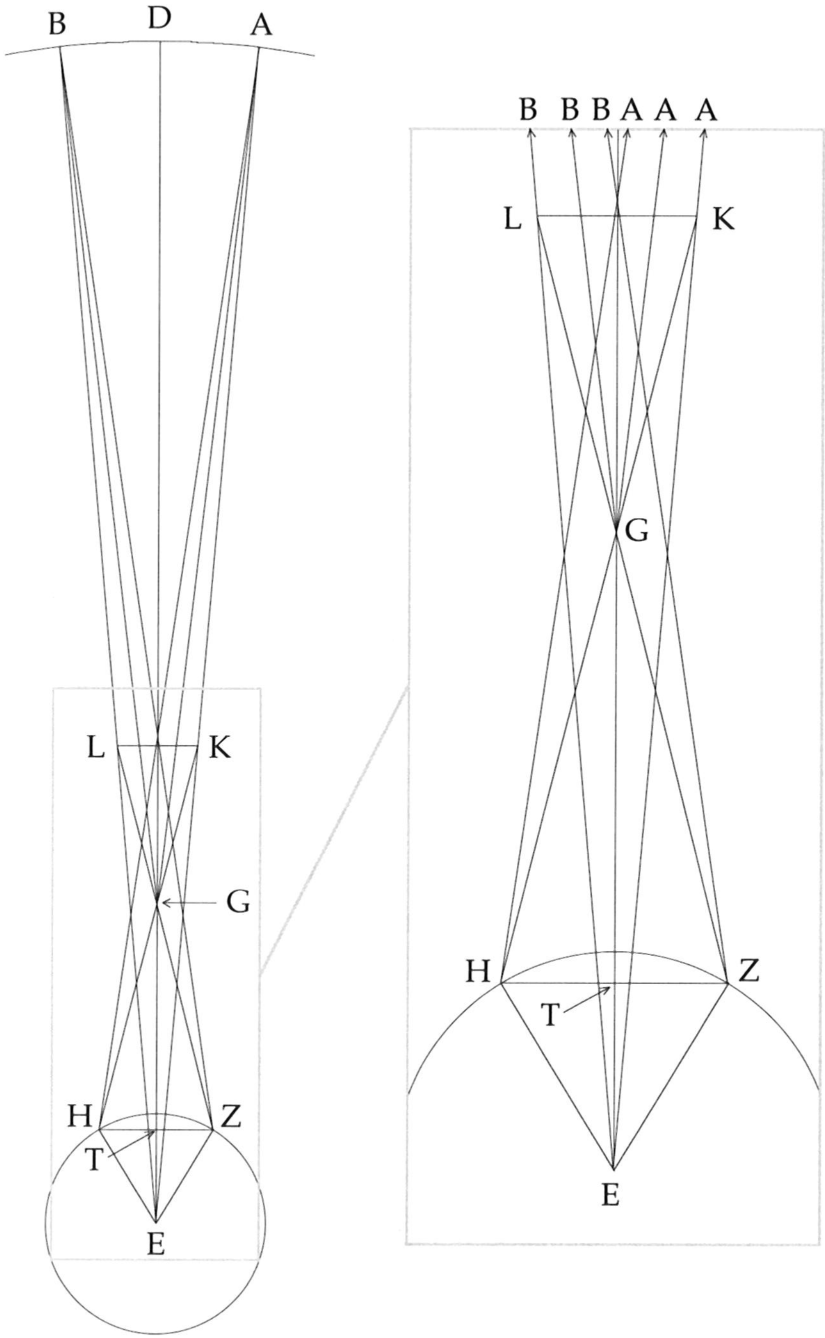

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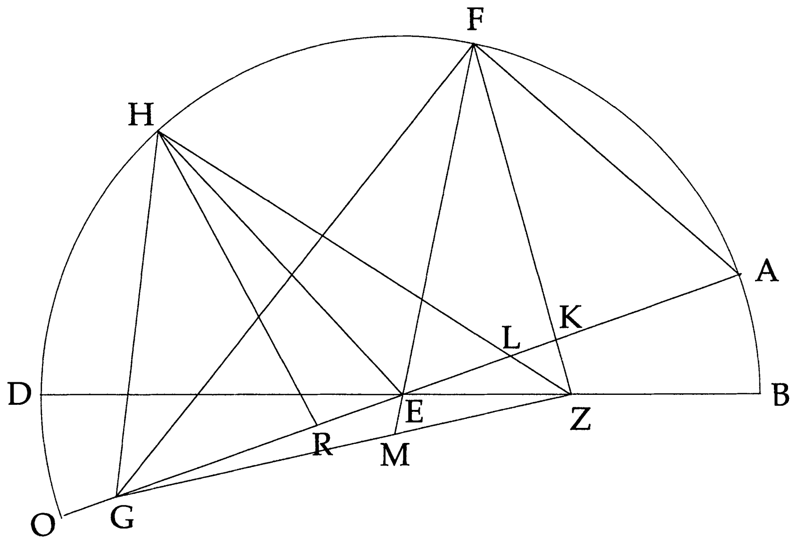

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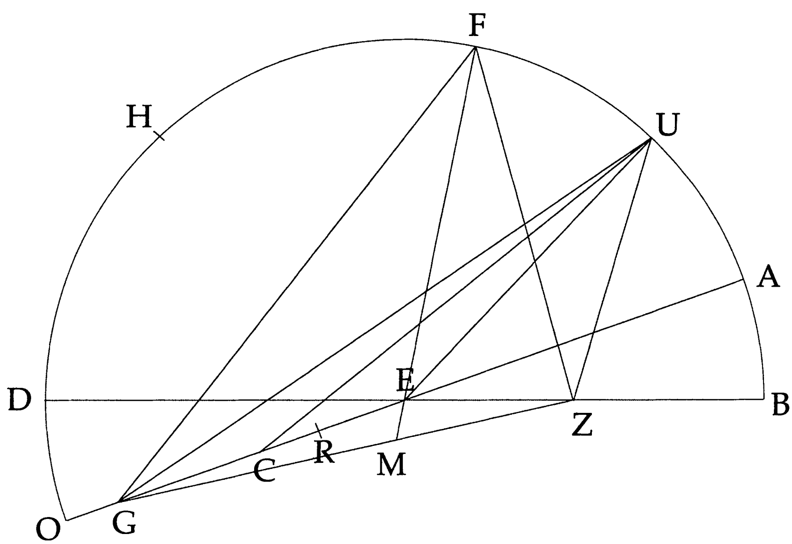

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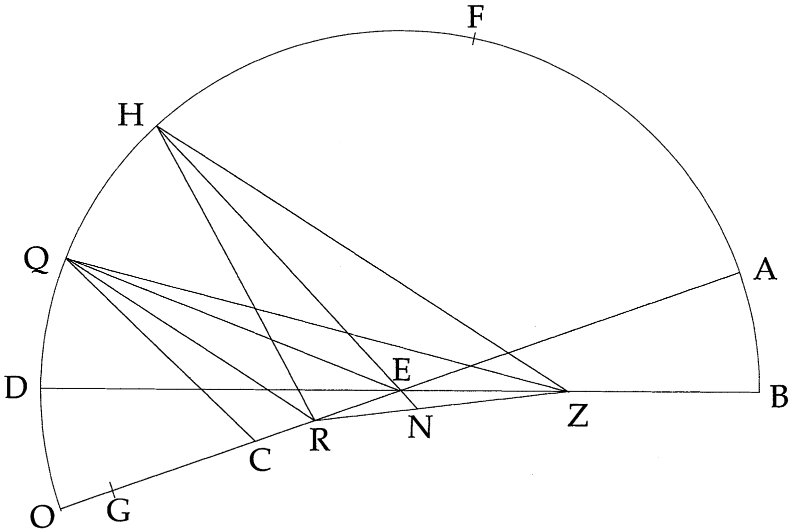

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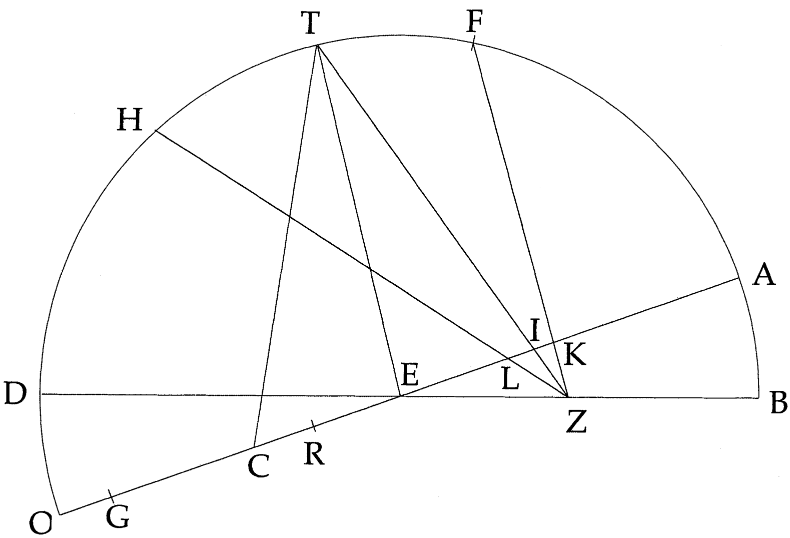

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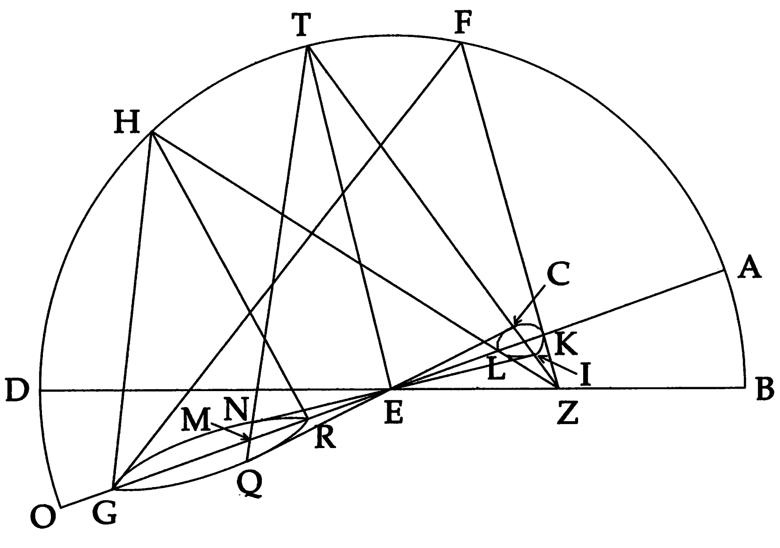

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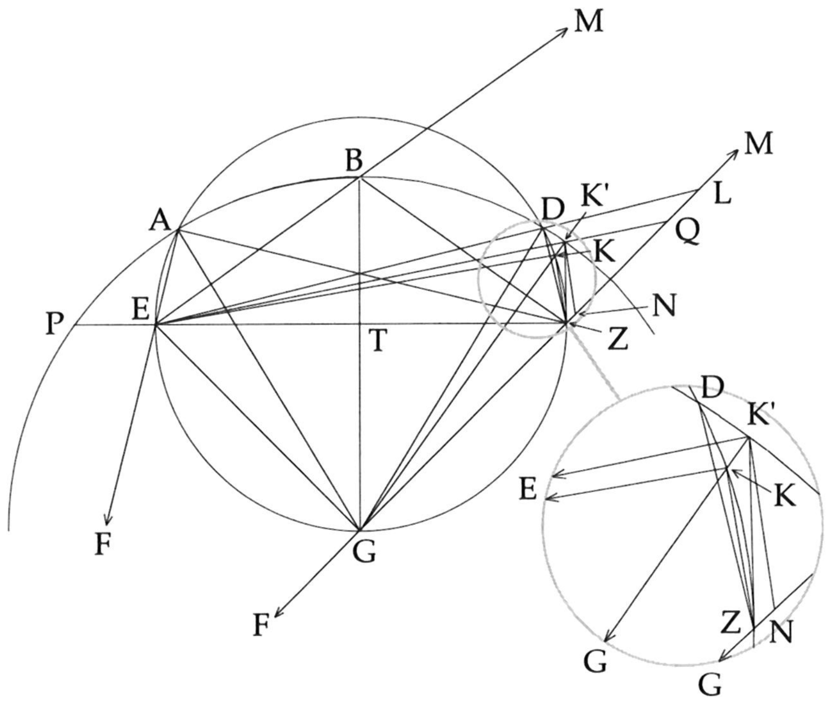

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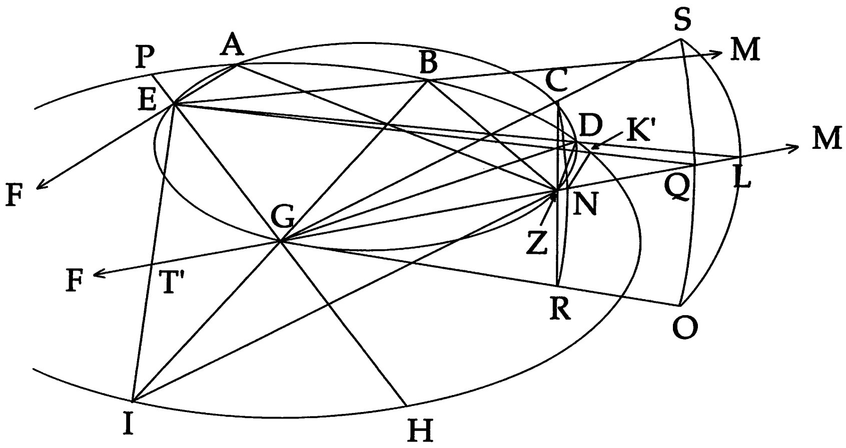

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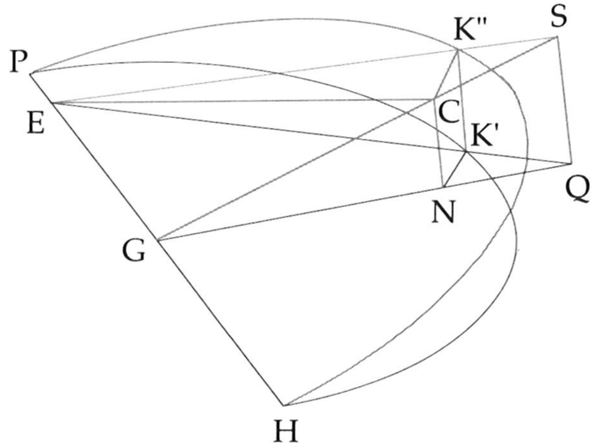

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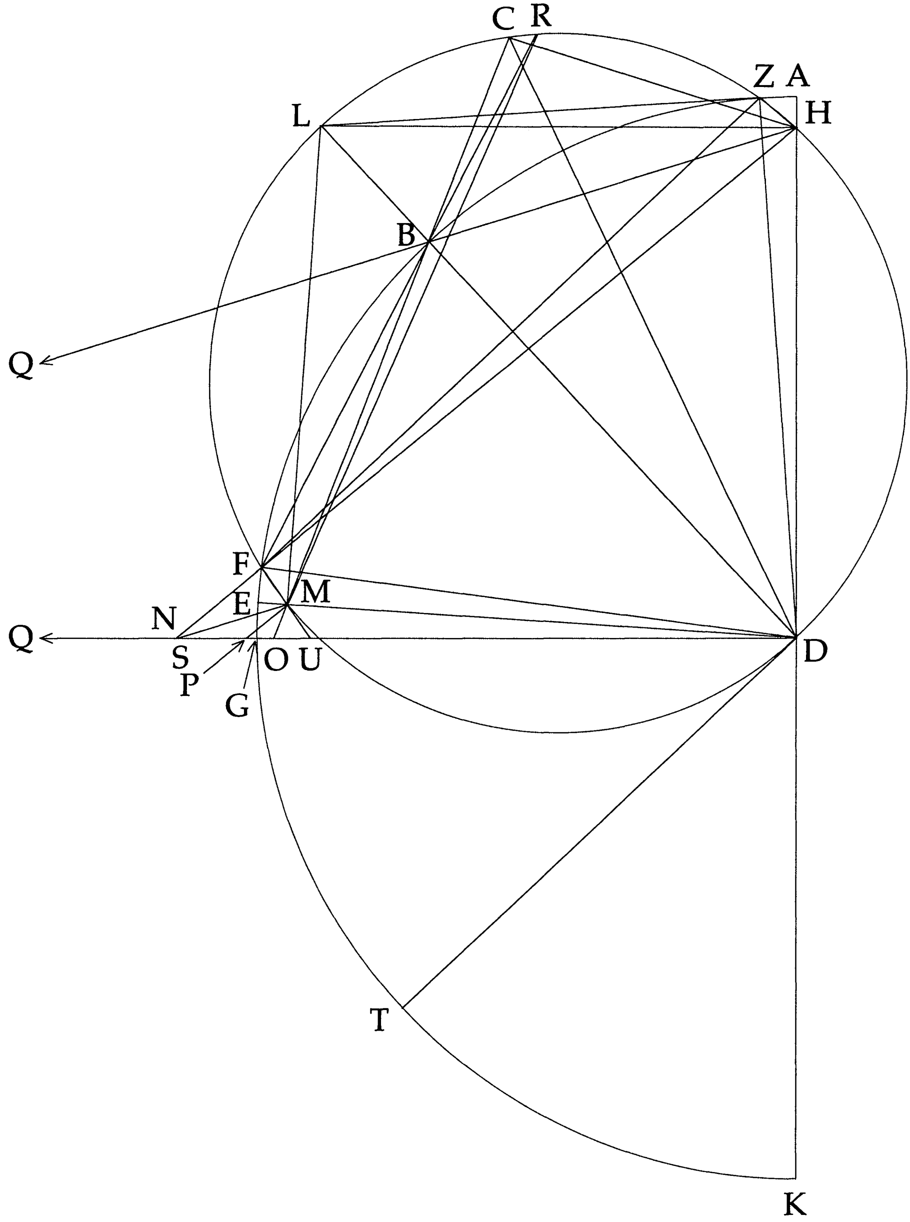

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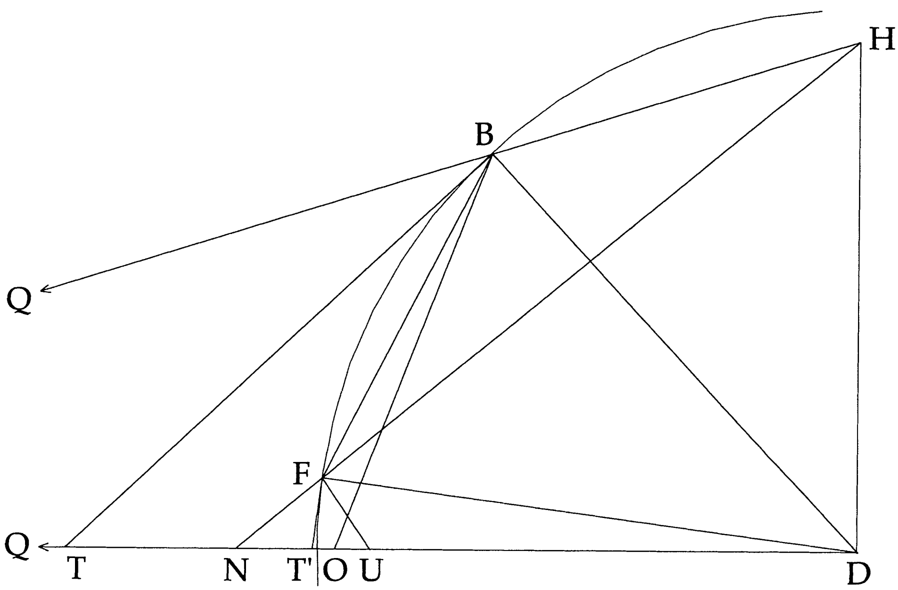

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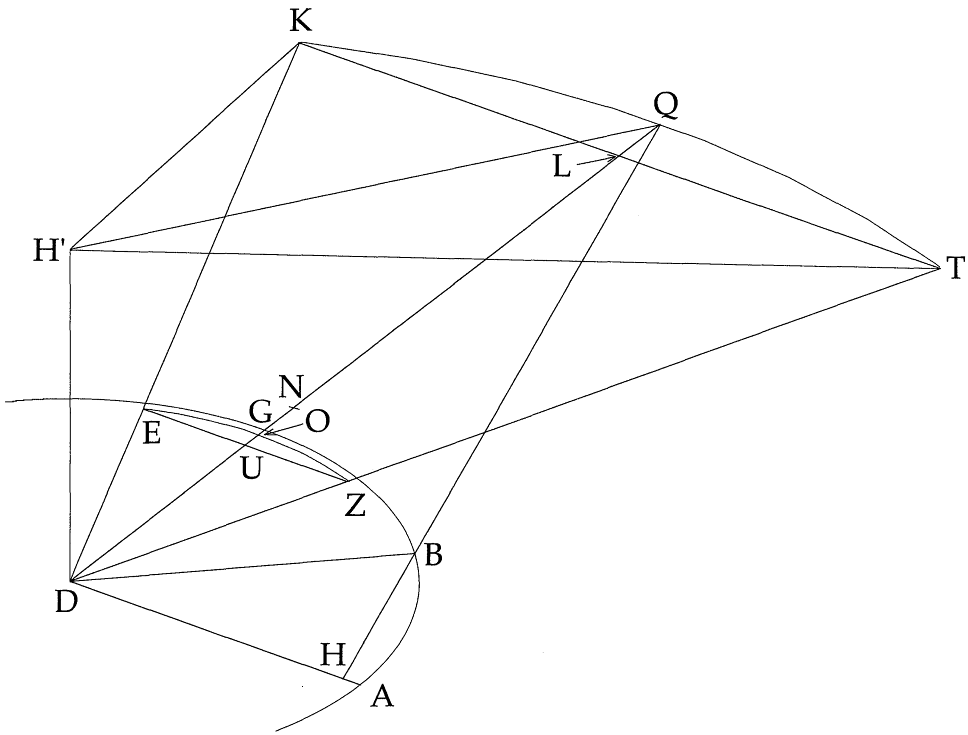

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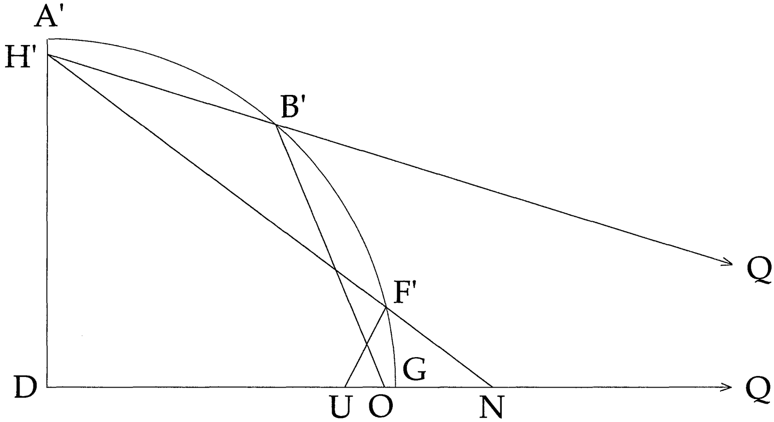

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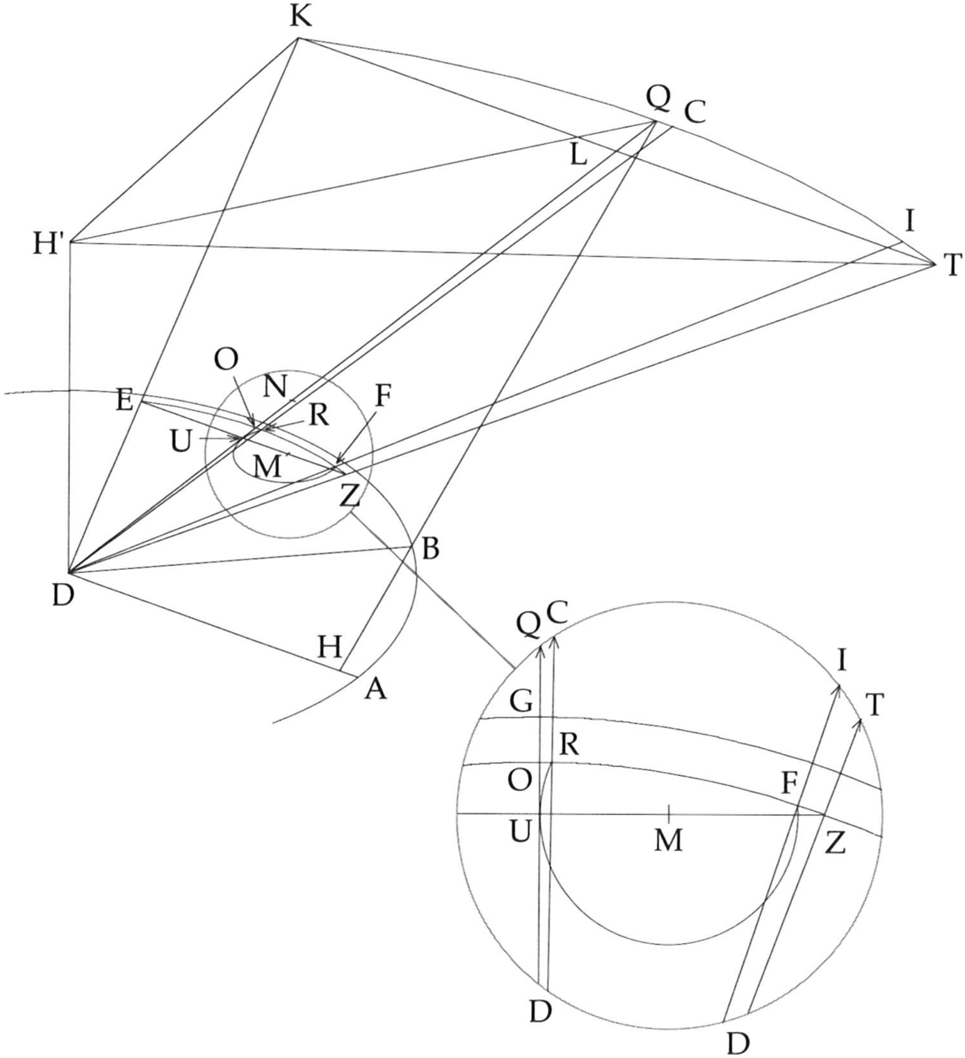

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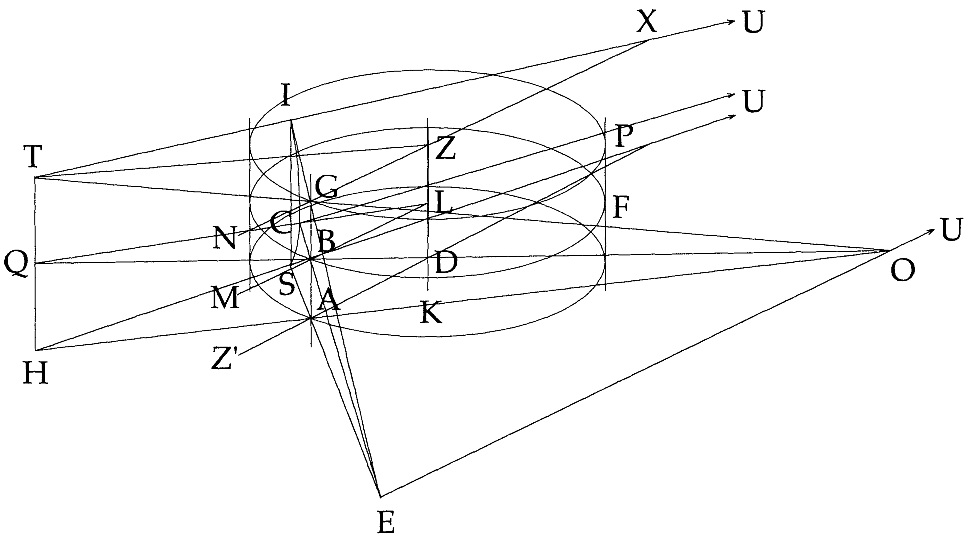

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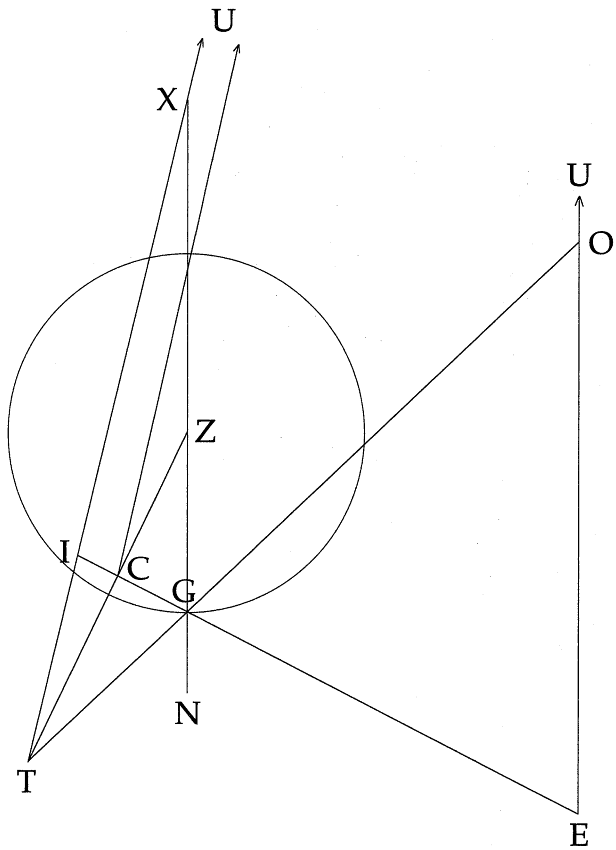

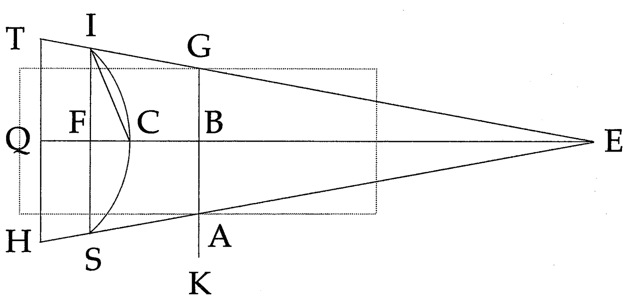

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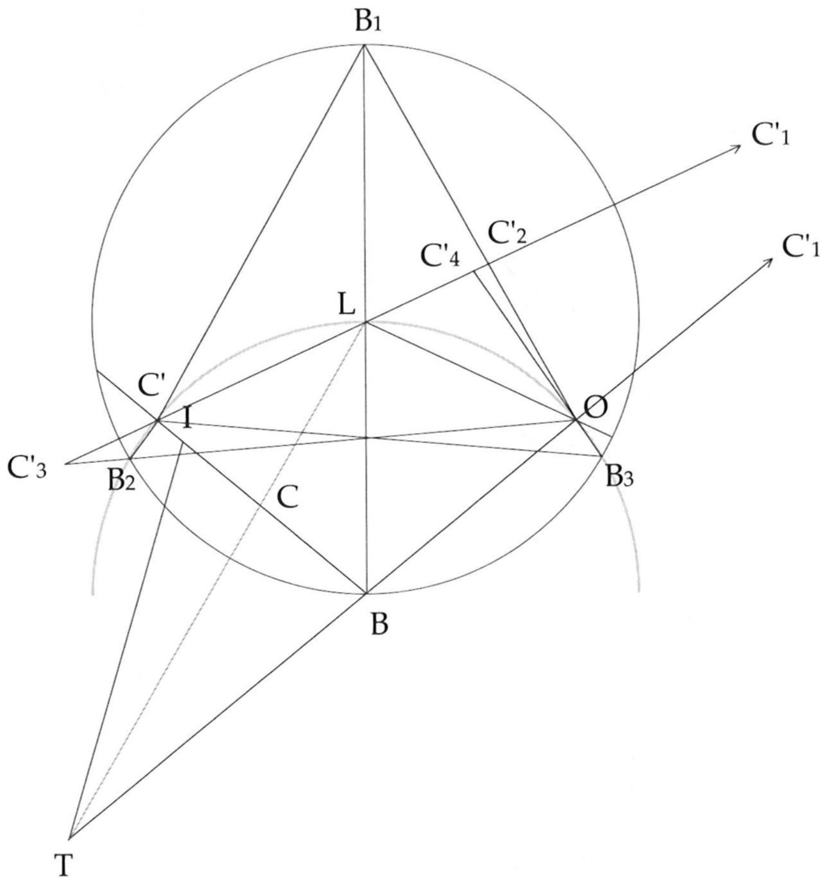

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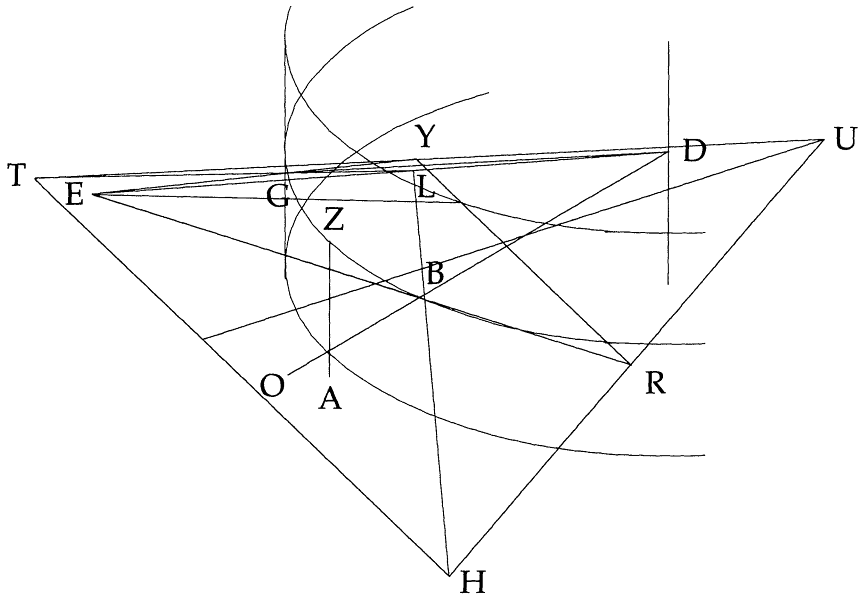

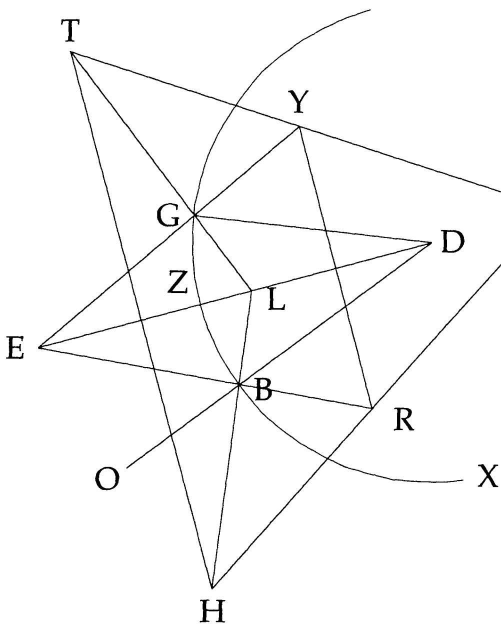

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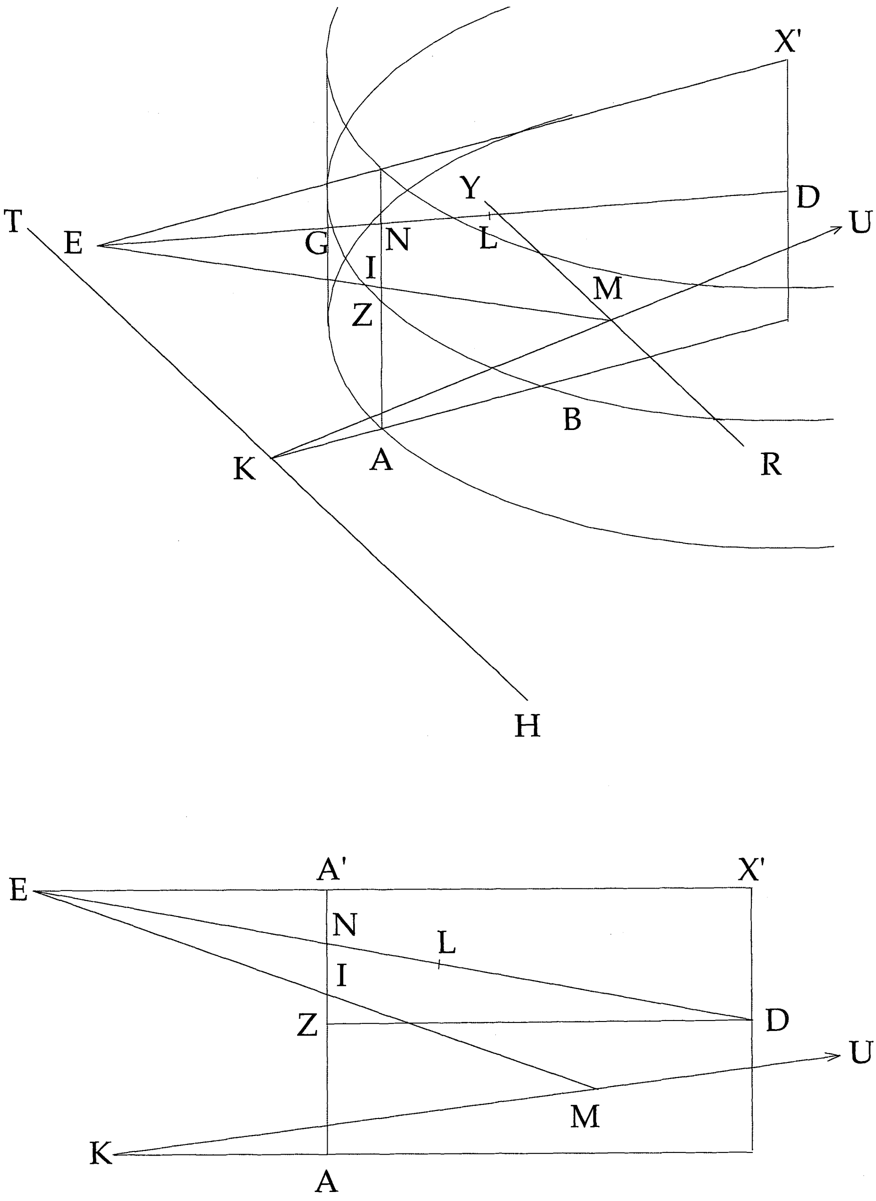

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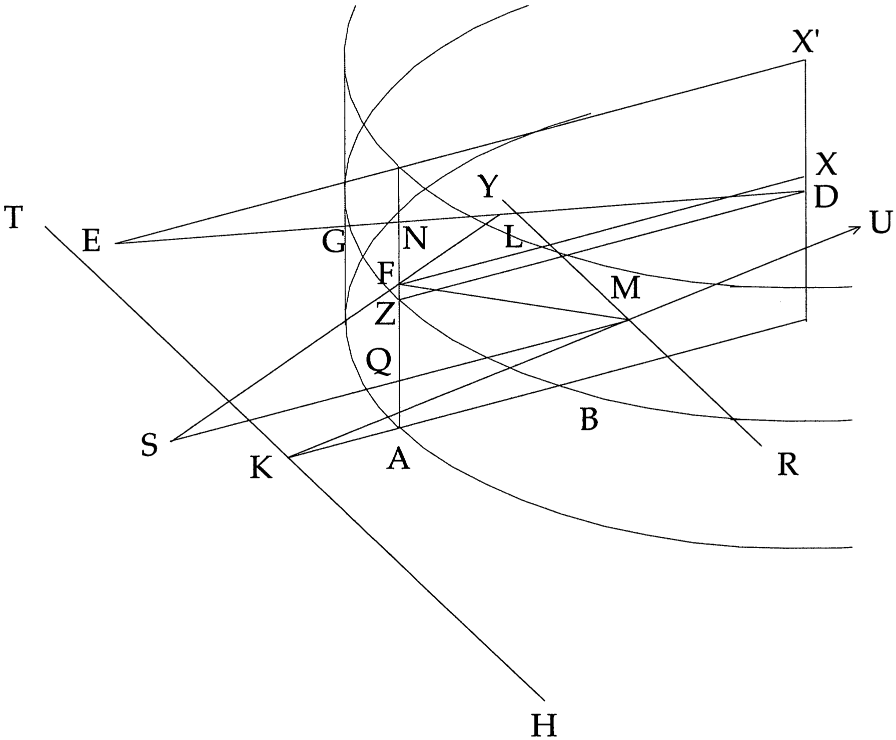

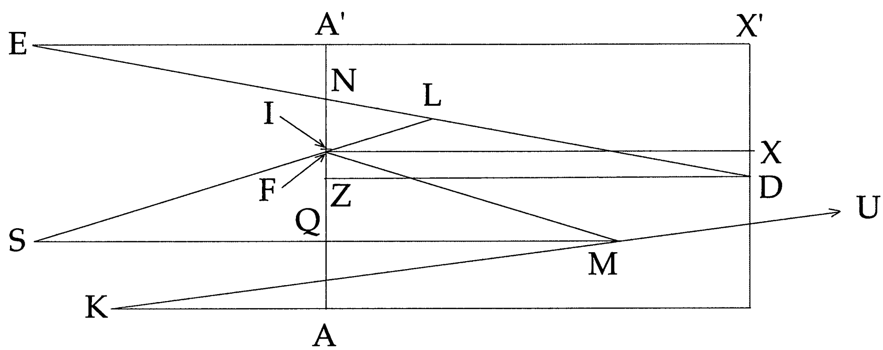

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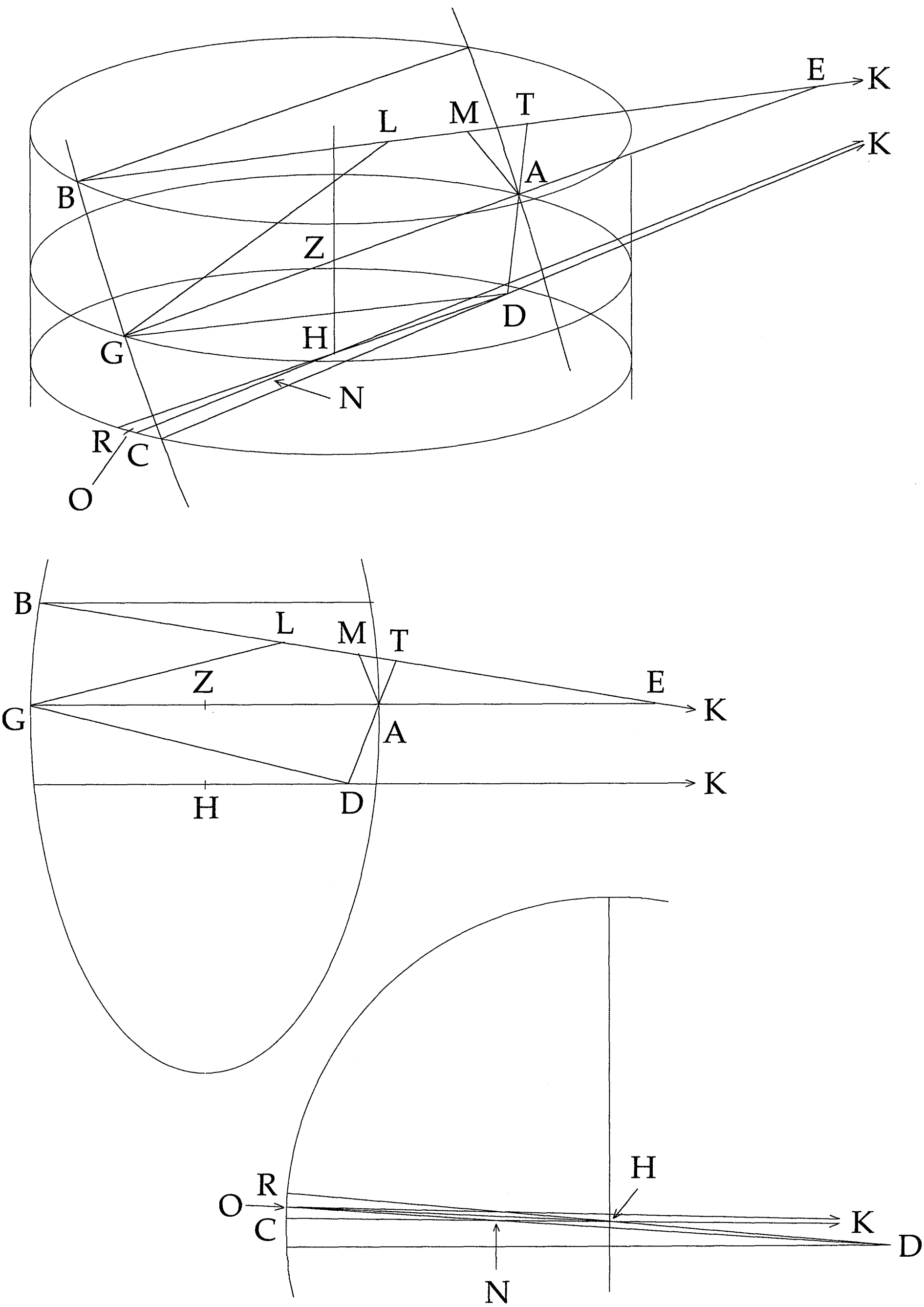

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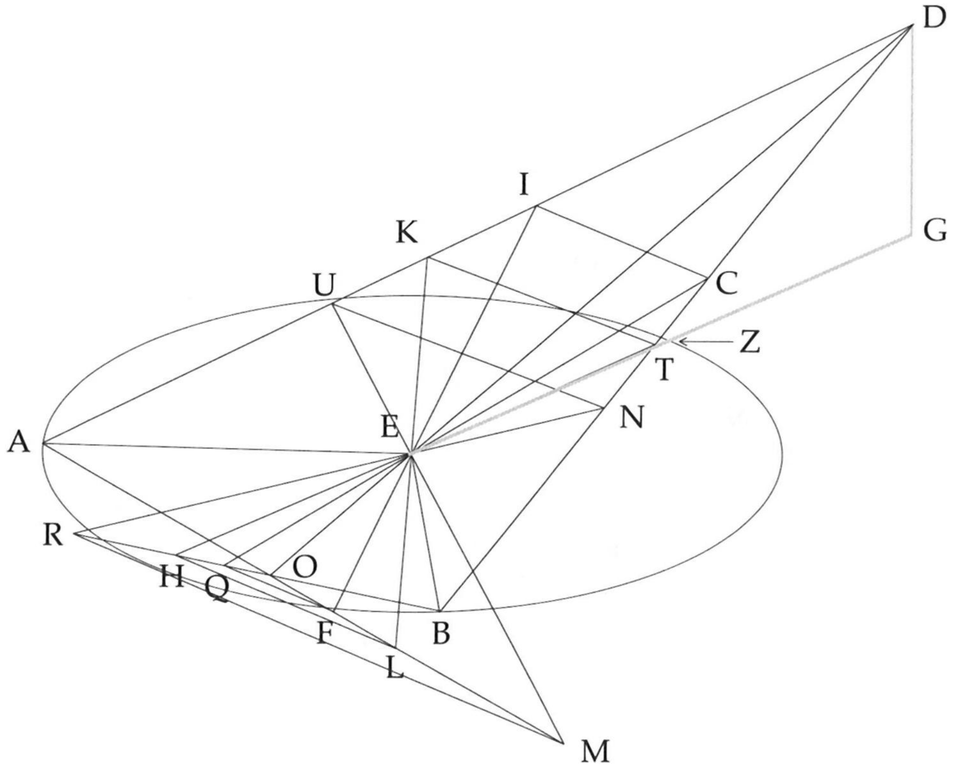

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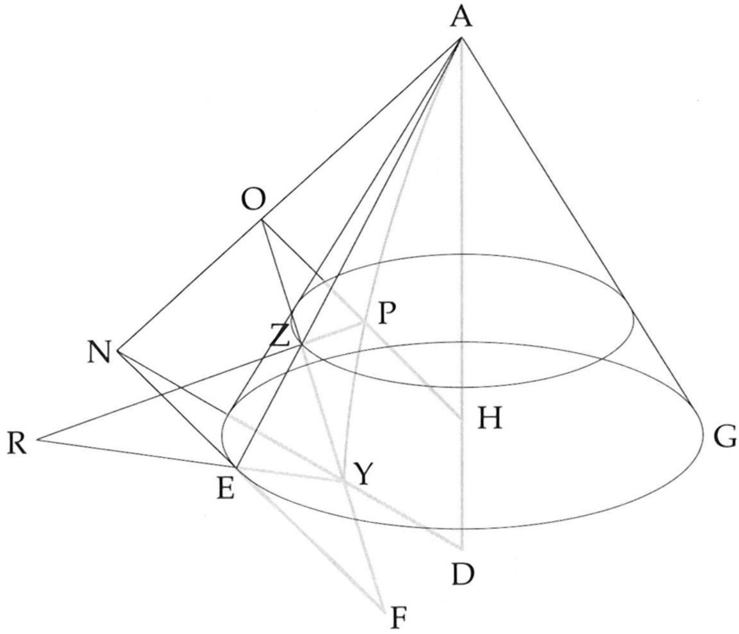

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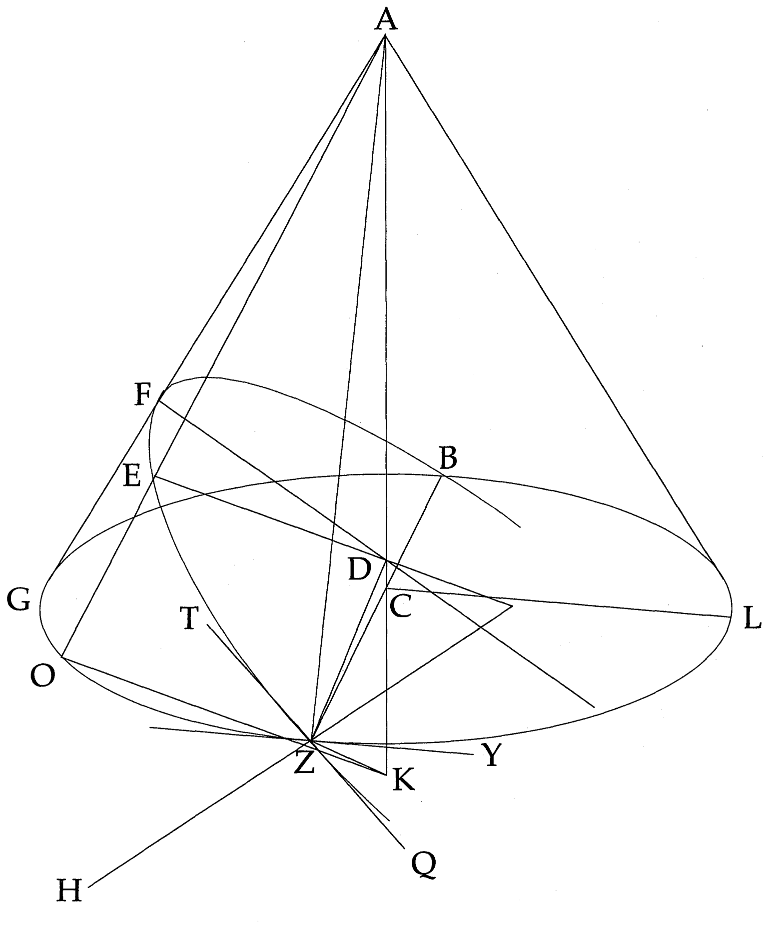

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